The Mind and Brain III :Audition, Language, Communication
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Neuromagnetic Source Reconstruction and Inverse Modeling

Part I: Introduction to adaptive spatial filter techniques

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This talk:

•formulates the neuromagnetic source reconstruction problem using spatial filters.

•introduces non-adaptive and adaptive spatial filter techniques.

•focuses on the adaptive spatial filter technique (adaptive beamformer).

Magnetoencephalography
(Neuromagnetic measurements)

•can provide a high temporal resolution.

•<u>cannot provide (adequate) information on the source spatial configuration.</u>

$\Downarrow$

Efficient numerical algorithms for estimating source configuration are need to be developed.

(Source localization problems)

Source localization problem

•Dipole modeling approach

•Image reconstruction approach

Tomographic reconstruction

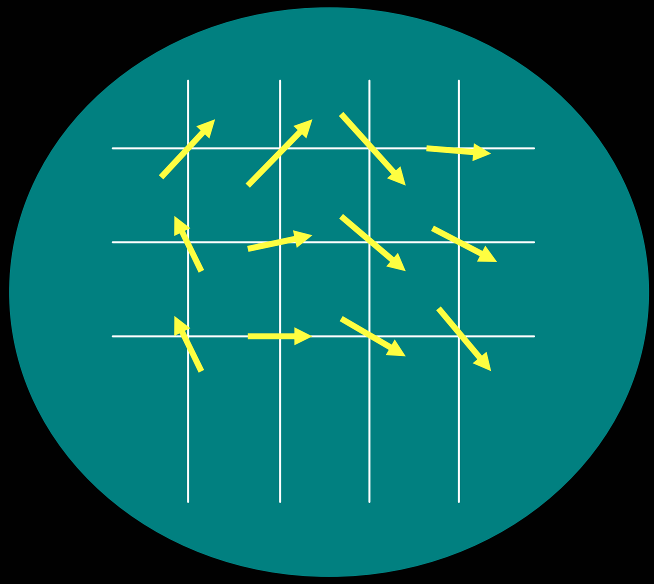
Spatial filter

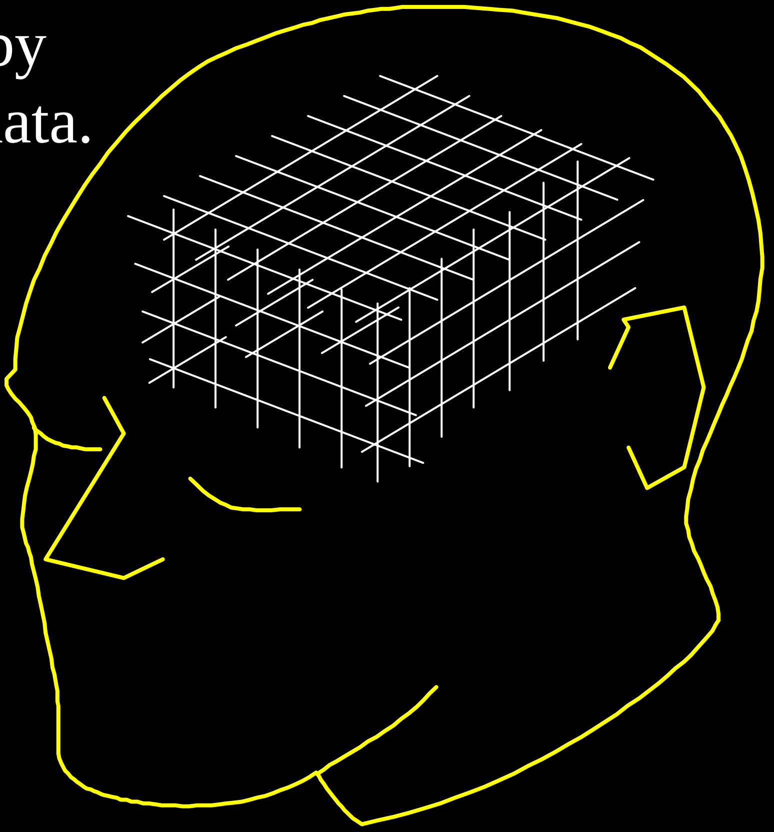
Tomographic reconstruction

- Assume pixel grids in the region of interest.

- Assume a source at each grid.

- Estimate the moment of each source by least-squares fitting to the measured data.

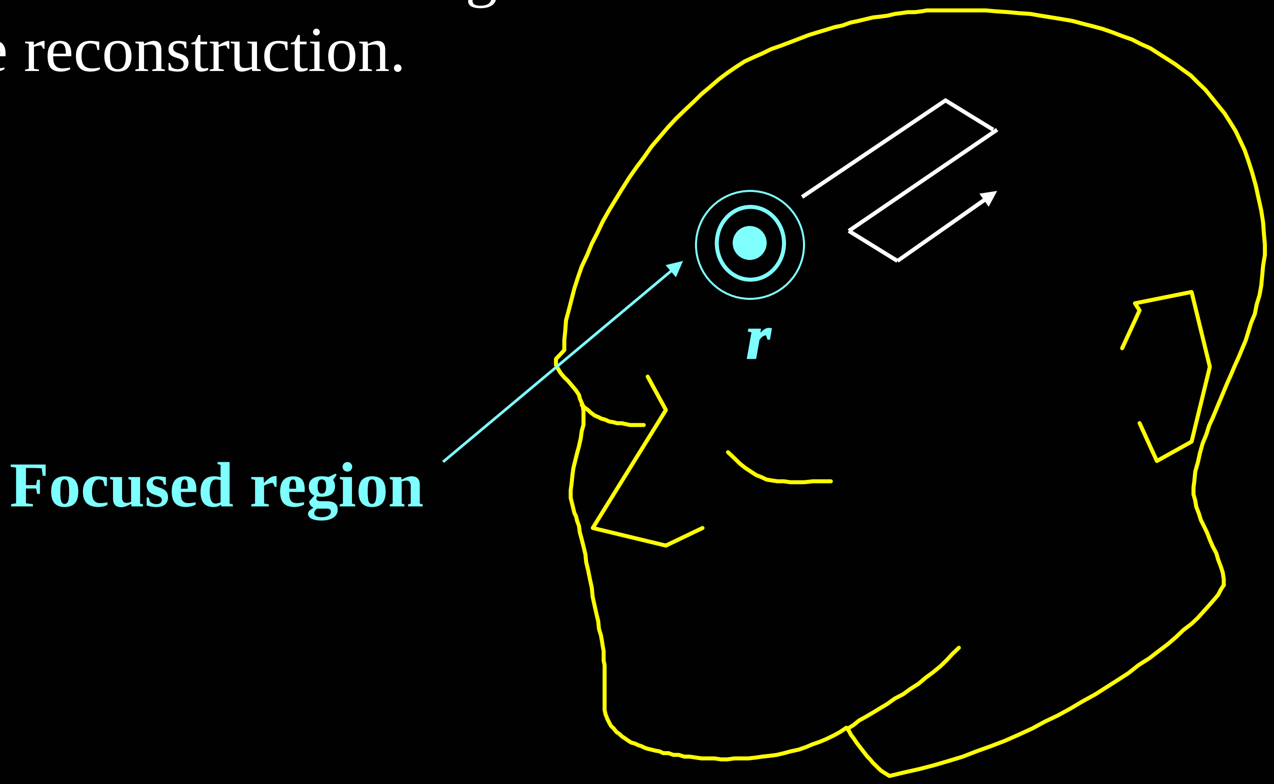




Spatial filter technique

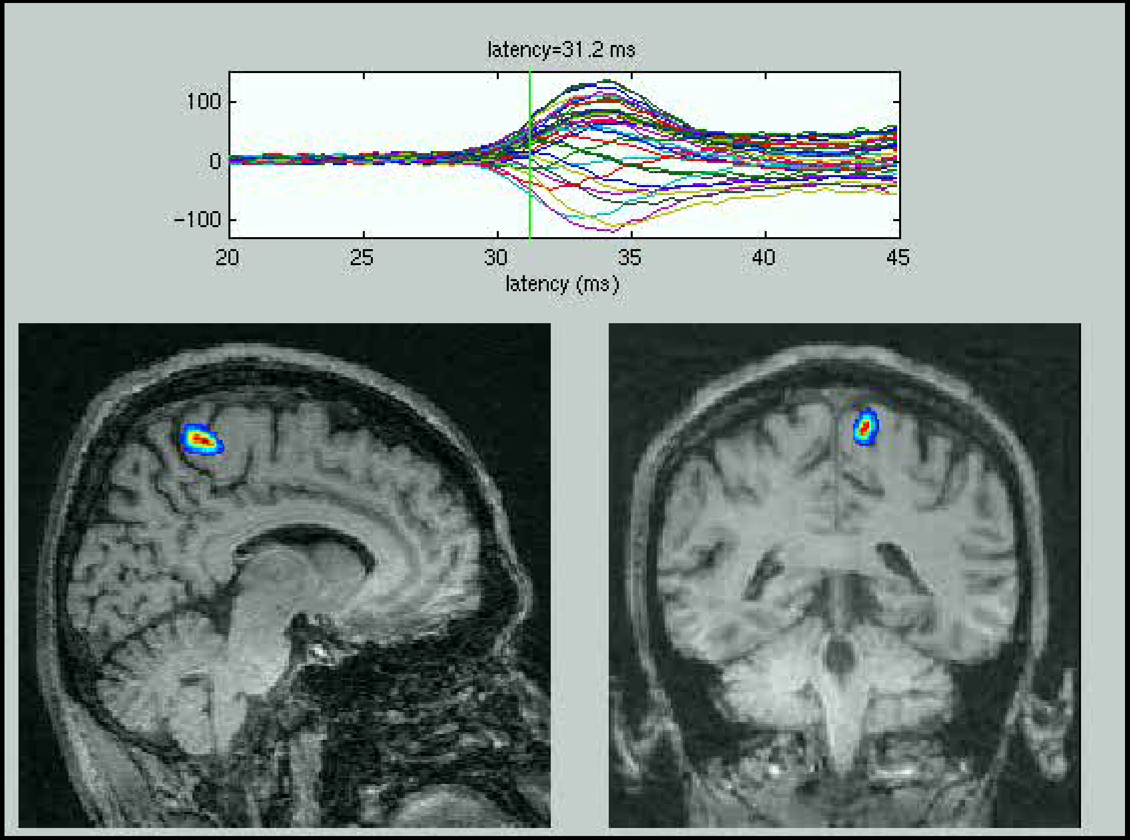
•Form spatial filter weight $\boldsymbol{w}(\boldsymbol{r})$ that focuses the sensitivity of the sensor array at a small area at $\boldsymbol{r}$.

•Scan this focused area over the region of interest to obtain source reconstruction.

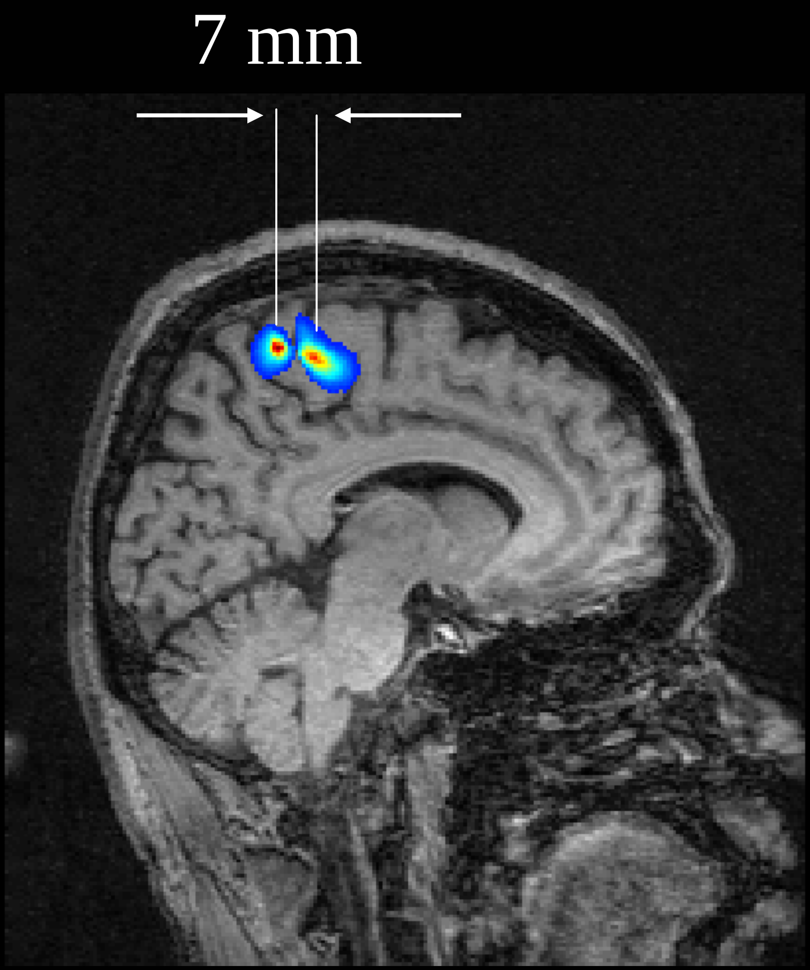


Right posterior tibial nerve stimulation

measured by a 37-channel sensor array

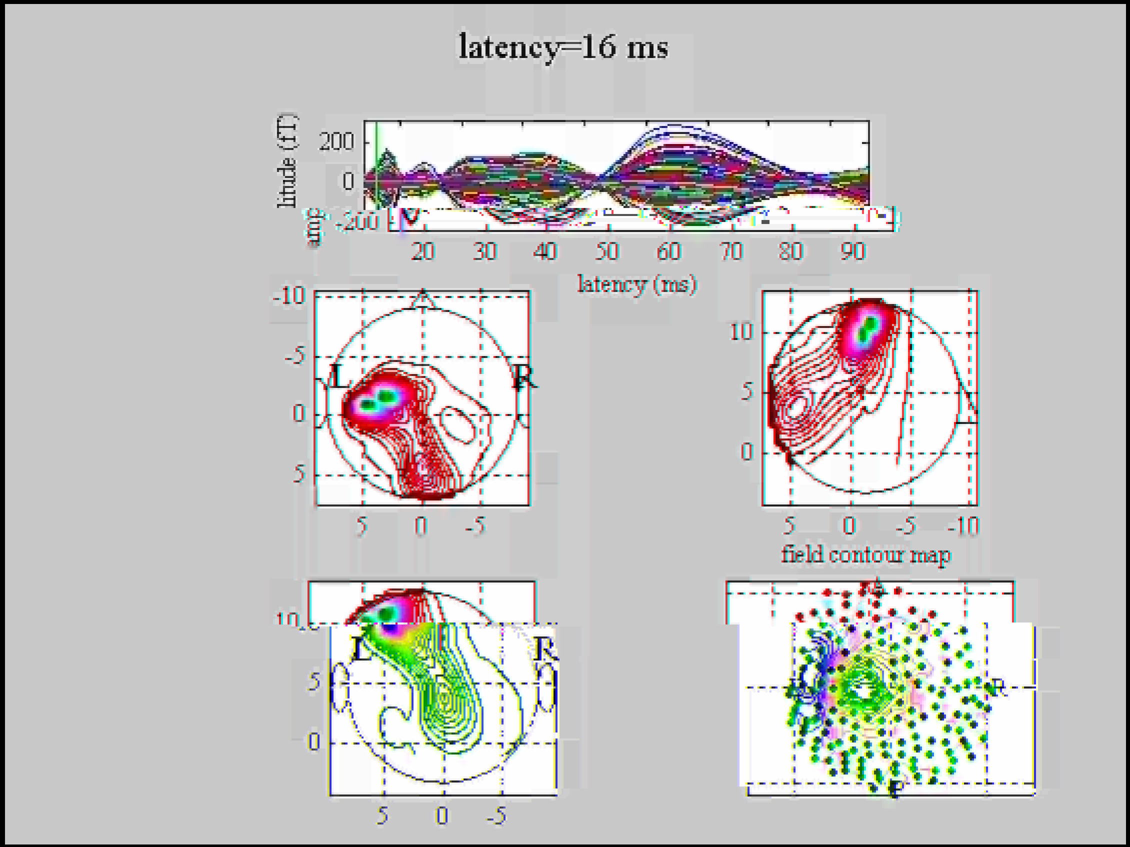


Hashimoto et al., NeuroReport 2001

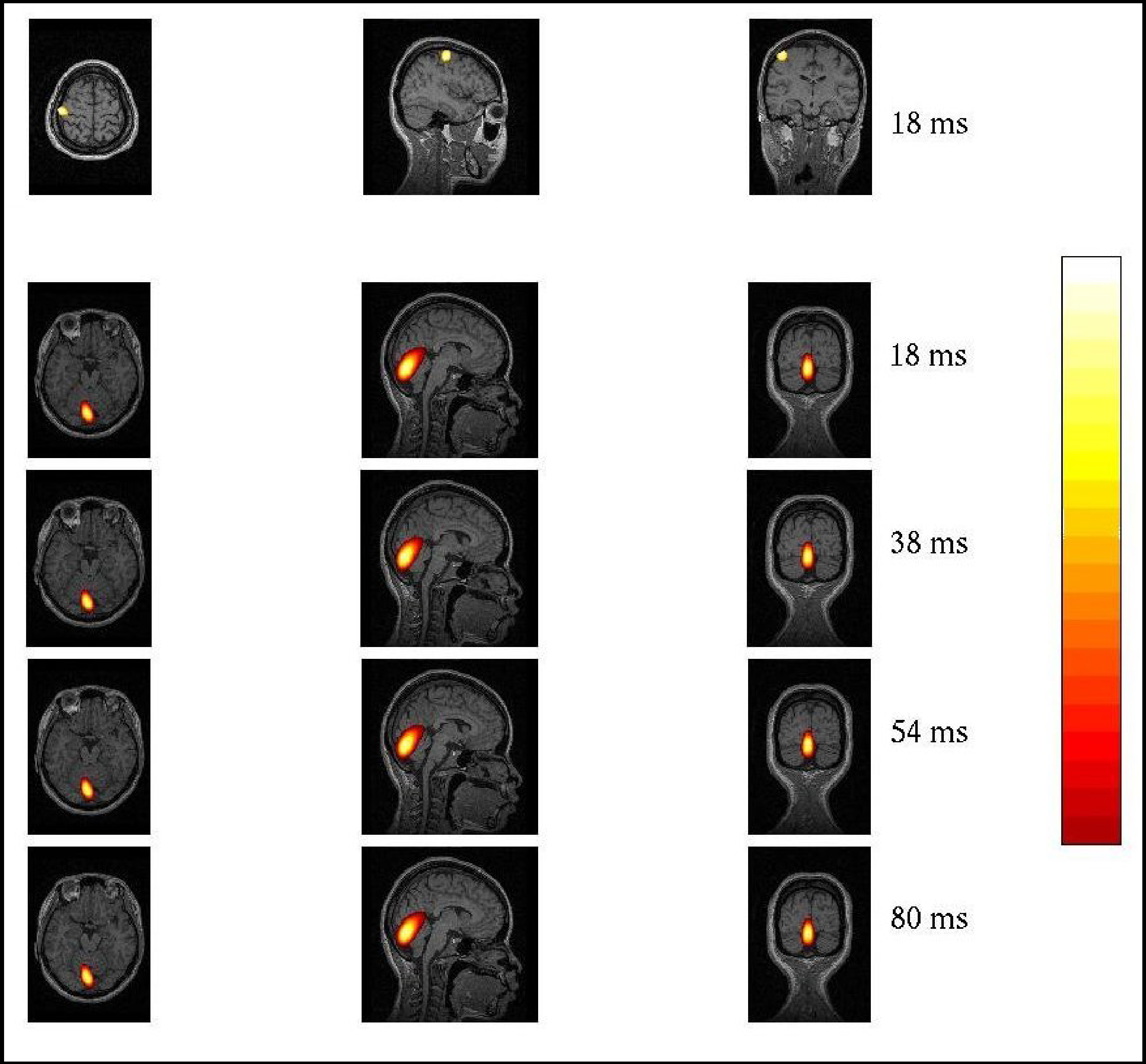

7 mm

Right median nerve stimulation

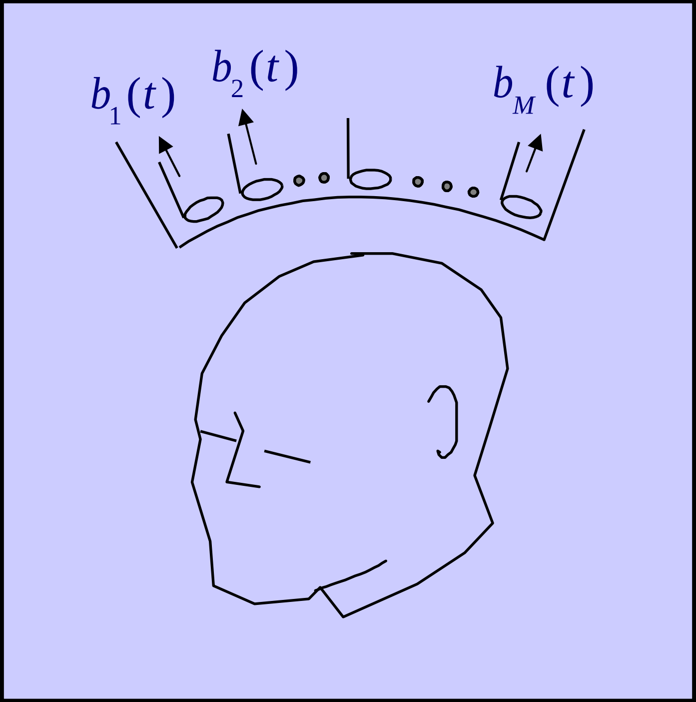
measured by a 160-channel whole-head sensor array



Hashimoto et al., J. Clinical Neurophysiology submitted for publication


18 ms
18 ms
38 ms
54 ms
80 ms

Definitions

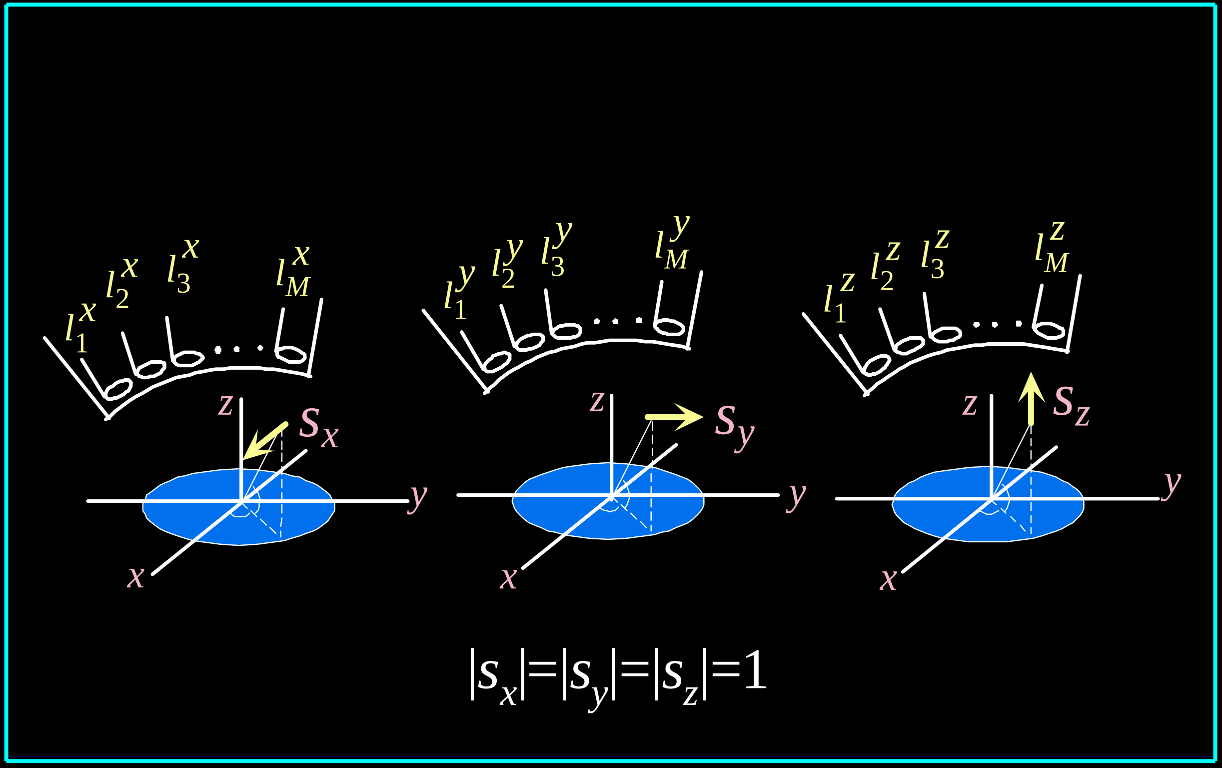


- data vector: $\boldsymbol{b}(\mathrm{t}) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$

- data covariance matrix: $\boldsymbol{R} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

- source magnitude: $s(\boldsymbol{r},t)$

- source orientation: $\boldsymbol{\eta}(\boldsymbol{r},t) = [\eta_X(\boldsymbol{r},t), \eta_y(\boldsymbol{r},t), \eta_Z(\boldsymbol{r},t)]^T$



Lead field vector for the source orientation $\boldsymbol{\eta}(\boldsymbol{r})$

$$\boldsymbol{L}(\boldsymbol{r}) = \begin{bmatrix} l_1^x(\boldsymbol{r}) & l_1^y(\boldsymbol{r}) & l_1^z(\boldsymbol{r}) \\ l_2^x(\boldsymbol{r}) & l_2^y(\boldsymbol{r}) & l_2^z(\boldsymbol{r}) \\ \vdots & \vdots & \vdots \\ l_M^x(\boldsymbol{r}) & l_M^y(\boldsymbol{r}) & l_M^z(\boldsymbol{r}) \end{bmatrix}, \quad \boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{L}(\boldsymbol{r}) \begin{bmatrix} \eta_x(\boldsymbol{r}) \\ \eta_y(\boldsymbol{r}) \\ \eta_z(\boldsymbol{r}) \end{bmatrix}$$

Basic relationship

$$b_j(t) = \int \boldsymbol{l}_j(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r},t)d\boldsymbol{r}$$

or

$$\boldsymbol{b}(t) = \int \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r},t)d\boldsymbol{r}$$

Problem of source localization:

Estimate $\boldsymbol{s}(\boldsymbol{r},t)$ from the measurement $\boldsymbol{b}(t)$

Spatial filter

$$\hat{\boldsymbol{s}}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = [w_1(\boldsymbol{r}),\ldots,w_M(\boldsymbol{r})]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t)$$

$\Uparrow$ estimate of $\boldsymbol{s}(\boldsymbol{r},t)$

$\Uparrow$ weight vector

How to evaluate an appropriateness of the weight ?

$$\left.\begin{aligned} \boldsymbol{b} &= \int \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r})d\boldsymbol{r} \\ \hat{\boldsymbol{s}}(\boldsymbol{r}) &= \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b} \end{aligned}\right\} \rightarrow \hat{\boldsymbol{s}}(\boldsymbol{r}) = \int \underbrace{\boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{L}(\boldsymbol{r}')}_{\mathbb{R}(\boldsymbol{r},\boldsymbol{r}')}\boldsymbol{s}(\boldsymbol{r}')d\boldsymbol{r}'$$

$\Uparrow$

Resolution kernel

$$\boxed{\hat{\boldsymbol{s}}(\boldsymbol{r}) = \int \mathbb{R}(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{s}(\boldsymbol{r}')d\boldsymbol{r}'}$$

(neglecting the explicit time notation)

Non-adaptive weight

$\boldsymbol{w}(\boldsymbol{r})$ is data independent

Adaptive weight

$\boldsymbol{w}(\boldsymbol{r})$ is data dependent

Data-independent (non-adaptive) weight

minimum-norm estimate (Hamalainen and Ilmoniemi)

The weight $\boldsymbol{w}(\boldsymbol{r})$ is obtained by

$$\min \int \left[\mathbb{R}(\boldsymbol{r}, \boldsymbol{r}') - \delta(\boldsymbol{r} - \boldsymbol{r}') \right]^2 d\boldsymbol{r}'$$

$$\Downarrow$$

$$\boldsymbol{w}^T(\boldsymbol{r}) = \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{G}^{-1}, \text{ where } \underbrace{G_{i,j} = \int \boldsymbol{l}_i(\boldsymbol{r})\boldsymbol{l}_j^T(\boldsymbol{r})d\boldsymbol{r}}_{\text{Gram matrix}}$$

Inverse solution: $\hat{\boldsymbol{s}}(\boldsymbol{r}) = \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{G}^{-1}\boldsymbol{b}$

$\Uparrow$

This is erroneous

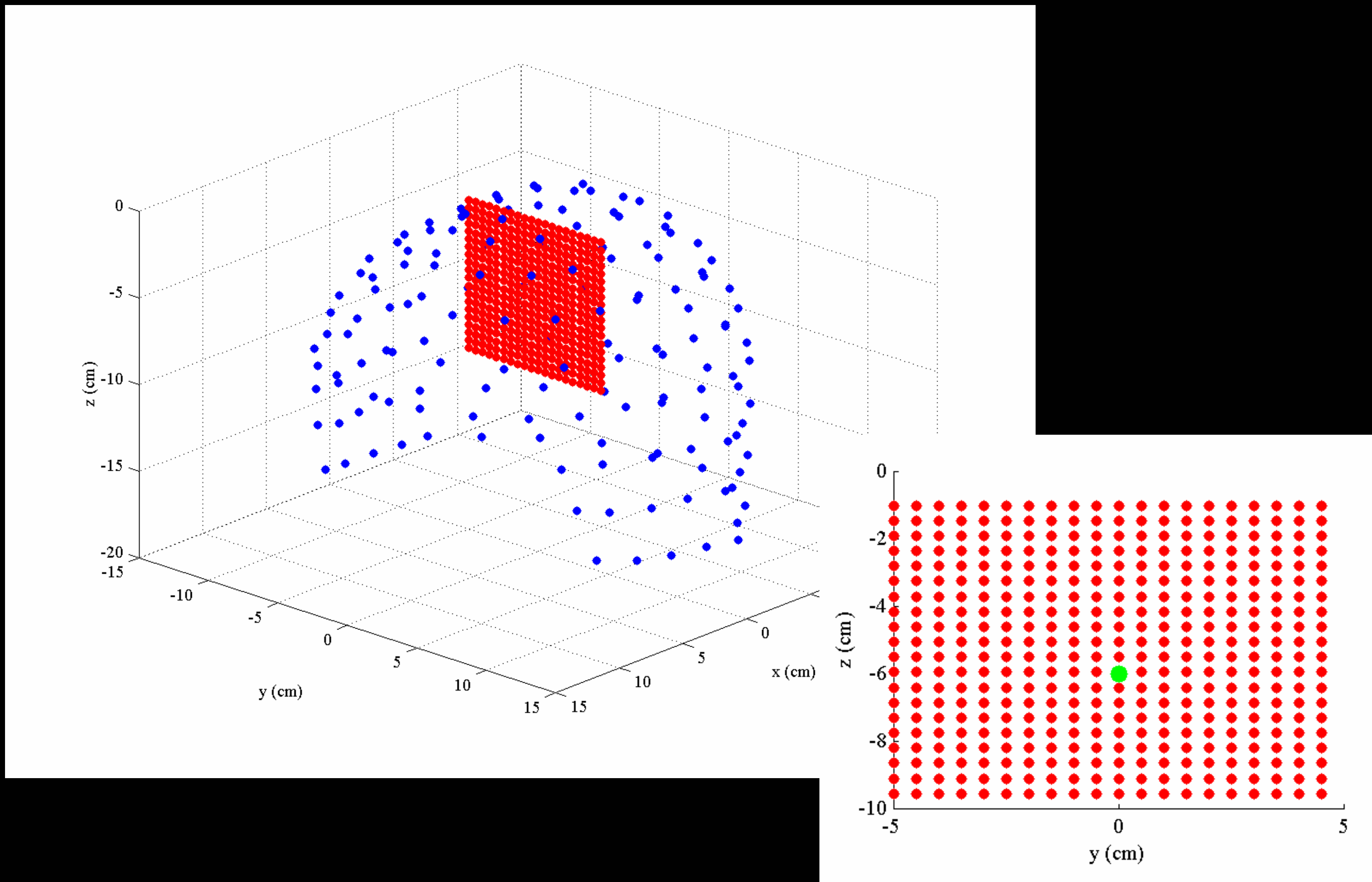
Gram matrix $\boldsymbol{G}$ is usually calculated by introducing pixel grid $\boldsymbol{r}_j$

$$\boldsymbol{b} = \int \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r})d\boldsymbol{r} = \sum_{j=1}^{N} \boldsymbol{L}(\boldsymbol{r}_j)\boldsymbol{s}(\boldsymbol{r}_j)$$

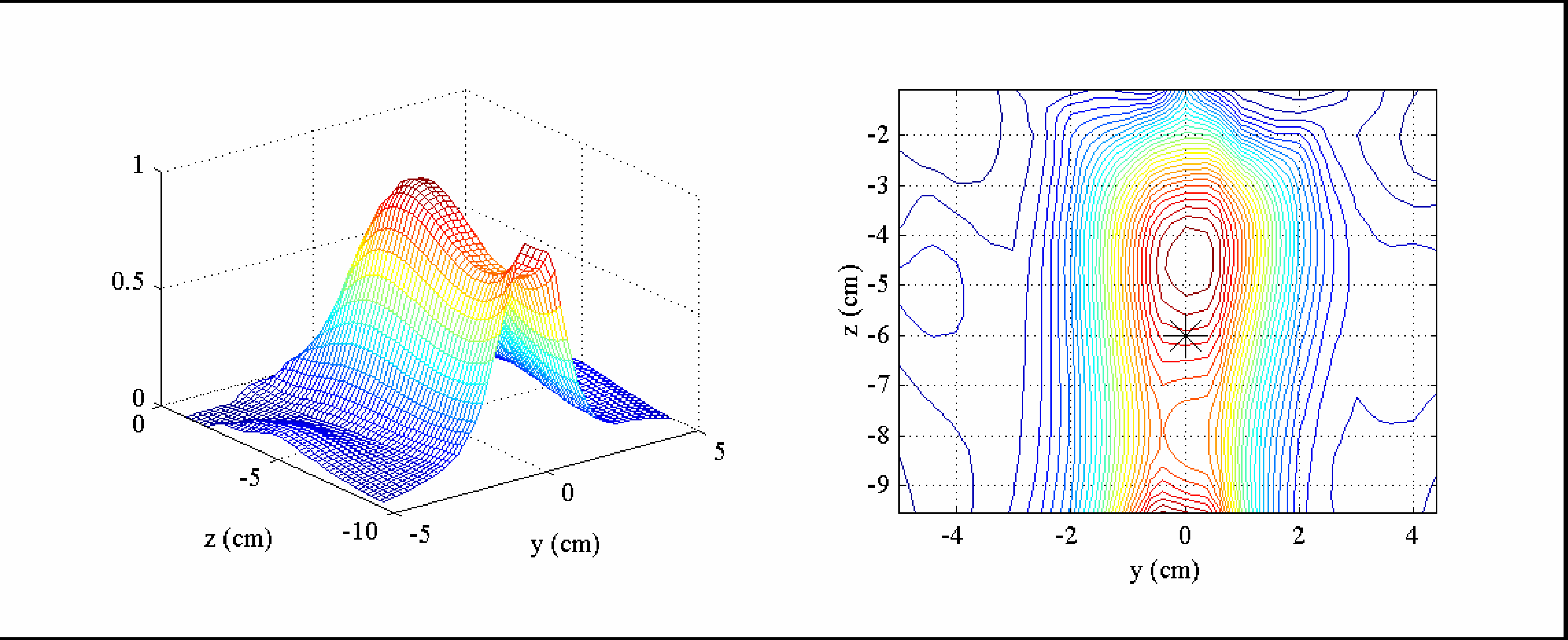
$$= \underbrace{\begin{bmatrix} \boldsymbol{L}(\boldsymbol{r}_1), & \cdots, & \boldsymbol{L}(\boldsymbol{r}_N) \end{bmatrix}}_{\boldsymbol{L}_N} \underbrace{\begin{bmatrix} \boldsymbol{s}(\boldsymbol{r}_1) \\ \vdots \\ \boldsymbol{s}(\boldsymbol{r}_N) \end{bmatrix}}_{\boldsymbol{s}_N} = \boldsymbol{L}_N \boldsymbol{s}_N$$

Therefore $\boldsymbol{G} = \boldsymbol{L}_N \boldsymbol{L}_N^T$ and

$$\boldsymbol{w}^T(\boldsymbol{r}) = \boldsymbol{L}^T(\boldsymbol{r})(\boldsymbol{L}_N \boldsymbol{L}_N^T)^{-1}$$


z (cm)
0
-5
-10
-15
-20
-15
-10
-5
0
5
10
15
y (cm)
15
10
5
0
x (cm)
z (cm)
0
-2
-4
-6
-8
-10
-5
0
5
y (cm)

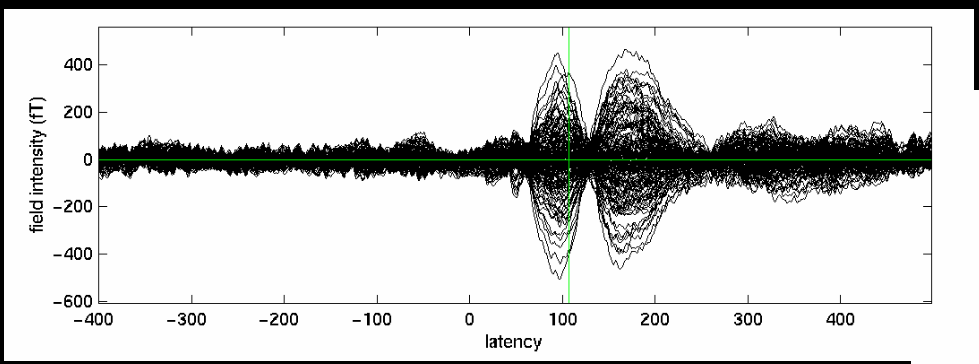
Resolution kernel for non-adaptive (minimum-norm) method

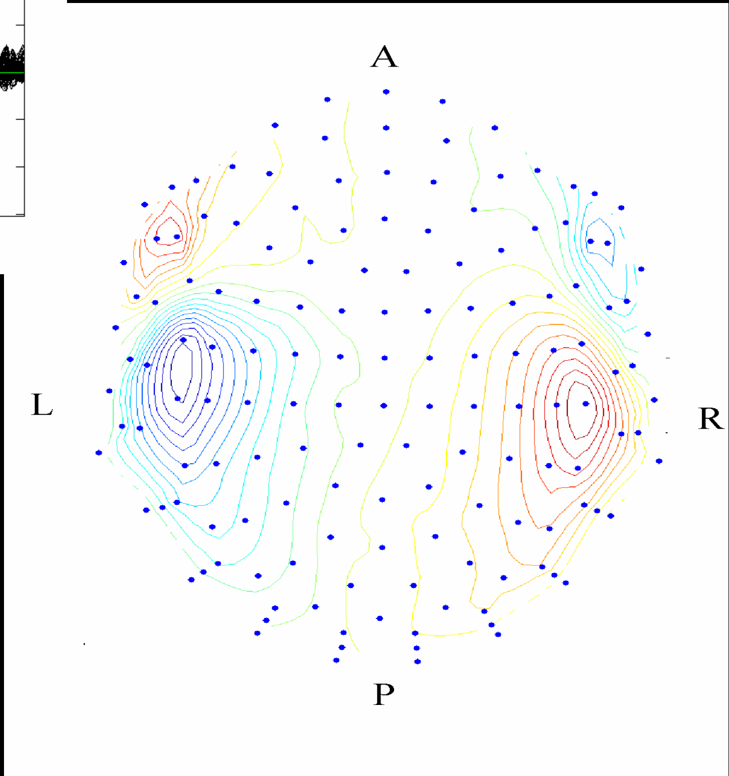


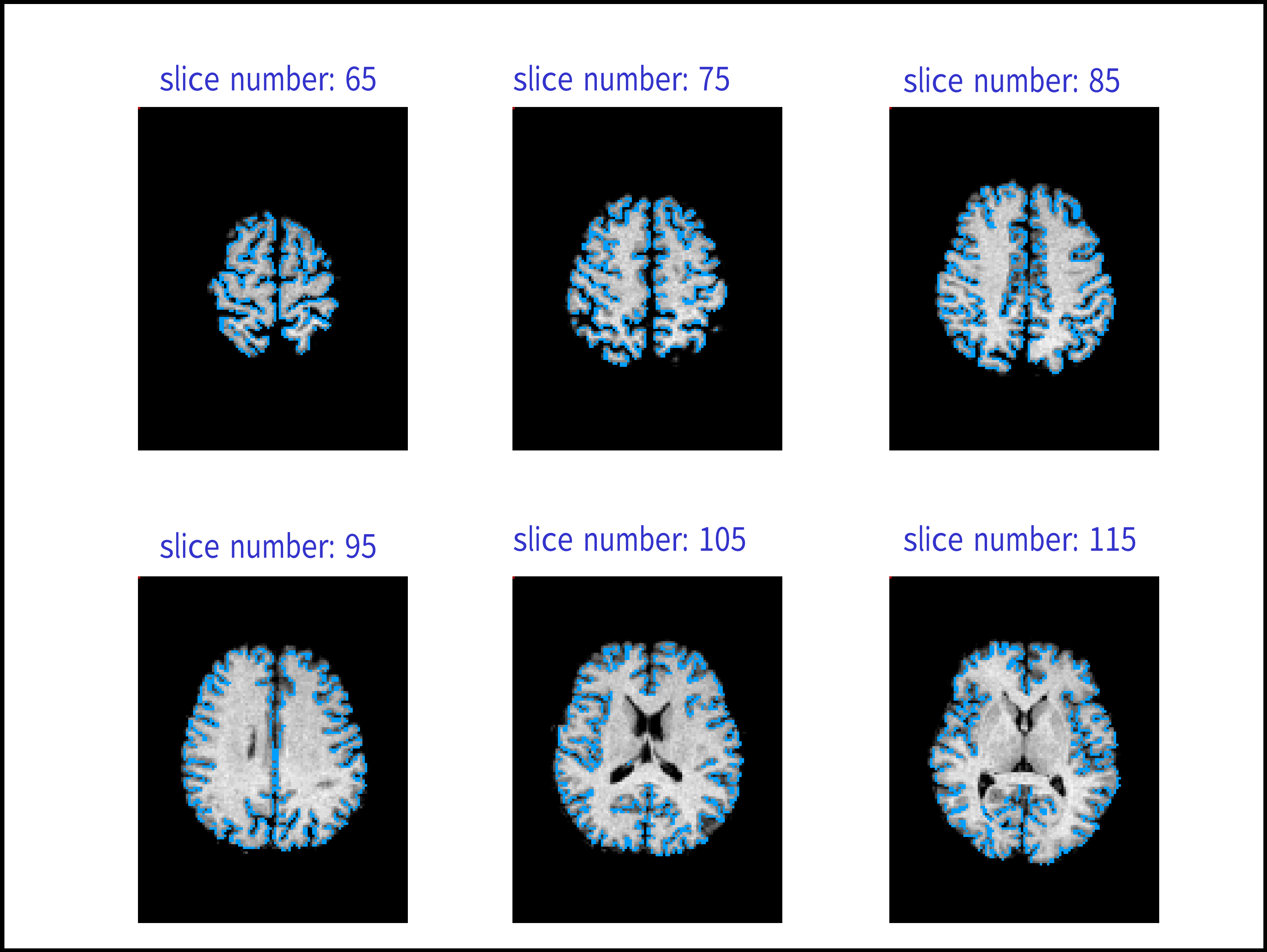
One example

- Auditory-evoked field were measured using 148-channel whole-head sensor array (Magnes 2500).

Stimulus: 1-kHz pure tone applied to subject's left ear

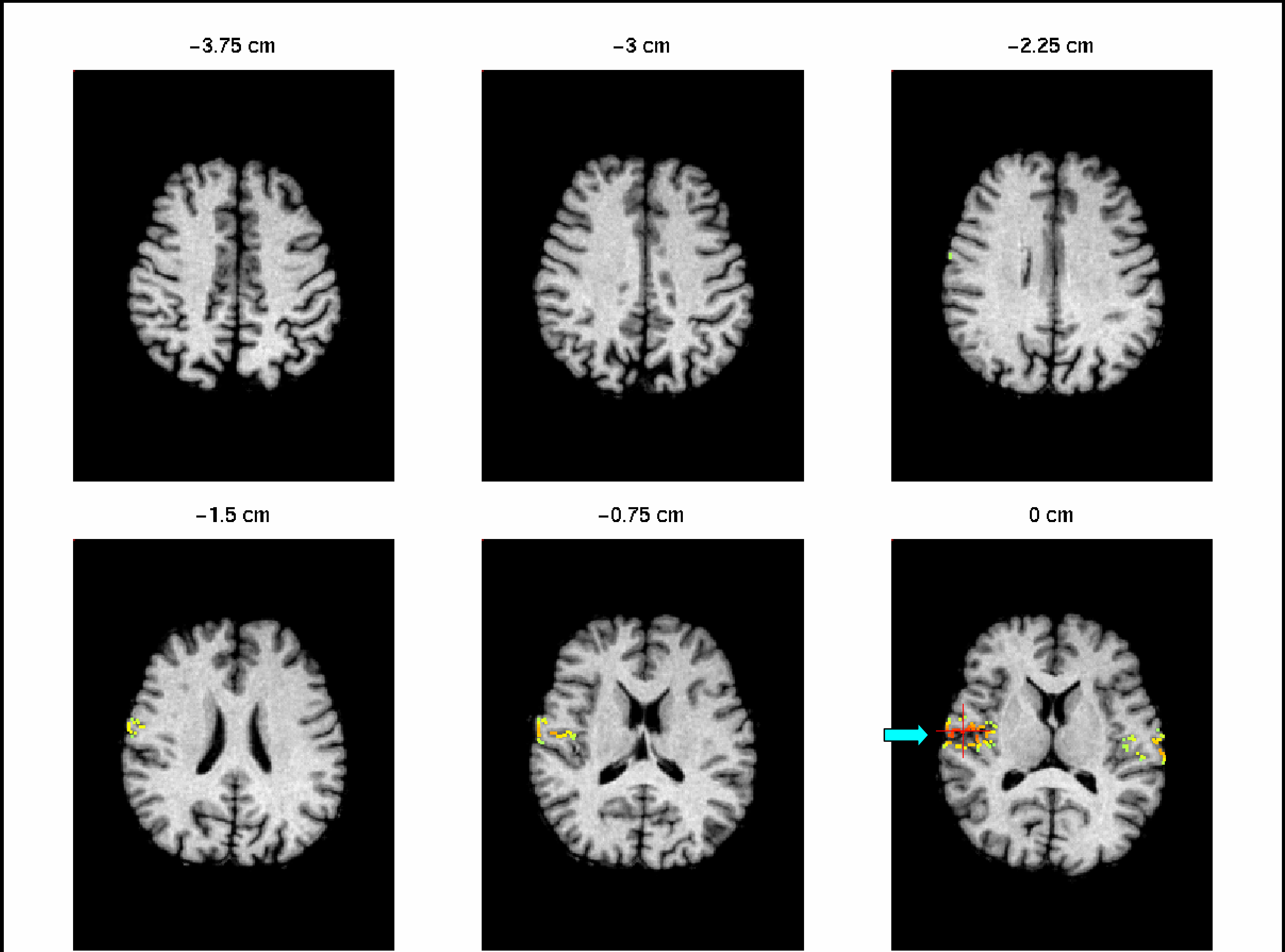




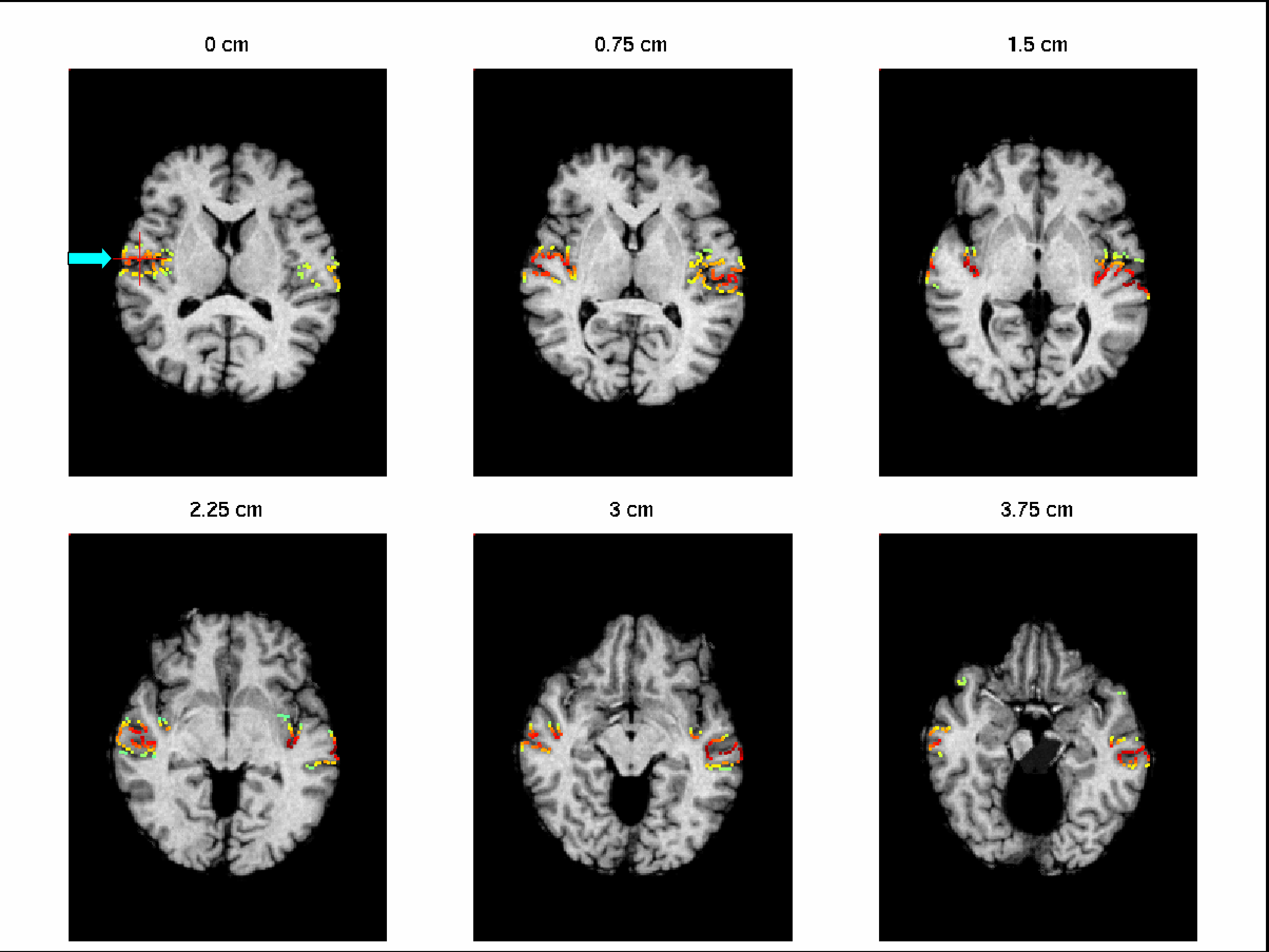


The number of pixels: 12940 points

The condition number of $\boldsymbol{G}$: $\sim 10^9$

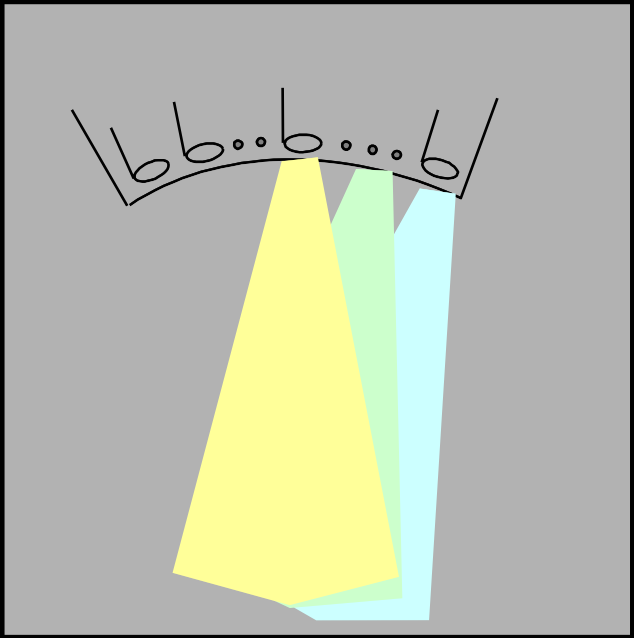


−3.75 cm
−3 cm
−2.25 cm
−1.5 cm
−0.75 cm
0 cm

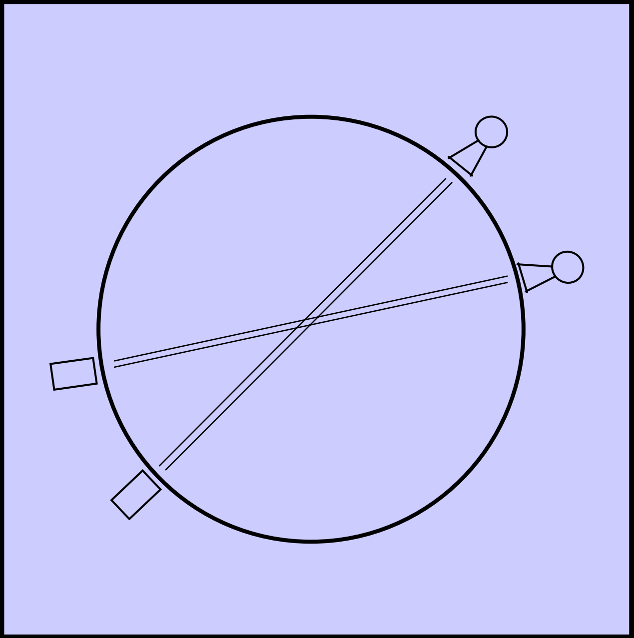

0 cm
0.75 cm
1.5 cm
2.25 cm
3 cm
3.75 cm

Property of the gram matrix $\boldsymbol{G}$

$$G_{i,j} = \int \boldsymbol{l}_i(\boldsymbol{r})\boldsymbol{l}_j(\boldsymbol{r})d\boldsymbol{r}$$



Biomagnetic instruments



X-ray computed tomography

Overlaps of sensor lead fields is large

$\boldsymbol{G}$ is poorly conditioned

$\boldsymbol{G} \approx$ unit matrix

$\boldsymbol{G}$ is poorly conditioned

- Apply regularization when calculating $\boldsymbol{G}^{-1}$

$\longrightarrow$ Baysian-type approach

$$\min_{\hat{\boldsymbol{s}}} F \ : F = \|\boldsymbol{b} - \boldsymbol{L}\hat{\boldsymbol{s}}\|^2 + \|\boldsymbol{H}\hat{\boldsymbol{s}}\|^2$$

$$\Updownarrow$$

$$\hat{\boldsymbol{s}} = \boldsymbol{H}\boldsymbol{L}^T (\underbrace{\boldsymbol{L}_N \boldsymbol{H}\boldsymbol{L}_N^T + \gamma \boldsymbol{I}}_{\text{modified } \boldsymbol{G}})^{-1} \boldsymbol{b}$$

- Do not use $\boldsymbol{G}$

$\longrightarrow$ Adaptive beamforming technique

Adaptive spatial filter

Minimum-variance beamformer

$$\hat{s}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = [w_1(\boldsymbol{r}),\ldots,w_M(\boldsymbol{r})]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t)$$

$$\Uparrow$$

weight vector

$$\min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T\boldsymbol{l}(\boldsymbol{r}) = 1 \quad \Rightarrow \quad \boldsymbol{w}^T(\boldsymbol{r}) = \frac{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

$$\left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

Assumption that source activities are uncorrelated

With constraint: $\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_p) = 1$,

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r}_p) = \left\langle s(\boldsymbol{r}_p,t)^2 \right\rangle + \sum_{q \neq p} \left\langle s(\boldsymbol{r}_q,t)^2 \right\rangle \left\| \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) \right\|$$

$$\Uparrow$$

$$\left\langle s(\boldsymbol{r}_p,t) s(\boldsymbol{r}_q,t) \right\rangle = 0 \quad \text{when} \quad p \neq q$$

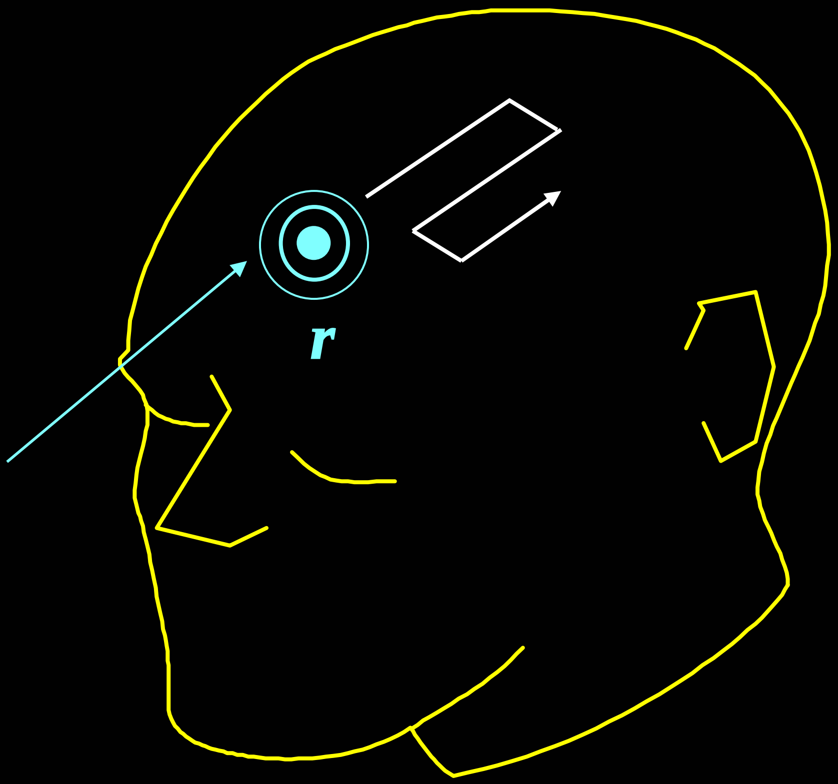
$$\min_{\boldsymbol{w}} \left[\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r}_p) \right] \Rightarrow \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = 0, \; q \neq p$$

Therefore, this minimization gives the weight satisfying

$$\begin{aligned} \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) &= 1 \quad \text{for } p = q \\ &= 0 \quad \text{for } p \neq q \end{aligned}$$

Spatial filter technique

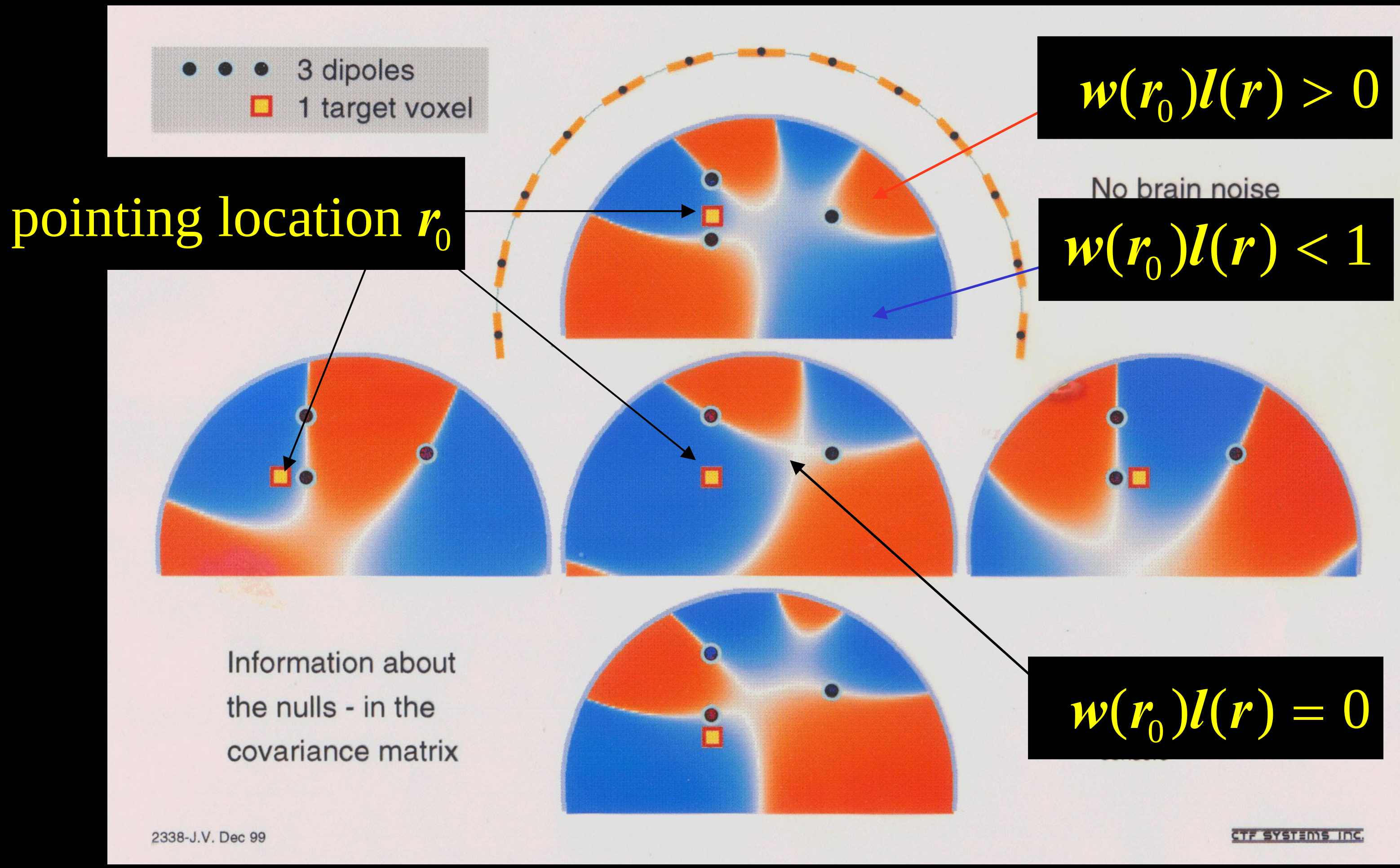
•Form spatial filter weight $\boldsymbol{w}(\boldsymbol{r})$ that focuses the sensitivity of the sensor array at a small area at $\boldsymbol{r}$.



Focused region

Adaptive beamformer sensitivity pattern: plot of $\boldsymbol{w}(\boldsymbol{r}_0)\boldsymbol{l}(\boldsymbol{r})$

The density of the colors is propotional to $\boldsymbol{w}(\boldsymbol{r}_0)\boldsymbol{l}(\boldsymbol{r})$.



The weight sets null-sensitivity at regions where sources exist.

Low-rank signal assumption

Consider a easiest case where we know locations and orientations of all Q sources

weight $\boldsymbol{w}(\boldsymbol{r}_1)$ (containing M unknowns) can be obtained by solving a set of Q linear equations:

$$\boldsymbol{w}^T(\boldsymbol{r}_1)\boldsymbol{l}(\boldsymbol{r}_1)=w_1(\boldsymbol{r}_1)l_1(\boldsymbol{r}_1)+\ldots+w_M(\boldsymbol{r}_1)l_M(\boldsymbol{r}_1)=1$$

$$\boldsymbol{w}^T(\boldsymbol{r}_1)\boldsymbol{l}(\boldsymbol{r}_2)=w_1(\boldsymbol{r}_1)l_1(\boldsymbol{r}_2)+\ldots+w_M(\boldsymbol{r}_1)l_M(\boldsymbol{r}_2)=0$$

$$\vdots$$

$$\boldsymbol{w}^T(\boldsymbol{r}_1)\boldsymbol{l}(\boldsymbol{r}_Q)=w_1(\boldsymbol{r}_1)l_1(\boldsymbol{r}_Q)+\ldots+w_M(\boldsymbol{r}_1)l_M(\boldsymbol{r}_Q)=0$$

when $Q > M$, there is no solution for $\boldsymbol{w}^T(\boldsymbol{r}_1)$

Low-rank signal

Number of sensors M > Number of sources Q

$$\boldsymbol{R} = \boldsymbol{U}\left[\begin{array}{ccc|cc} \lambda_1 & 0 & \cdots & \cdot & 0 \\ 0 & \ddots & & 0 & \cdot \\ \vdots & & \lambda_Q & & \vdots \\ \hline \cdot & 0 & & \ddots & 0 \\ 0 & \cdot & \cdots & 0 & \lambda_M \end{array}\right]\boldsymbol{U}^T = \boldsymbol{U}\begin{bmatrix} \boldsymbol{\Lambda}_S & 0 \\ 0 & \boldsymbol{\Lambda}_N \end{bmatrix}\boldsymbol{U}^T$$

$$\boldsymbol{U} = [\underbrace{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_Q}_{\boldsymbol{E}_S} \mid \underbrace{\boldsymbol{e}_{Q+1}, \ldots, \boldsymbol{e}_M}_{\boldsymbol{E}_N}] = [\boldsymbol{E}_S \mid \boldsymbol{E}_N]$$

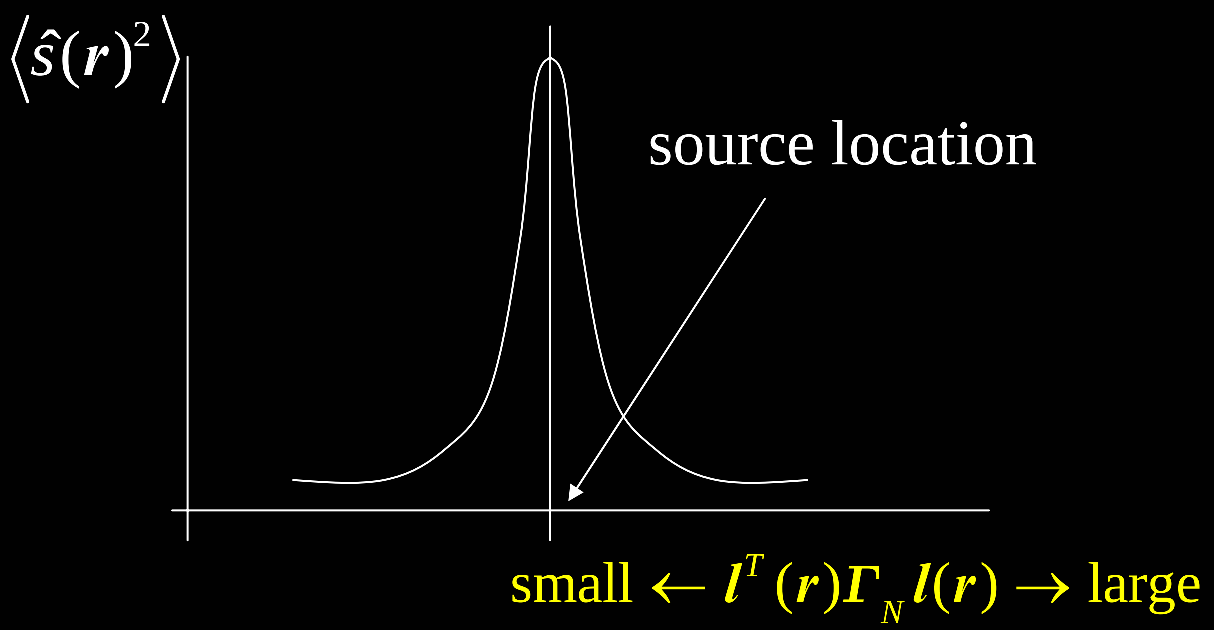
$$\boxed{\boldsymbol{\Gamma}_S^{-1} = \boldsymbol{E}_S \boldsymbol{\Lambda}_S^{-1} \boldsymbol{E}_S^T \text{ and } \boldsymbol{\Gamma}_N^{-1} = \boldsymbol{E}_N \boldsymbol{\Lambda}_N^{-1} \boldsymbol{E}_N^T \Rightarrow \boldsymbol{R}^{-1} = \boldsymbol{\Gamma}_S^{-1} + \boldsymbol{\Gamma}_N^{-1}}$$

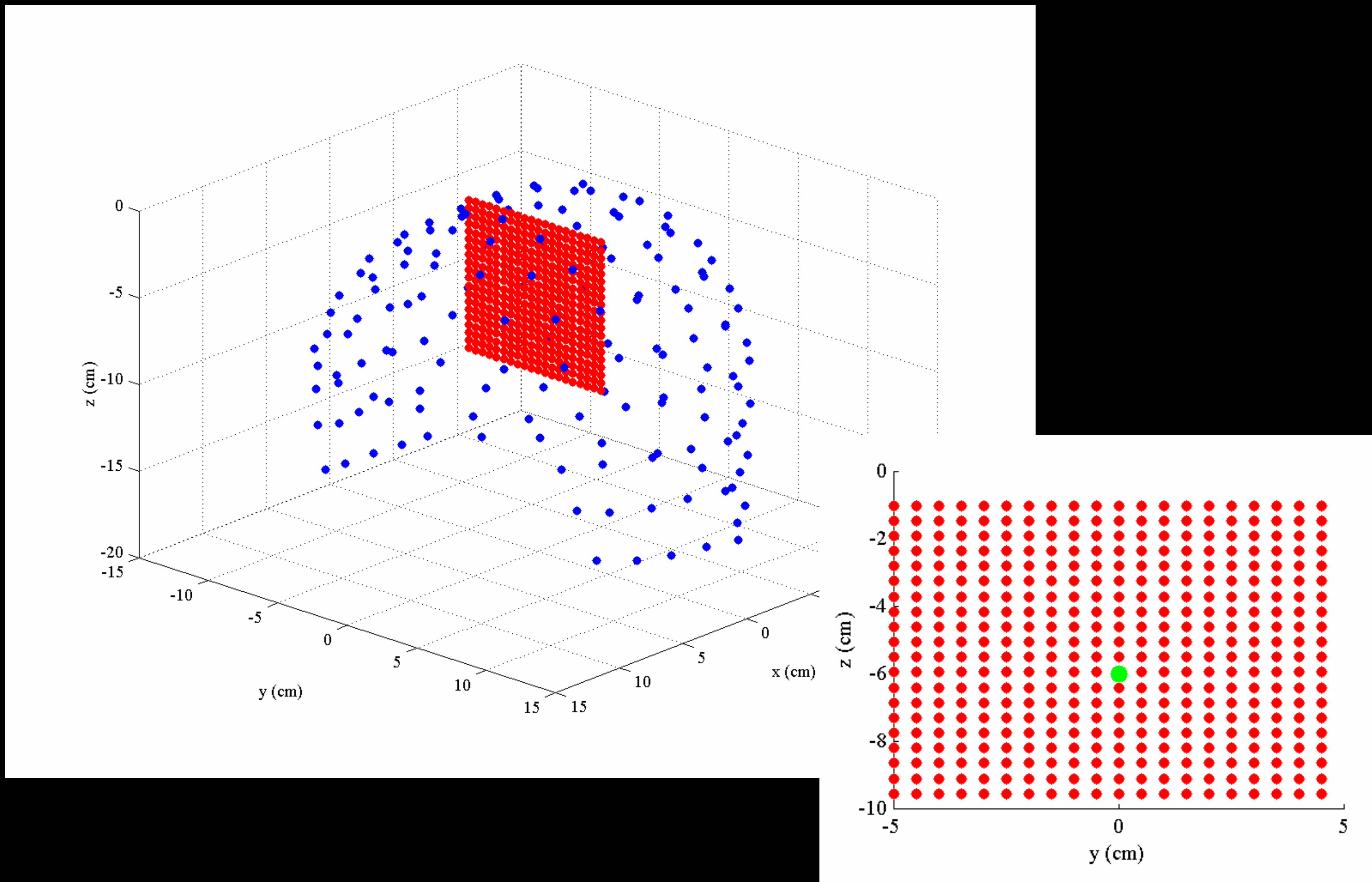
Orthogonality principle

$$\boxed{\boldsymbol{E}_N^T \boldsymbol{l}(\boldsymbol{r}_q) = \boldsymbol{\Gamma}_N^{-1} \boldsymbol{l}(\boldsymbol{r}_q) = 0 \text{ at any source location } \boldsymbol{r}_q}$$

Minimum-variance spatial filter output:

$$\left\langle \hat{s}(\boldsymbol{r})^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})} = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S^{-1}\boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r})}$$




z (cm)
y (cm)
x (cm)
0
-5
-10
-15
-20
-15
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-5
0
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10
15
15
10
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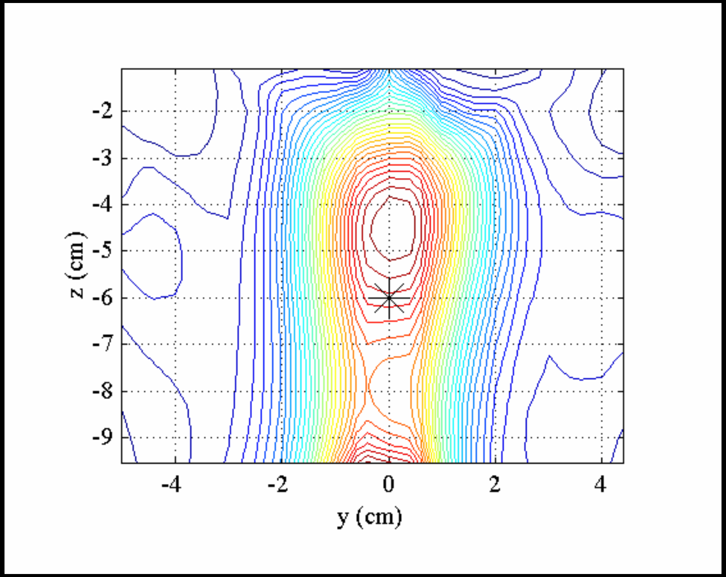
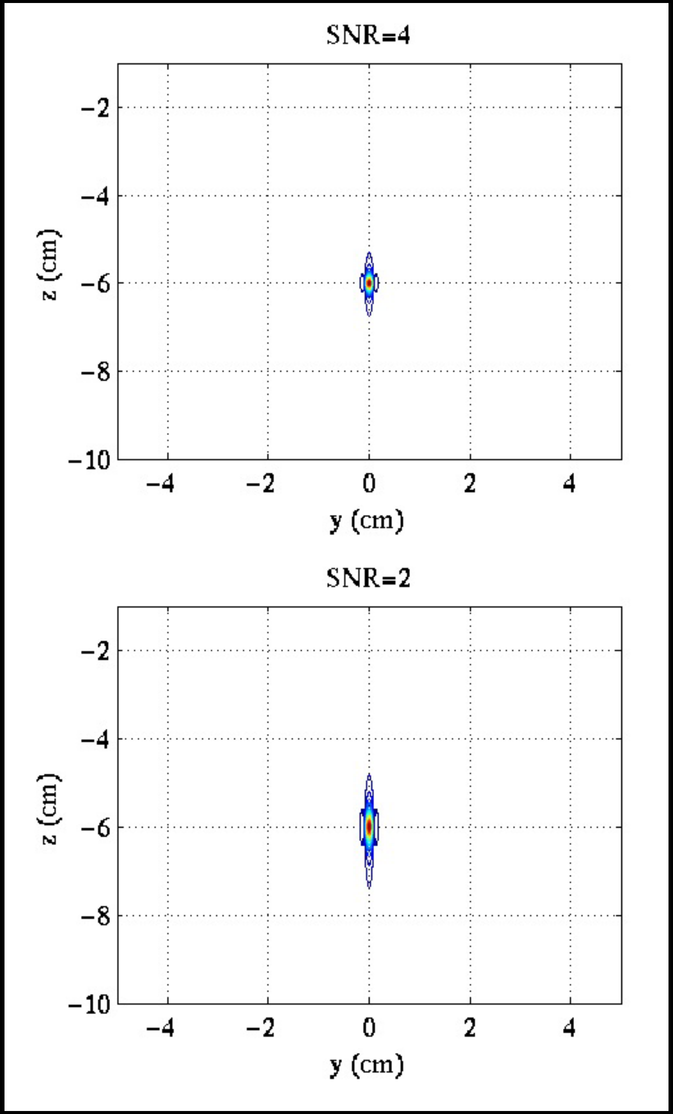
0
z (cm)
y (cm)
0
-2
-4
-6
-8
-10
-5
0
5

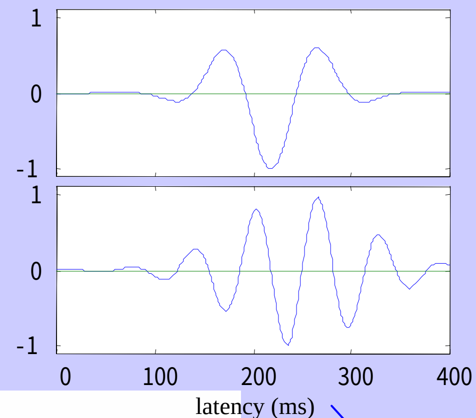
Non-adaptive spatial filter

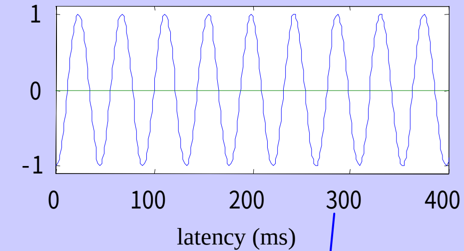


Adaptive spatial filter

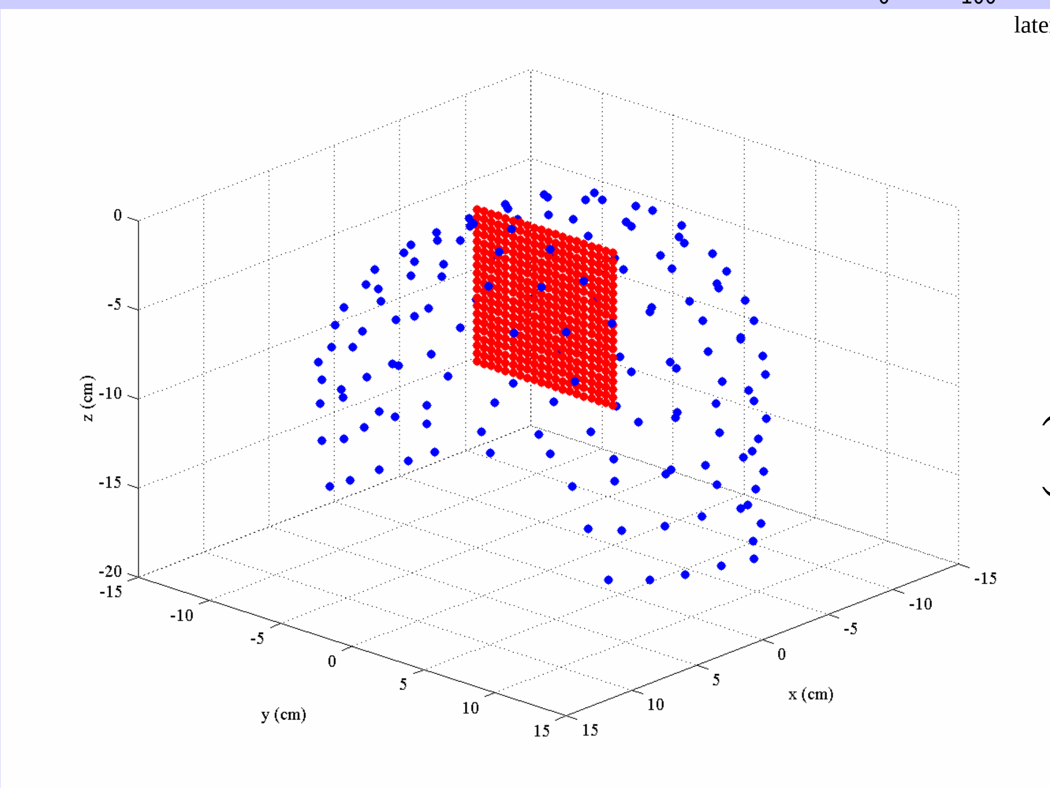


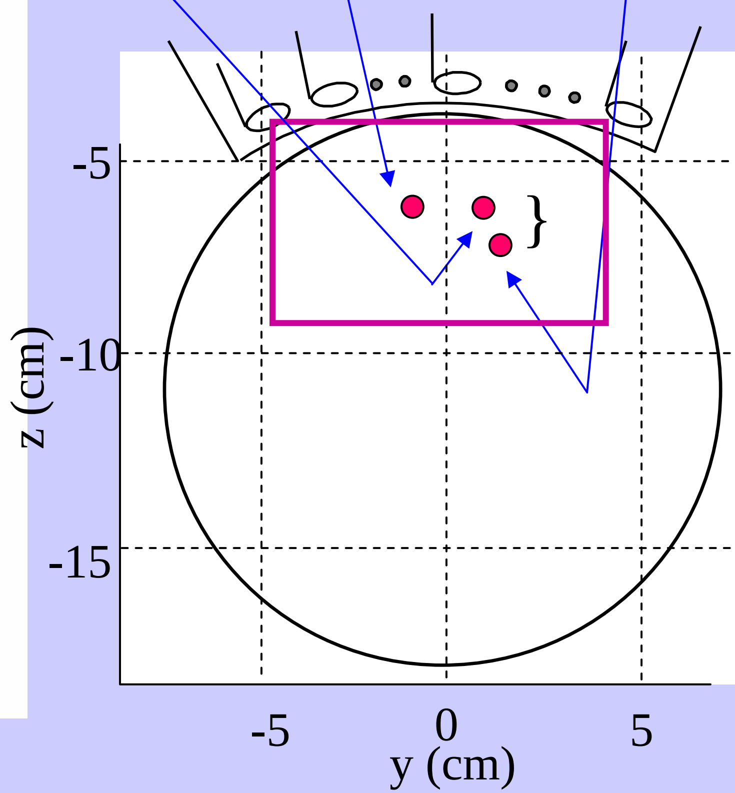
assumed source waveform



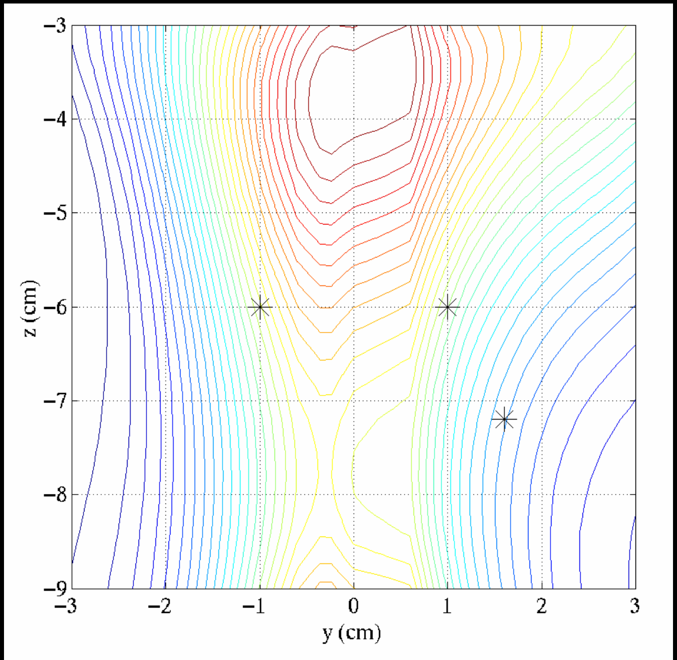


148-channel sensor array

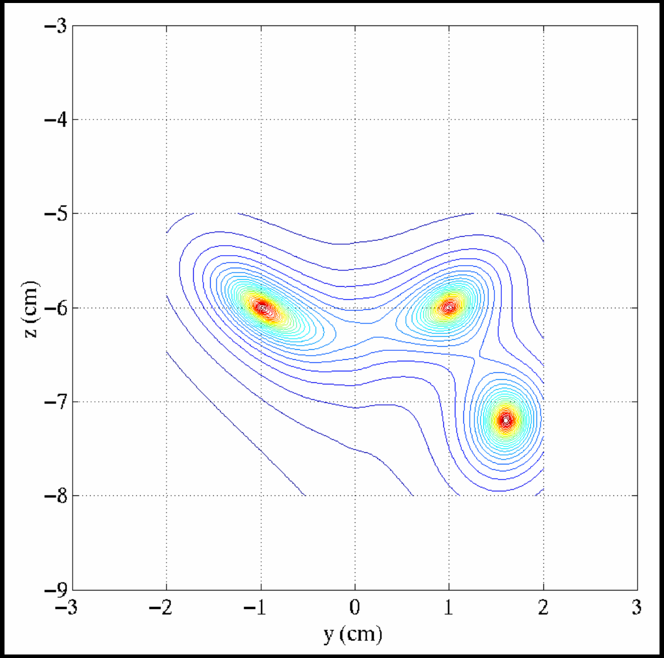




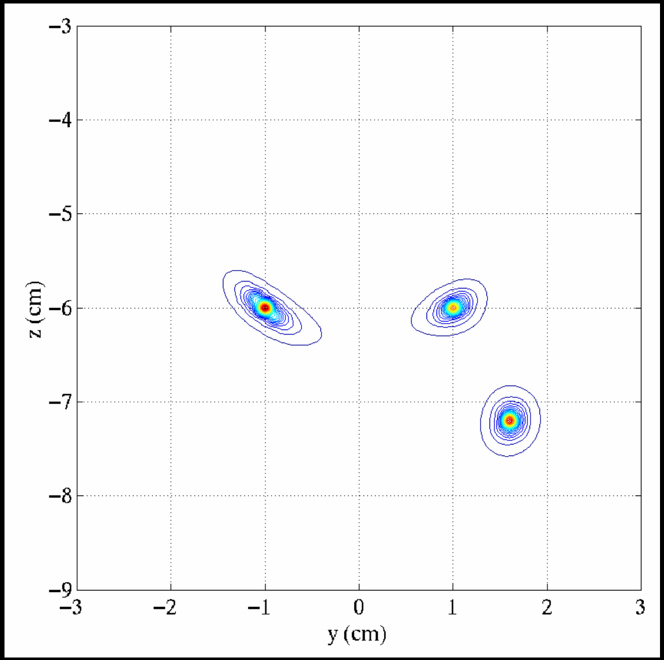
Minimum-norm reconstruction



Minimum-variance spatial filter reconstruction



MUSIC reconstruction



Adaptive-beamformer techniques were originally developed in the fields of array signal processing, including radar, sonar, and seismic exploration.

Two major problems arise when applying minimum-variance beamformer to MEG source localization.

(1) Vector source detection.

(2) Output SNR degradation.

Vector source detection

The neuromagnetic sources are three dimensional vectors.

$\Downarrow$

The minimum-variance beamformer formulation should be extended to incorporate the vector nature of sources.

Two-types of extensions has been proposed: scalar and vector formulations.

Scalar MV beamformer formulation

$$\boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta}) = \frac{\boldsymbol{l}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r},\boldsymbol{\eta})} = \frac{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}}{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}$$

The weight depends not only on $\boldsymbol{r}$ but also on $\boldsymbol{\eta}$.

Vector MV beamformer formulation

$$[\boldsymbol{w}_x(\boldsymbol{r}),\boldsymbol{w}_y(\boldsymbol{r}),\boldsymbol{w}_z(\boldsymbol{r})]^T = [\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}$$

The three weight vectors detect x, y, and z components.
Calculation of the weight does not require $\boldsymbol{\eta}$.

Scalar MV beamformer formulation

$$\boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta}) = \frac{\boldsymbol{l}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r},\boldsymbol{\eta})} = \frac{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}}{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}$$

The weight depends not only on $\boldsymbol{r}$ but also on $\boldsymbol{\eta}$.

Vector MV beamformer formulation

$$[\boldsymbol{w}_x(\boldsymbol{r}),\boldsymbol{w}_y(\boldsymbol{r}),\boldsymbol{w}_z(\boldsymbol{r})]^T = [\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}$$

The three weight vectors detect x, y, and z components.
Calculation of the weight does not require $\boldsymbol{\eta}$.

How to derive the optimum $\boldsymbol{\eta}$ in scalar formulation?

$$\Downarrow$$

Choose $\boldsymbol{\eta}$ that gives the maximum power output

$$\boxed{\max_{\eta} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \max_{\eta} \frac{1}{\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r}) \boldsymbol{\eta}} = \left[\min_{\eta} (\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r}) \boldsymbol{\eta}) \right]^{-1}}$$

Eigendecomposition: $[\boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})] = \sum_{j=1}^{3} \gamma_j \boldsymbol{v}_j \boldsymbol{v}_j^T, \ (\gamma_1 > \gamma_2 > \gamma_3)$

$$\Downarrow$$

$$\min_{\eta} (\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r}) \boldsymbol{\eta}) = \underbrace{\gamma_3}_{\uparrow}$$

minimum eigenvalue

$$\max_{\eta} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = 1/\gamma_3 = S_{opt}$$

Vector beamformer formulation

Each weight vector, $\boldsymbol{w}_x(\boldsymbol{r}), \boldsymbol{w}_y(\boldsymbol{r}),$ or $\boldsymbol{w}_z(\boldsymbol{r})$ is obtained by using the following multiple constraints.

$$\min \boldsymbol{w}_x^T \boldsymbol{R} \boldsymbol{w}_x \text{ subject to } \boldsymbol{w}_x^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_x) = 1,\ \boldsymbol{w}_x^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_y) = 0,\ \boldsymbol{w}_x^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_z) = 0$$

$$\min \boldsymbol{w}_y^T \boldsymbol{R} \boldsymbol{w}_y \text{ subject to } \boldsymbol{w}_y^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_x) = 0,\ \boldsymbol{w}_y^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_y) = 1,\ \boldsymbol{w}_y^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_z) = 0$$

$$\min \boldsymbol{w}_z^T \boldsymbol{R} \boldsymbol{w}_z \text{ subject to } \boldsymbol{w}_z^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_x) = 0,\ \boldsymbol{w}_z^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_y) = 0,\ \boldsymbol{w}_z^T \boldsymbol{l}(\boldsymbol{r}, \boldsymbol{e}_z) = 1$$

$\boldsymbol{e}_x,\ \boldsymbol{e}_y,\ \boldsymbol{e}_z$: unit vectors in the x, y, z directions.

$$\Downarrow$$

$$[\boldsymbol{w}_x(\boldsymbol{r}), \boldsymbol{w}_y(\boldsymbol{r}), \boldsymbol{w}_z(\boldsymbol{r})]^T = [\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}$$

(The weight is not equal to the scalar weight with $\boldsymbol{\eta} = \boldsymbol{e}_x, \boldsymbol{e}_y$, or $\boldsymbol{e}_z$.)

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van Veen et al., 1996 Spencer et al., 1992

In vector formulation, $\boldsymbol{\eta}$ that gives the maximum power output can be obtained using

$$\max_{\boldsymbol{\eta}} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \max_{\boldsymbol{\eta}} \left\| [\hat{s}_x(\boldsymbol{r}), \hat{s}_y(\boldsymbol{r}), \hat{s}_z(\boldsymbol{r})]\boldsymbol{\eta} \right\|^2 = \max_{\boldsymbol{\eta}} \boldsymbol{\eta}^T [\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{\eta}$$

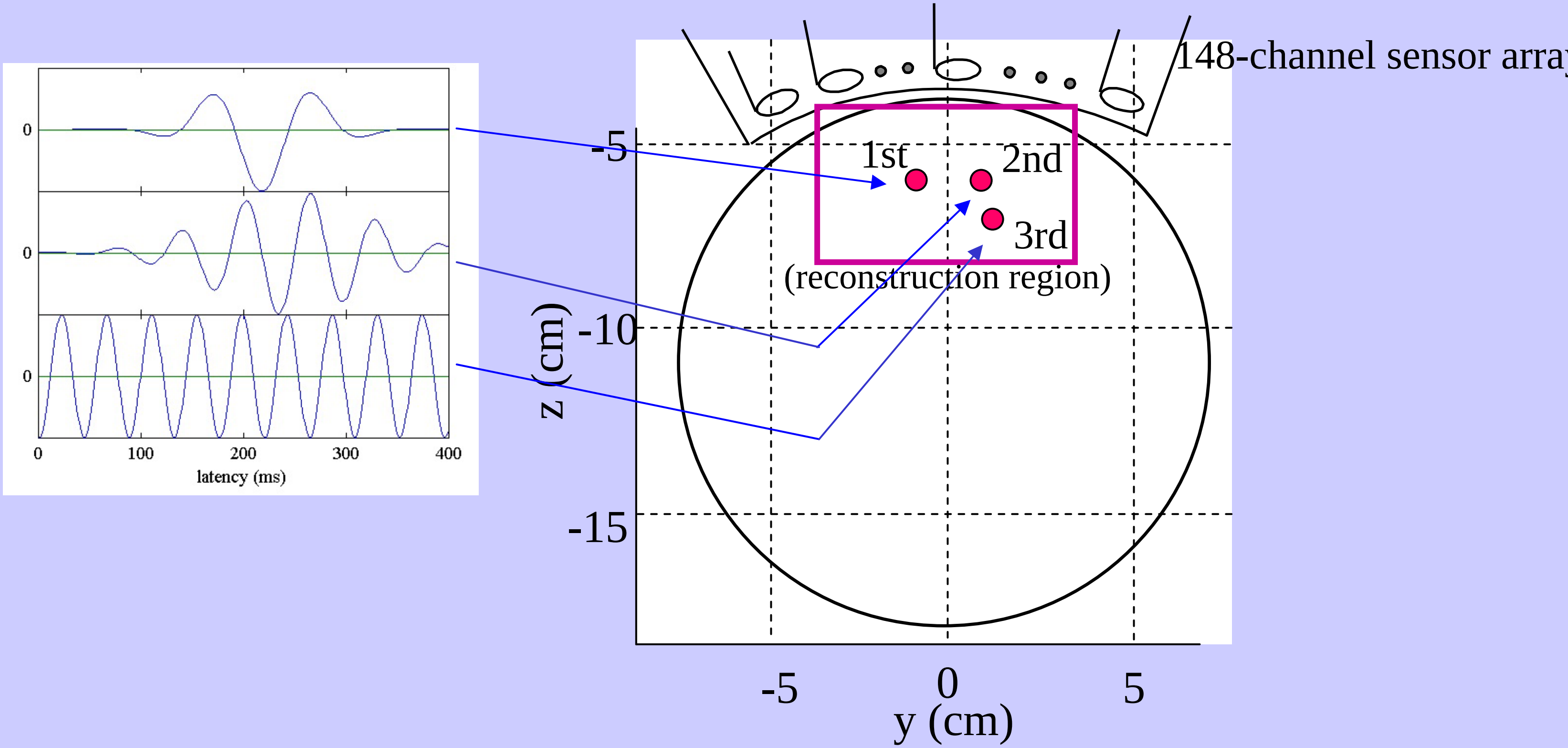
$$\Downarrow$$

$$\max_{\boldsymbol{\eta}} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = 1/\gamma_3 = S_{opt}$$

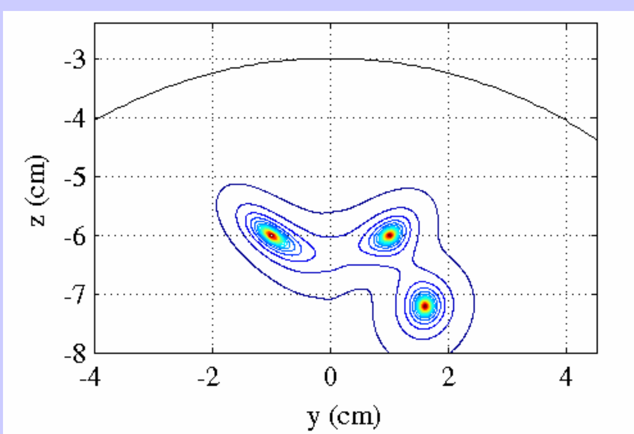
Either types of formulations attain S_{opt} when the beamformer pointing direction is optimized.

$$\Downarrow$$

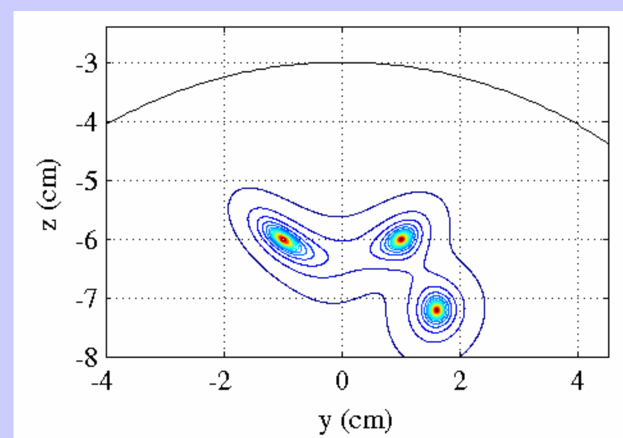
$$\boxed{\text{Two types of formulations are mathematically equivalent.}}$$



scalar formulation



vector formulation



Output SNR degradation.

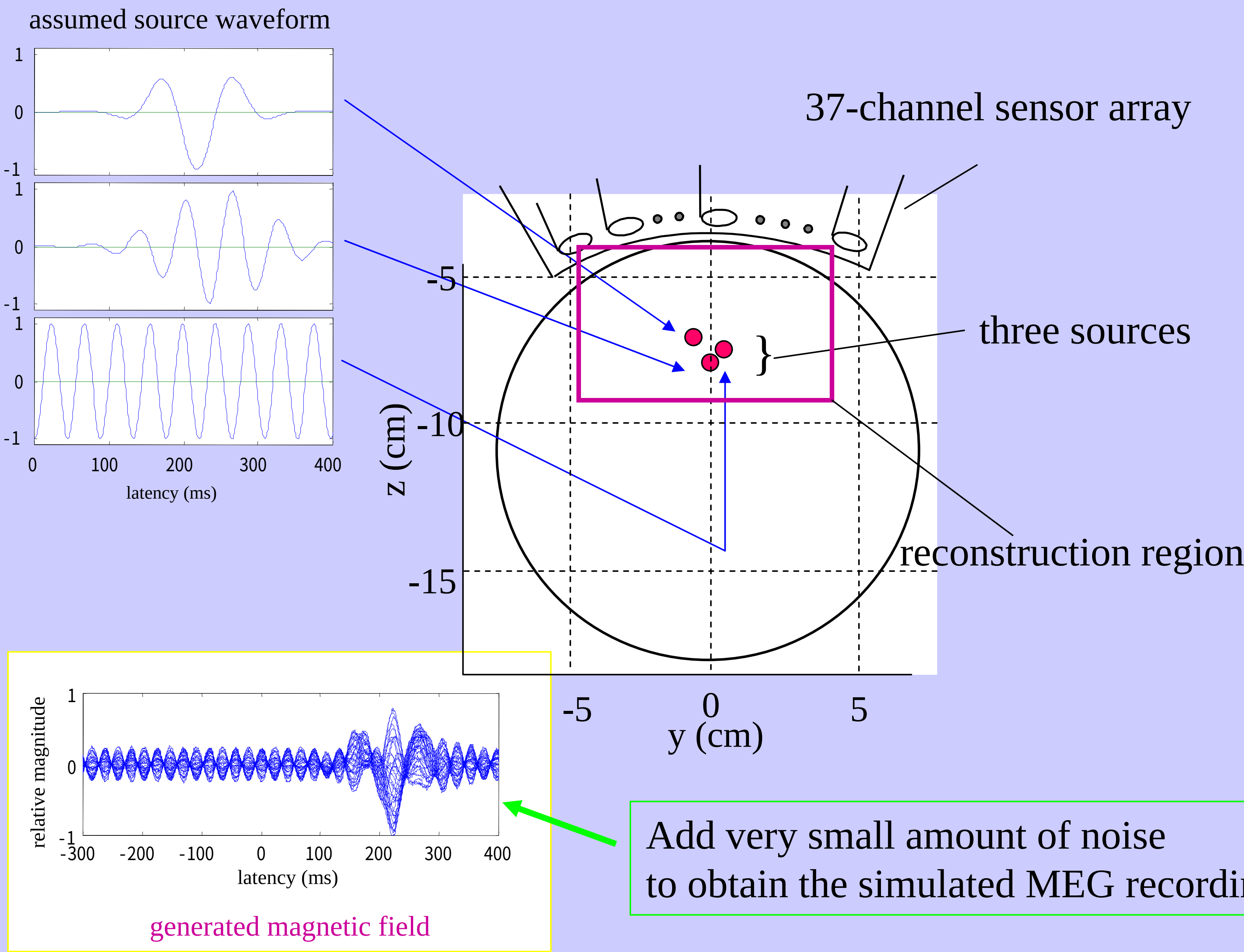
Minimum-variance beamformer is very sensitive to errors in forward modeling or errors in sample covariance matrix.

$\Downarrow$

Because such errors are almost inevitable in neuromagnetic measurements, minimum-variance beamformer generally provides noisy spatio-temporal reconstruction results.

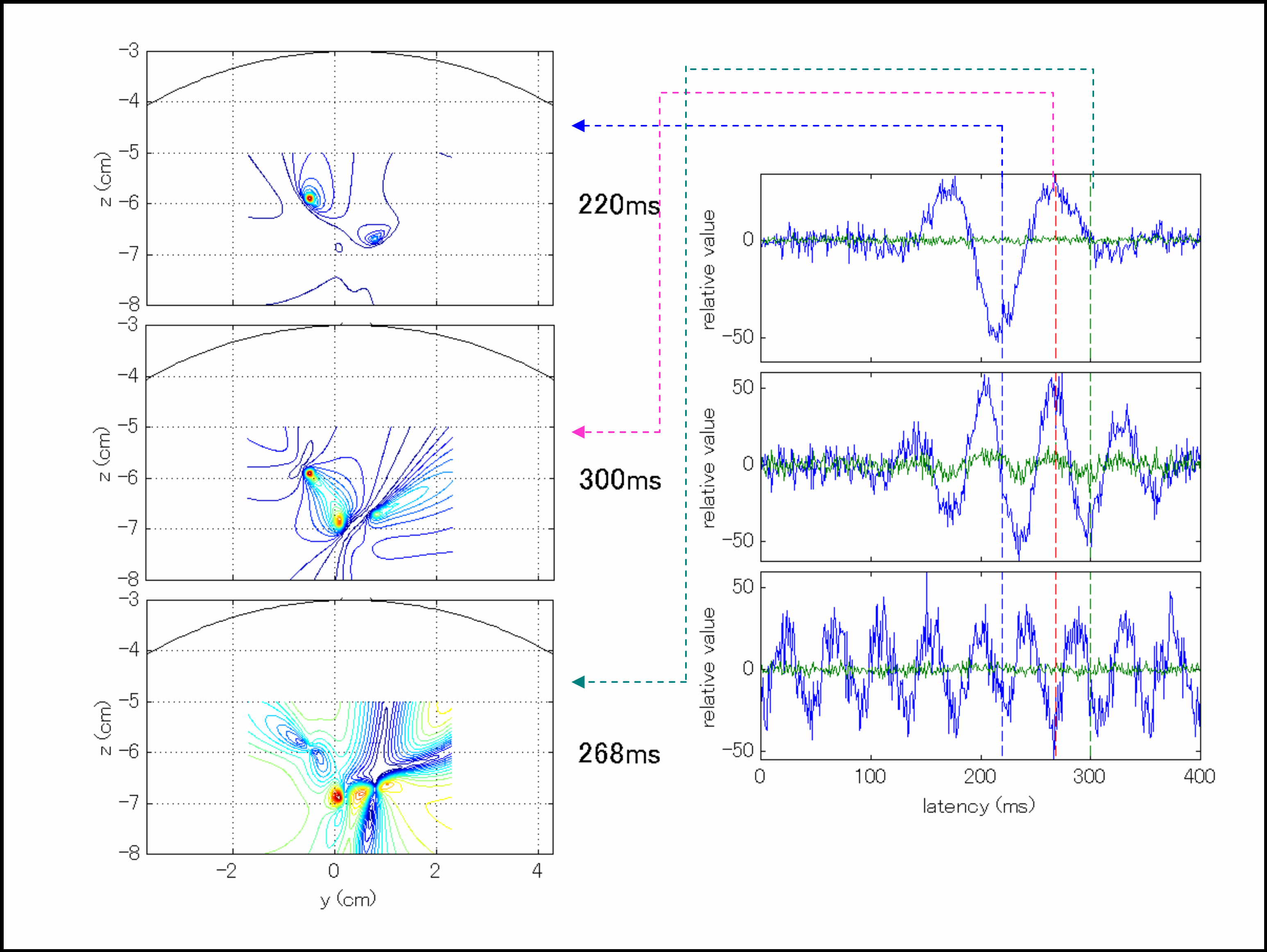
$\Downarrow$

Introducing eigenspace projection



Add very small amount of noise
to obtain the simulated MEG recordings

Spatio-tempotal reconstruction



Recall some definitions:

$$\boldsymbol{R} = \boldsymbol{U}\left[\begin{array}{ccc|cc} \lambda_1 & 0 & \cdots & \cdot & 0 \\ 0 & \ddots & & 0 & \cdot \\ \vdots & & \lambda_P & & \vdots \\ \hline \cdot & 0 & & \ddots & 0 \\ 0 & \cdot & \cdots & 0 & \lambda_M \end{array}\right]\boldsymbol{U}^T = \boldsymbol{U}\begin{bmatrix} \boldsymbol{\Lambda}_S & 0 \\ 0 & \boldsymbol{\Lambda}_N \end{bmatrix}\boldsymbol{U}^T, \quad \text{and} \quad \boldsymbol{U} = [\underbrace{\boldsymbol{e}_1, \dots, \boldsymbol{e}_P}_{\boldsymbol{E}_S} \mid \underbrace{\boldsymbol{e}_{P+1}, \dots, \boldsymbol{e}_M}_{\boldsymbol{E}_N}]$$

Also, $\boldsymbol{\Gamma}_{\boldsymbol{S}}^{-1} = \boldsymbol{E}_{\boldsymbol{S}}\boldsymbol{\Lambda}_{\boldsymbol{S}}^{-1}\boldsymbol{E}_{\boldsymbol{S}}^{\boldsymbol{T}}$, $\boldsymbol{\Gamma}_{\boldsymbol{N}}^{-1} = \boldsymbol{E}_{\boldsymbol{N}}\boldsymbol{\Lambda}_{\boldsymbol{N}}^{-1}\boldsymbol{E}_{\boldsymbol{N}}^{\boldsymbol{T}}$

---

overall error in estimating $\boldsymbol{l}(\boldsymbol{r})$

$$\text{Output SNR} \propto \frac{[\boldsymbol{l}^{\boldsymbol{T}}(\boldsymbol{r})\boldsymbol{\Gamma}_{\boldsymbol{S}}^{-1}\boldsymbol{l}(\boldsymbol{r})]^2}{[\boldsymbol{l}^{\boldsymbol{T}}(\boldsymbol{r})\boldsymbol{\Gamma}_{\boldsymbol{S}}^{-2}\boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{\varepsilon}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{N}}^{-2}\boldsymbol{\varepsilon}]}$$

Even when $\boldsymbol{\varepsilon}$ is small, $\boldsymbol{\varepsilon}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{N}}^{-2}\boldsymbol{\varepsilon}$ may not be small,
because $\boldsymbol{\varepsilon}^{\boldsymbol{T}}\boldsymbol{\Gamma}_{\boldsymbol{N}}^{-2}\boldsymbol{\varepsilon} \approx \|\boldsymbol{\varepsilon}\|^2 / \lambda_{\boldsymbol{p}+\boldsymbol{j}}^2 \leftarrow$ noise level eigenvalue

Eigenspace projection

The error term $\boldsymbol{\varepsilon}^{T}\boldsymbol{\Gamma}_{N}^{-2}\boldsymbol{\varepsilon}$ arises from the noise subspace component of $\boldsymbol{w}(\boldsymbol{r})$.

Extension to eigenspace projection beamformer

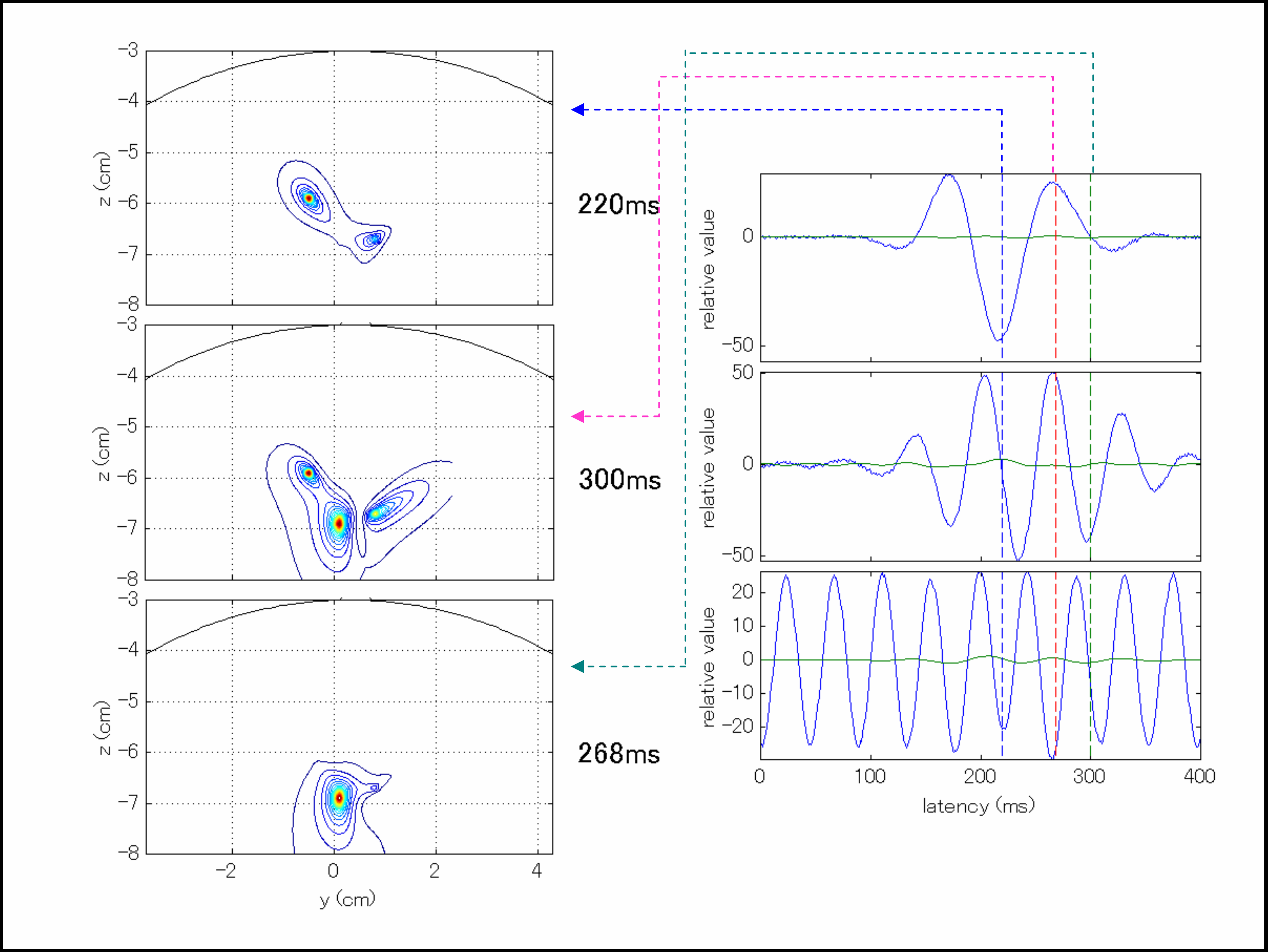
$$\bar{\boldsymbol{w}}_{\mu} = \boldsymbol{E}_{S}\boldsymbol{E}_{S}^{T}\boldsymbol{w}_{\mu}, \quad \text{where } \mu = x, y \text{ or } z$$

$$\text{Output SNR} \propto \frac{[\boldsymbol{l}^{T}(\boldsymbol{r})\boldsymbol{\Gamma}_{S}^{-1}\boldsymbol{l}(\boldsymbol{r})]^{2}}{[\boldsymbol{l}^{T}(\boldsymbol{r})\boldsymbol{\Gamma}_{S}^{-2}\boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{\varepsilon}^{T}\boldsymbol{\Gamma}_{N}^{-2}\boldsymbol{\varepsilon}]} \quad \text{(non-eigenspace projected)}$$

$$\Downarrow$$

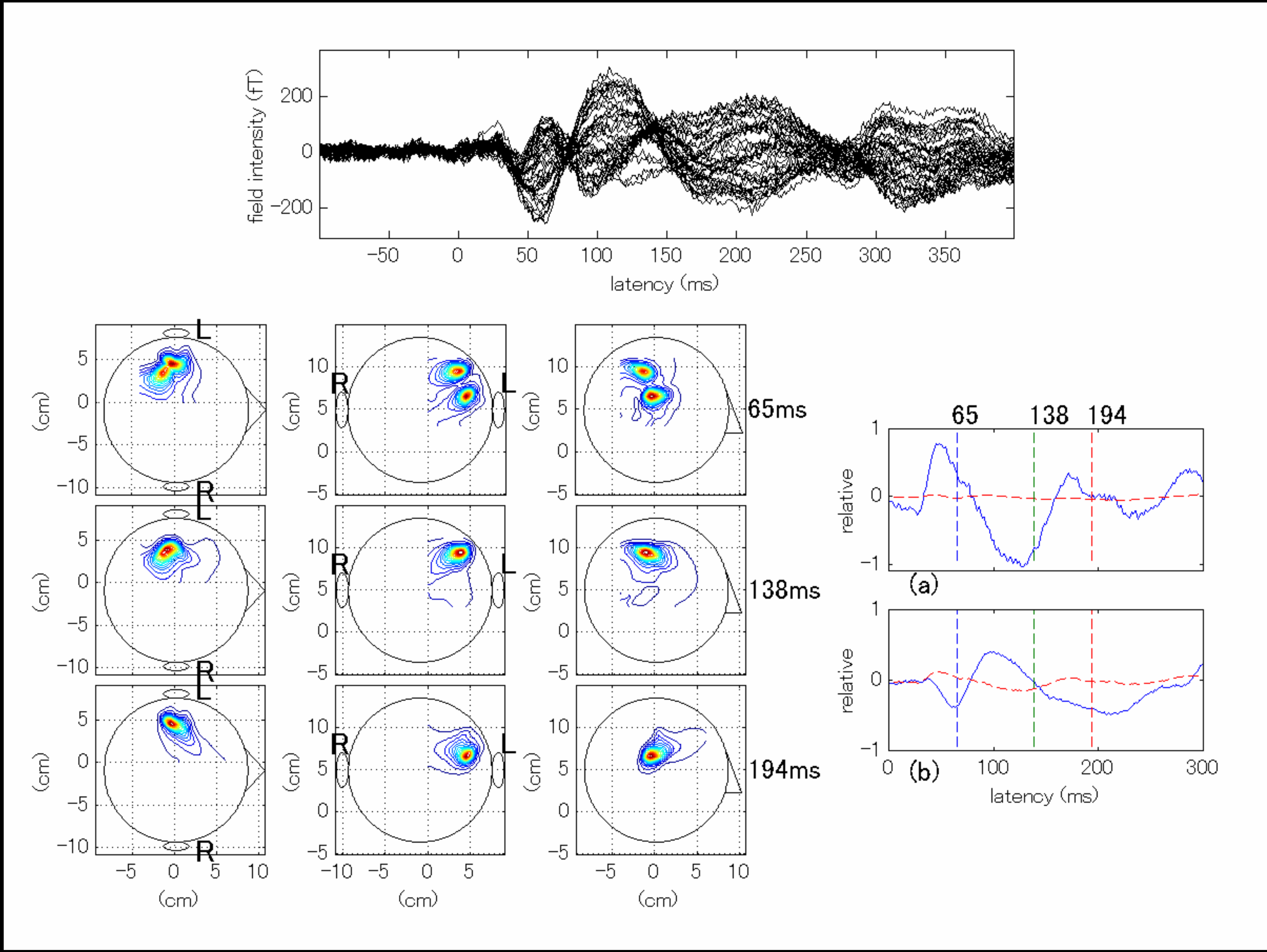
$$\text{Output SNR} \propto \frac{[\boldsymbol{l}^{T}(\boldsymbol{r})\boldsymbol{\Gamma}_{S}^{-1}\boldsymbol{l}(\boldsymbol{r})]^{2}}{[\boldsymbol{l}^{T}(\boldsymbol{r})\boldsymbol{\Gamma}_{S}^{-2}\boldsymbol{l}(\boldsymbol{r})]} \quad \text{(eigenspace projected)}$$

Spatio-tempotal reconstruction with eigen-space projection



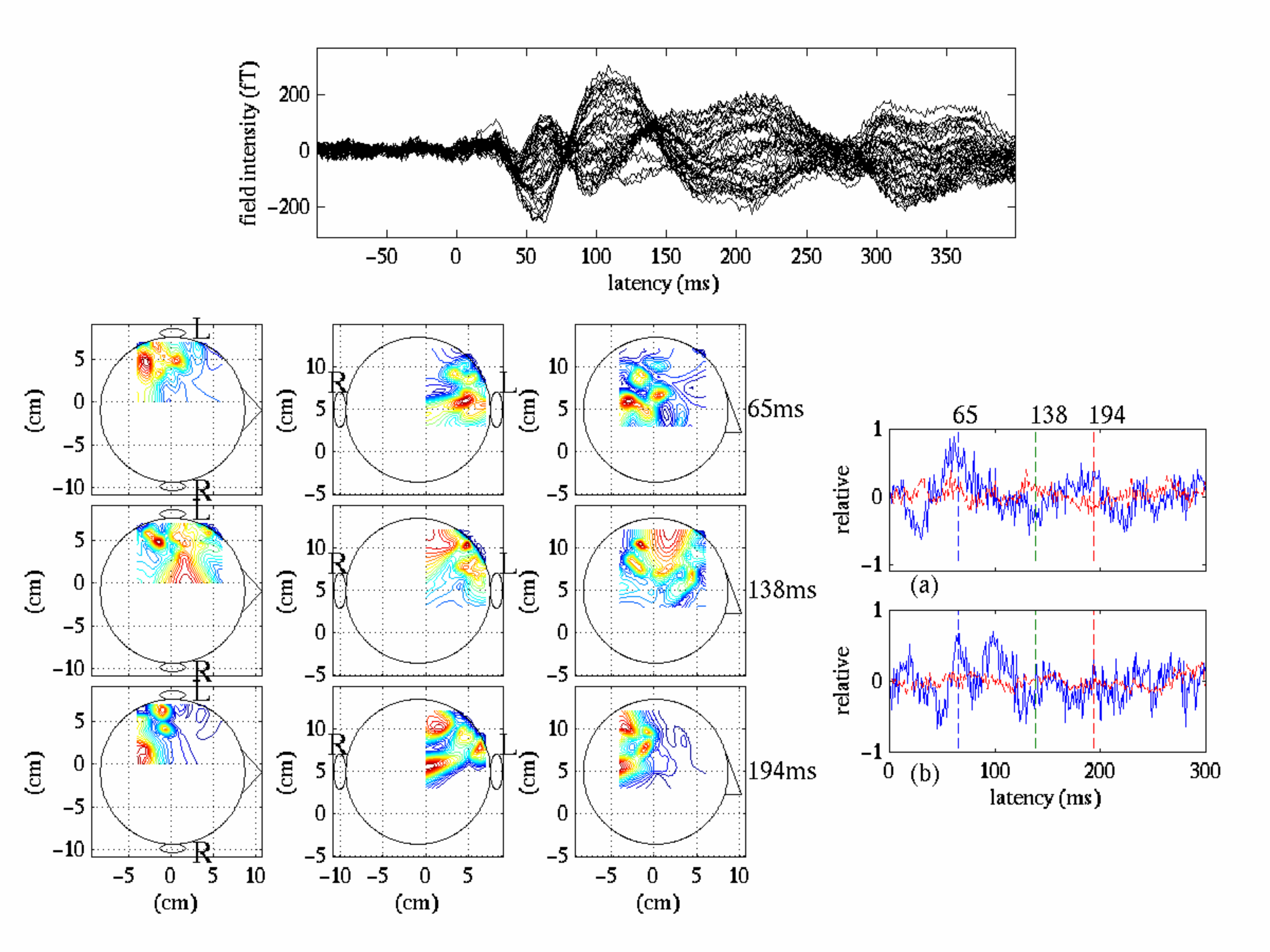
Application to 37-channel auditory-somatosensory recording

eigenspace-projection results

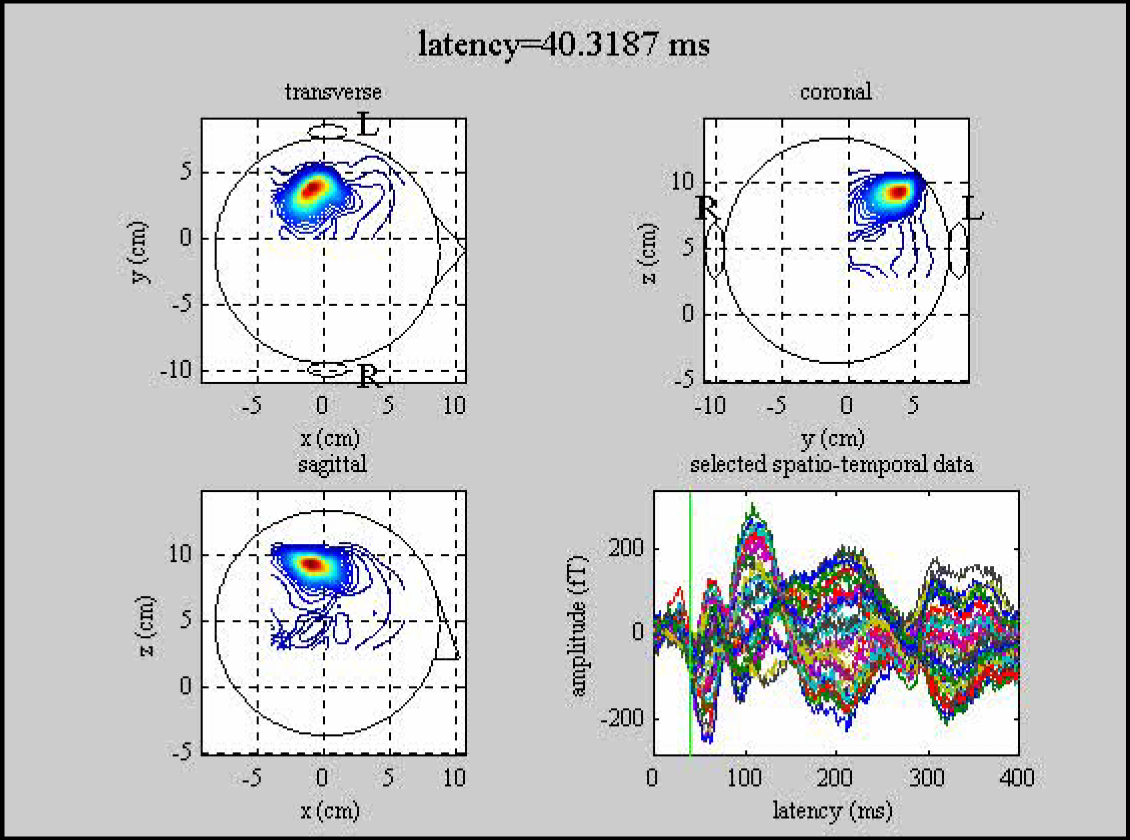


Application to 37-channel auditory-somatosensory recording

Non-eigenspace projected results



Application to 37-channel auditory-somatosensory recording
eigenspace-projection results



Summary

•This talk reviews the application of adaptive beamformer to reconstruction of brain activities, with some reference to the non-adaptive methods.

•Two types of extensions, scalar and vector extensions to incorporate the vector nature of the sources, are shown to be mathematically equivalent.

•Eigenspace projection is shown to overcome the SNR degradation problem.

•The implicit assumptions for the adaptive beamformer are that sources are uncorrelated and that the signal is low rank. The cases where these assumptions are invalidated will be discussed in Part II.

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