The Mind and Brain III :Audition, Language, Communication
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# Neuromagnetic Source Reconstruction and Inverse Modeling

## Part II: Performance analysis of adaptive beamformer techniques

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This talk discusses:

- Influence of source correlation.
- Influence of various types of noise.
  - Sensor noise
  - Artificial (non-biological) interference field
  - Cortical background activity

# Source correlation problem

$$\left\langle s(\boldsymbol{r}_p, t) s(\boldsymbol{r}_q, t) \right\rangle = 0 \quad \text{when} \quad p \neq q$$

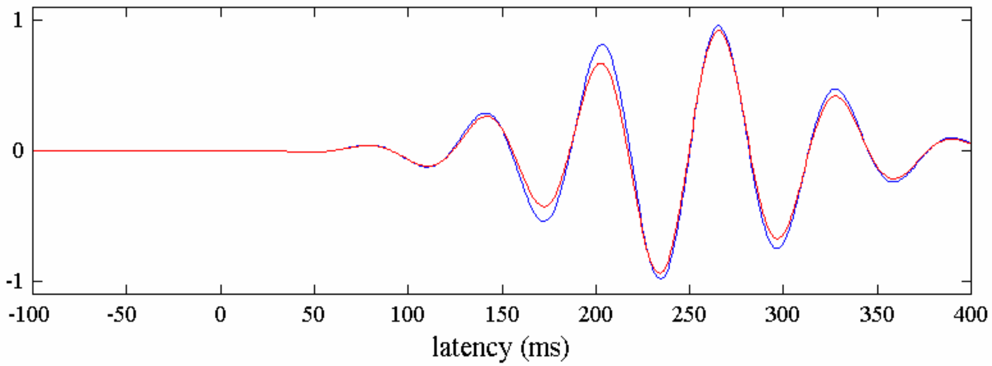
$$\Downarrow$$

$$\min_{\boldsymbol{w}} \left[ \boldsymbol{w}^T(\boldsymbol{r}_p) \boldsymbol{R} \boldsymbol{w}(\boldsymbol{r}_p) \right] \Rightarrow \boldsymbol{w}^T(\boldsymbol{r}_p) \boldsymbol{l}(\boldsymbol{r}_q) = 0,\ q \neq p$$

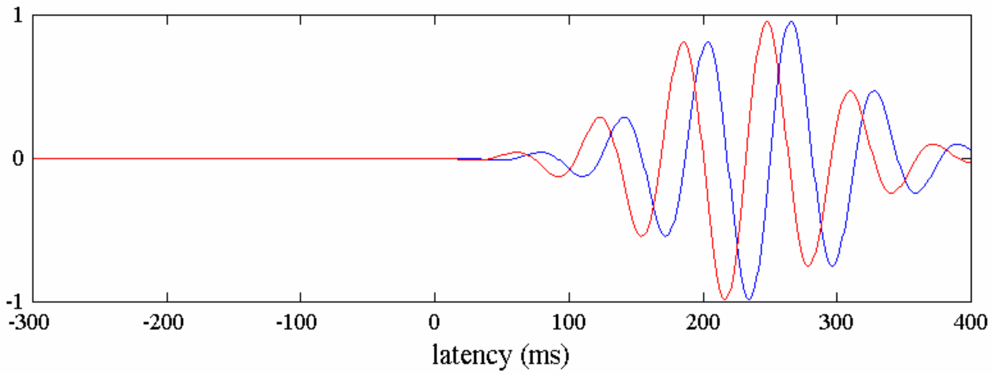
The weight satisfy:

$$\boxed{\begin{aligned} \boldsymbol{w}^T(\boldsymbol{r}_p) \boldsymbol{l}(\boldsymbol{r}_q) &= 1 \quad \text{for } p = q \\ &= 0 \quad \text{for } p \neq q \end{aligned}}$$

## correlation coefficient=0.99




## correlation coefficient=0.21



# Source correlation influence:

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \delta_{pq}$$

$$\Downarrow$$

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}}$$

$\boldsymbol{R}_S$ : source covariance matrix, $[\boldsymbol{R}_S^{-1}]_{pq}$:the $(p,q)$ element of $\boldsymbol{R}_S^{-1}$

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If there are discrete Q sources and their activities are expressed as $\boldsymbol{s}(\boldsymbol{r}_1,t), \ldots, \boldsymbol{s}(\boldsymbol{r}_Q,t)$

$$\boldsymbol{R}_S = \begin{bmatrix} \langle \boldsymbol{s}^2(\boldsymbol{r}_1,t)\rangle & \cdots & \langle \boldsymbol{s}(\boldsymbol{r}_Q,t)\boldsymbol{s}(\boldsymbol{r}_1,t)\rangle \\ \vdots & \ddots & \vdots \\ \langle \boldsymbol{s}(\boldsymbol{r}_1,t)\boldsymbol{s}(\boldsymbol{r}_Q,t)\rangle & \cdots & \langle \boldsymbol{s}^2(\boldsymbol{r}_Q,t)\rangle \end{bmatrix}$$

Assume that $Q$ sources are correlated with the $p$th source,

$$\tilde{s}(\boldsymbol{r}_p, t) = s(\boldsymbol{r}_p, t) + \sum_{q=1}^{Q} \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}} s(\boldsymbol{r}_q, t)$$

output for the *p*th source

*p*th source time course

leakage from other correlated sources

Consider the simplest case where two sources are correlated

$$R_S = \begin{bmatrix} \alpha_1^2 & \mu \\ \mu & \alpha_2^2 \end{bmatrix} \quad \Rightarrow \quad \boldsymbol{R}_S^{-1} = \frac{1}{\alpha_1^2\alpha_2^2(1-\mu^2)} \begin{bmatrix} \alpha_2^2 & -\mu\alpha_1\alpha_2 \\ -\mu\alpha_1\alpha_2 & \alpha_1^2 \end{bmatrix}$$

where $\alpha_j^2 = \left\langle s(\boldsymbol{r}_j,t)^2 \right\rangle, \quad \mu = \left\langle s(\boldsymbol{r}_1,t)s(\boldsymbol{r}_2,t) \right\rangle / \sqrt{\left\langle s(\boldsymbol{r}_1,t)^2 \right\rangle \left\langle s(\boldsymbol{r}_2,t)^2 \right\rangle}$

---

Then

$$\tilde{s}(\boldsymbol{r}_1,t) = s(\boldsymbol{r}_1,t) - (\frac{\alpha_1\mu}{\alpha_2})s(\boldsymbol{r}_2,t)$$

$$\tilde{s}(\boldsymbol{r}_2,t) = -(\frac{\alpha_2\mu}{\alpha_1})s(\boldsymbol{r}_1,t) + s(\boldsymbol{r}_2,t)$$

## Signal cancellation

$$\tilde{s}(\boldsymbol{r}_1, t) = s(\boldsymbol{r}_1, t) - (\frac{\alpha_1 \mu}{\alpha_2}) s(\boldsymbol{r}_2, t)$$

$$\tilde{s}(\boldsymbol{r}_2, t) = -(\frac{\alpha_2 \mu}{\alpha_1}) s(\boldsymbol{r}_1, t) + s(\boldsymbol{r}_2, t)$$

$$\Downarrow$$

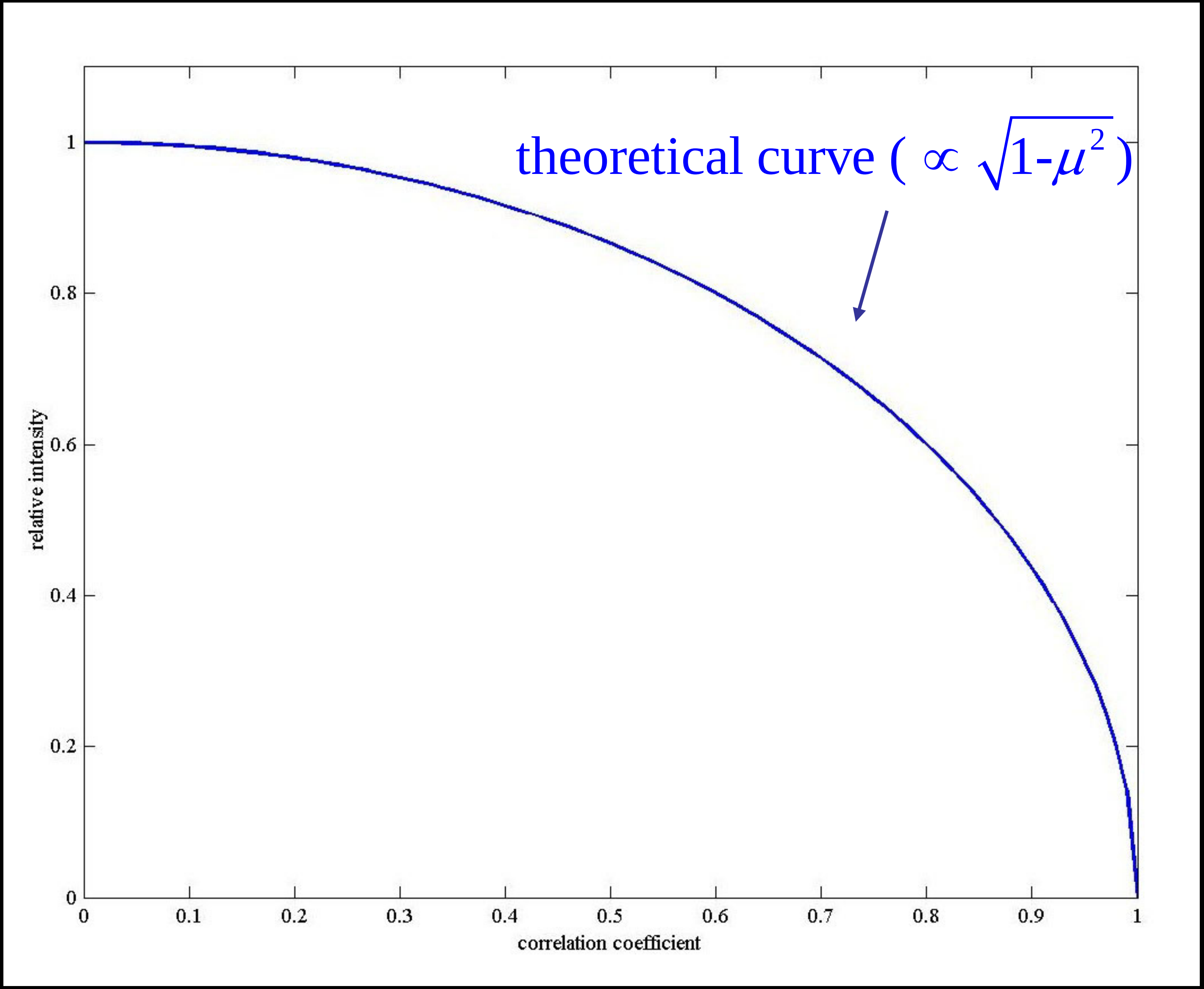
$$\left\langle \tilde{s}(\boldsymbol{r}_1, t)^2 \right\rangle = \alpha_1^2 (1 - \mu^2) = (1 - \mu^2) \left\langle s(\boldsymbol{r}_1, t)^2 \right\rangle$$

$$\left\langle \tilde{s}(\boldsymbol{r}_2, t)^2 \right\rangle = \alpha_2^2 (1 - \mu^2) = (1 - \mu^2) \left\langle s(\boldsymbol{r}_2, t)^2 \right\rangle$$

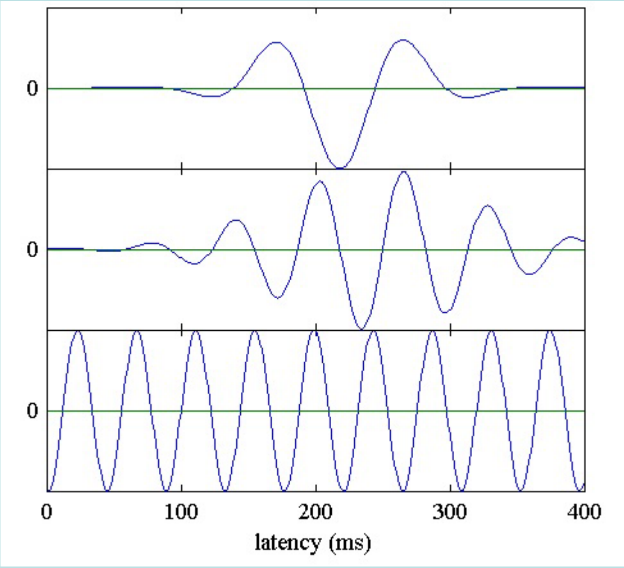
$$\Uparrow$$

Source power decreases by a factor of $(1 - \mu^2)$

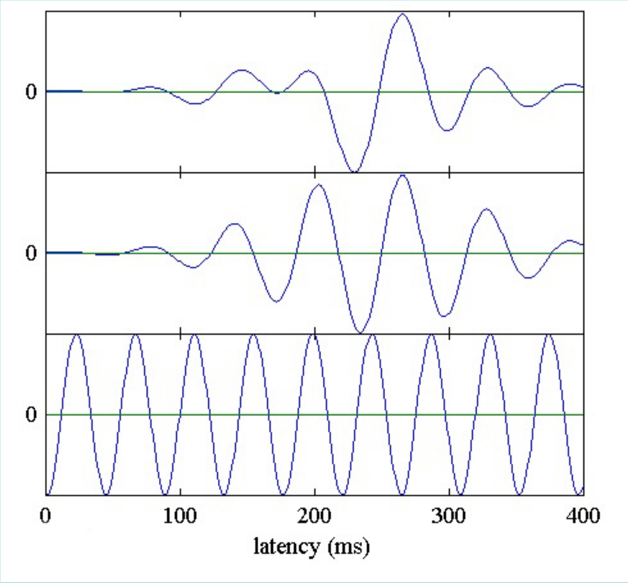
# Intensity vs. correlation



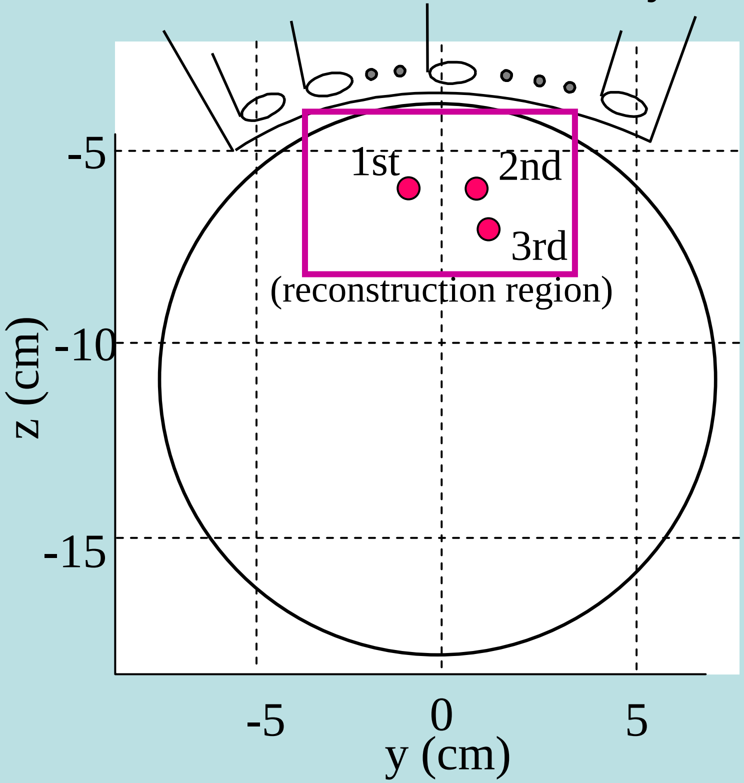
source correlation: zero

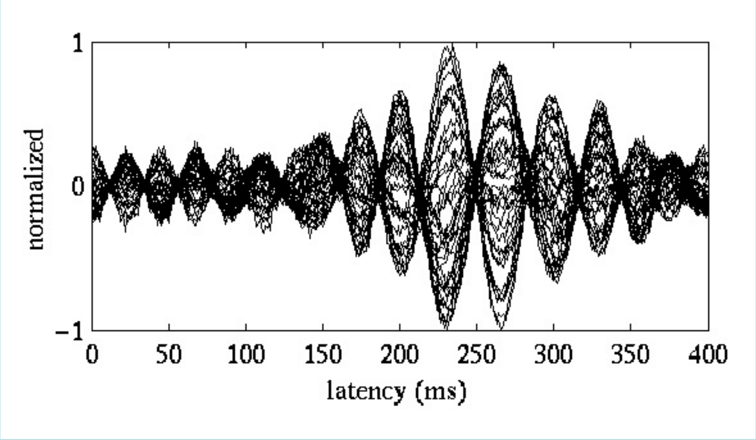



source correlation: 0.8




37-channel sensor array



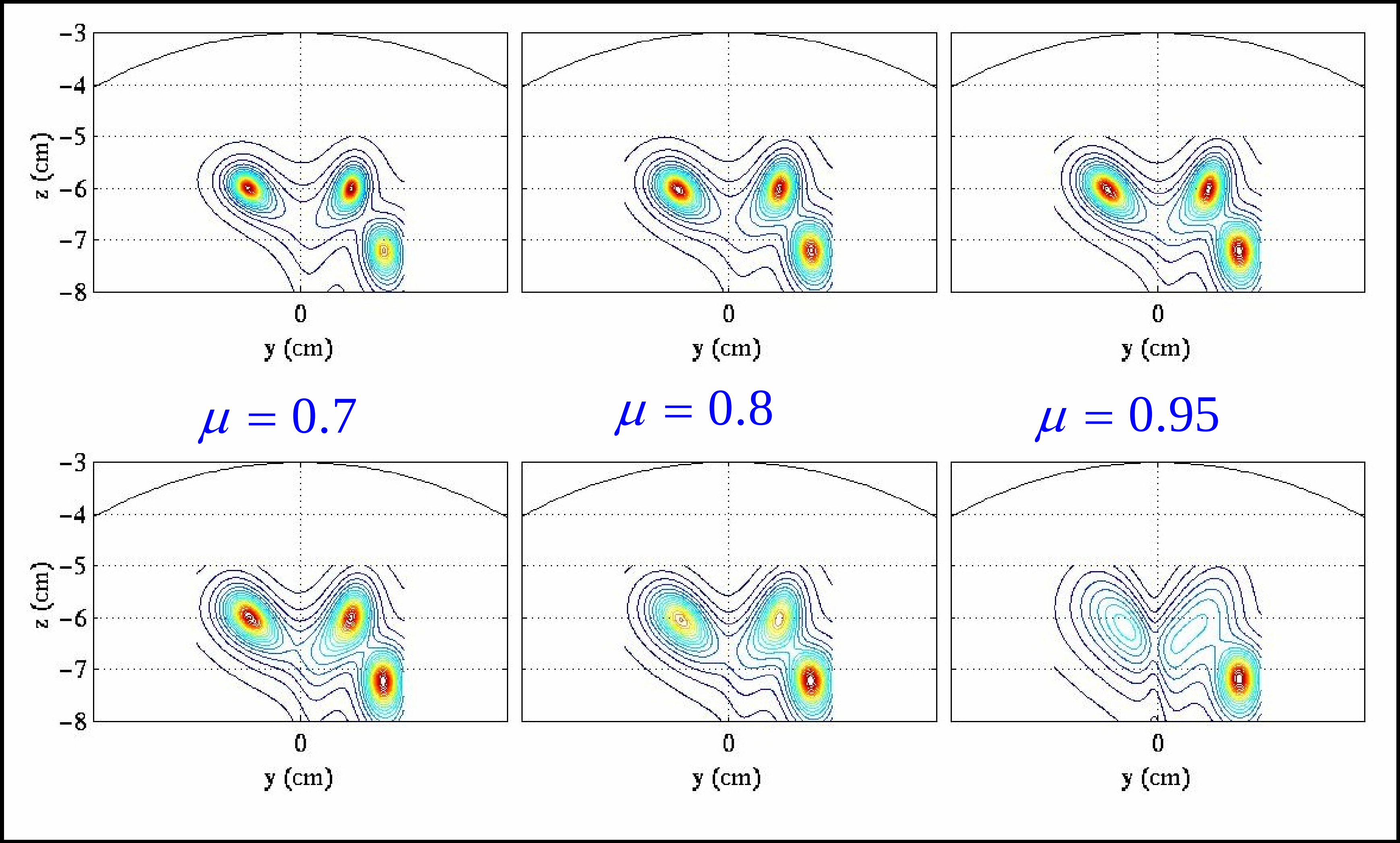




SNR=8

# Reconstruction results

$\mu = 0$ $\mu = 0.5$ $\mu = 0.6$

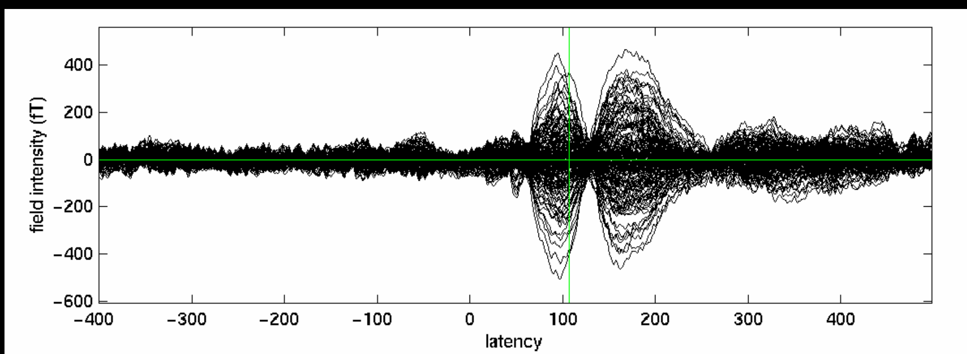
$\mu = 0.7$ $\mu = 0.8$ $\mu = 0.95$

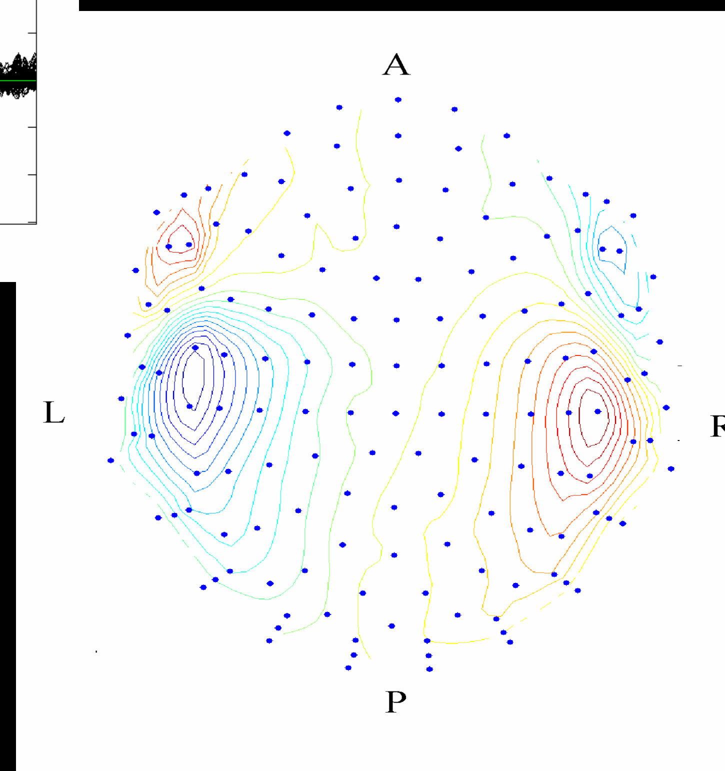


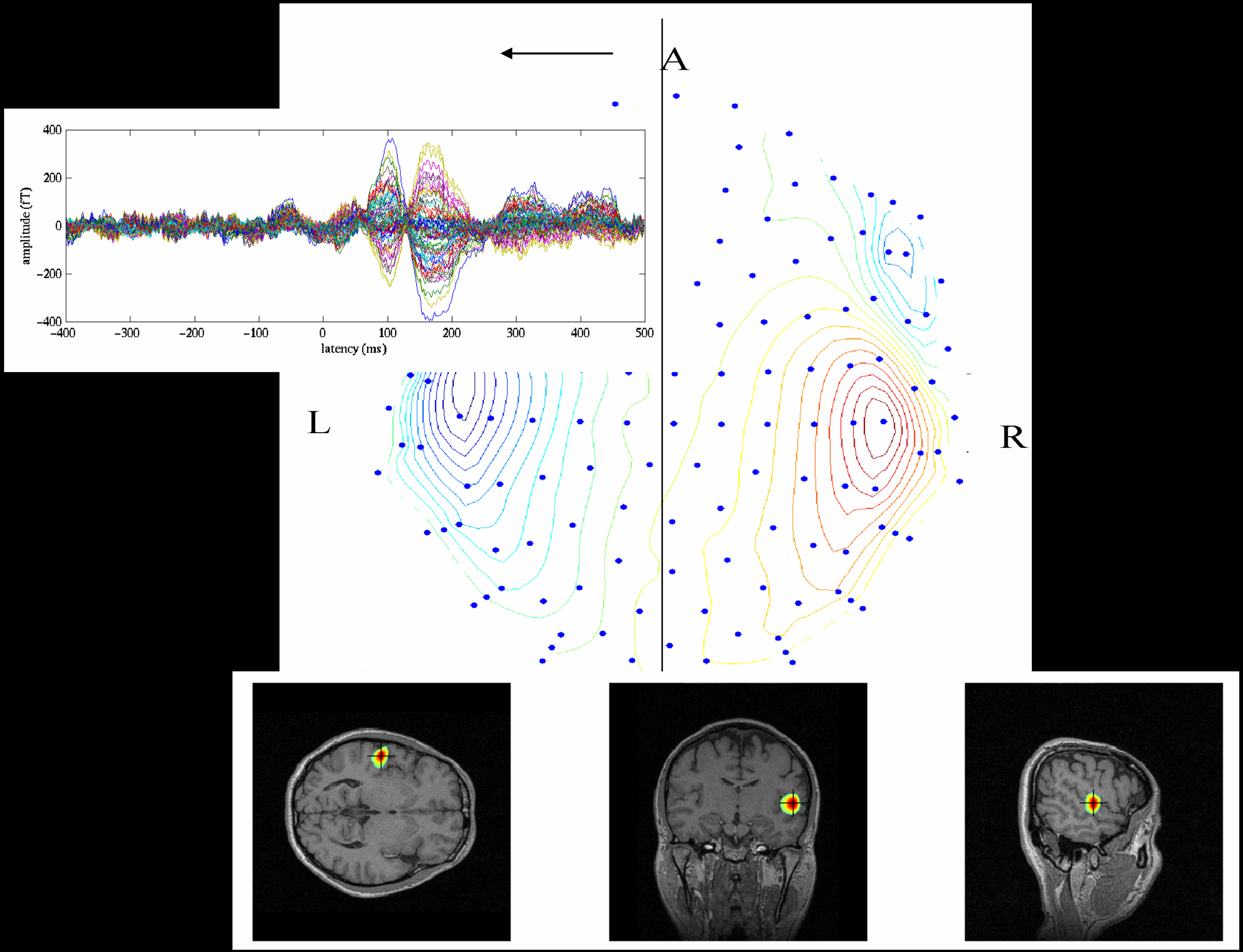
# Extreme case example

• Auditory-evoked field were measured using 148-channel whole-head sensor array (Magnes 2500).

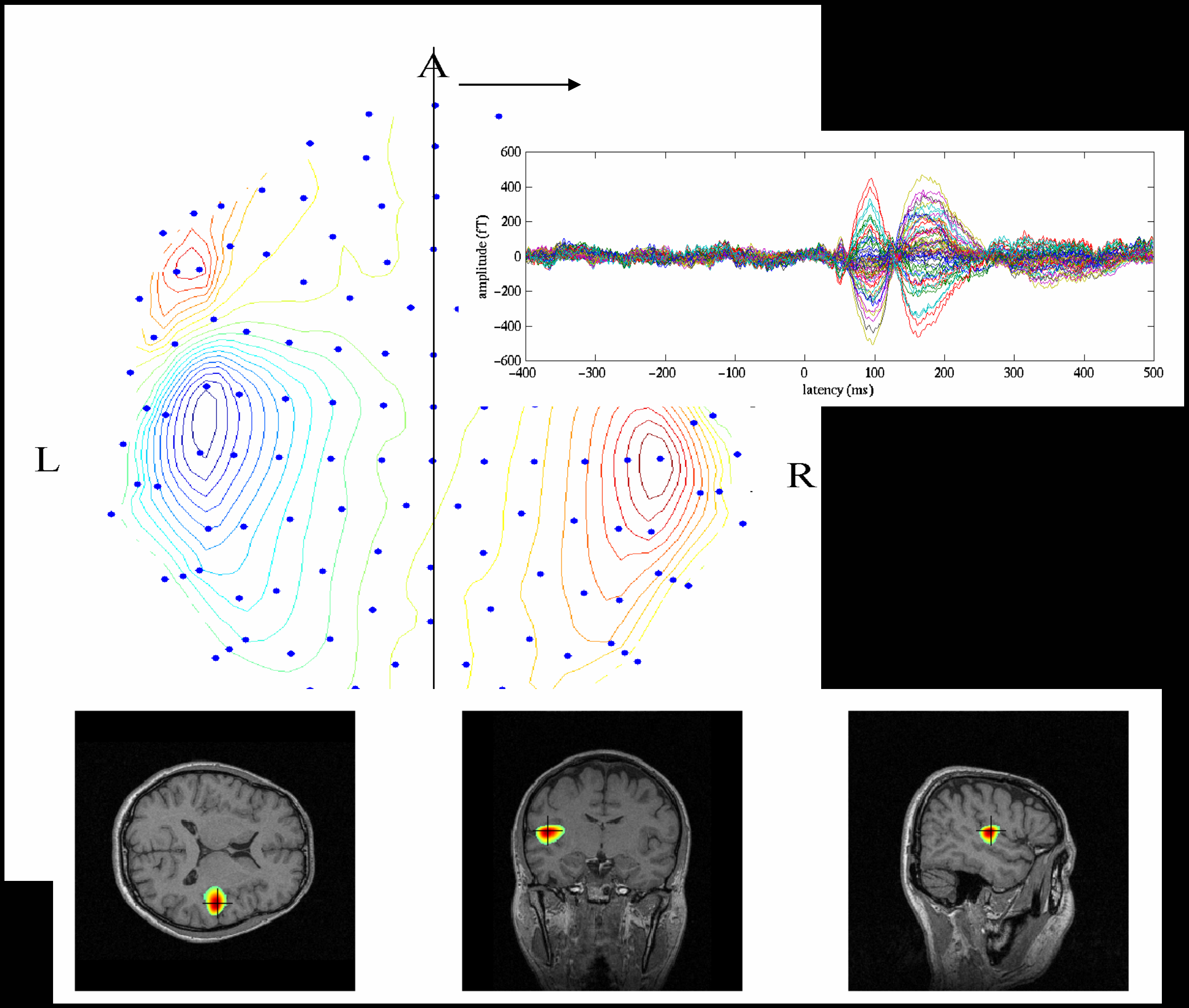
Stimulus: 1-kHz pure tone applied to subject's left ear



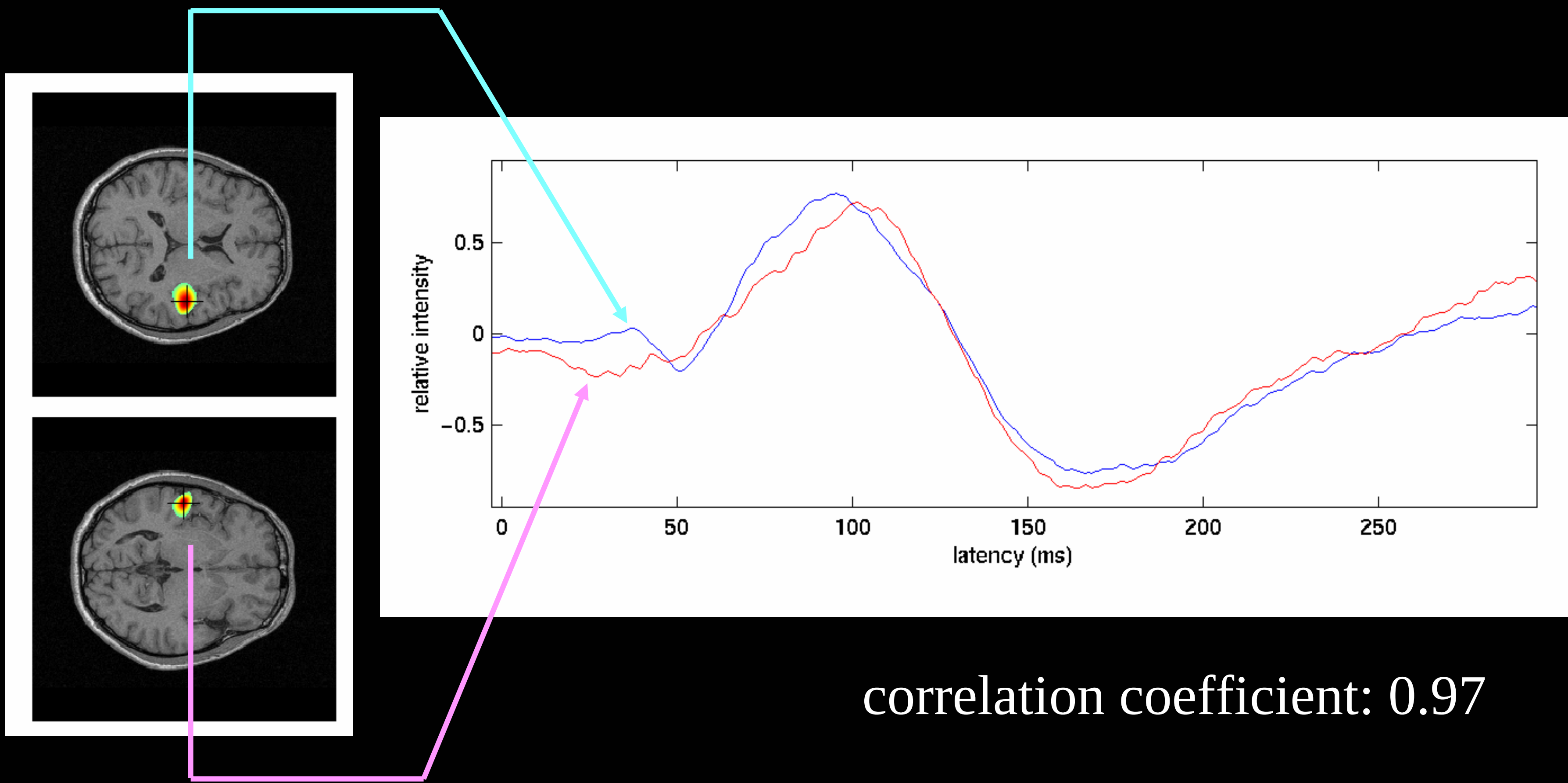



Reconstruction from left-hemisphere data only



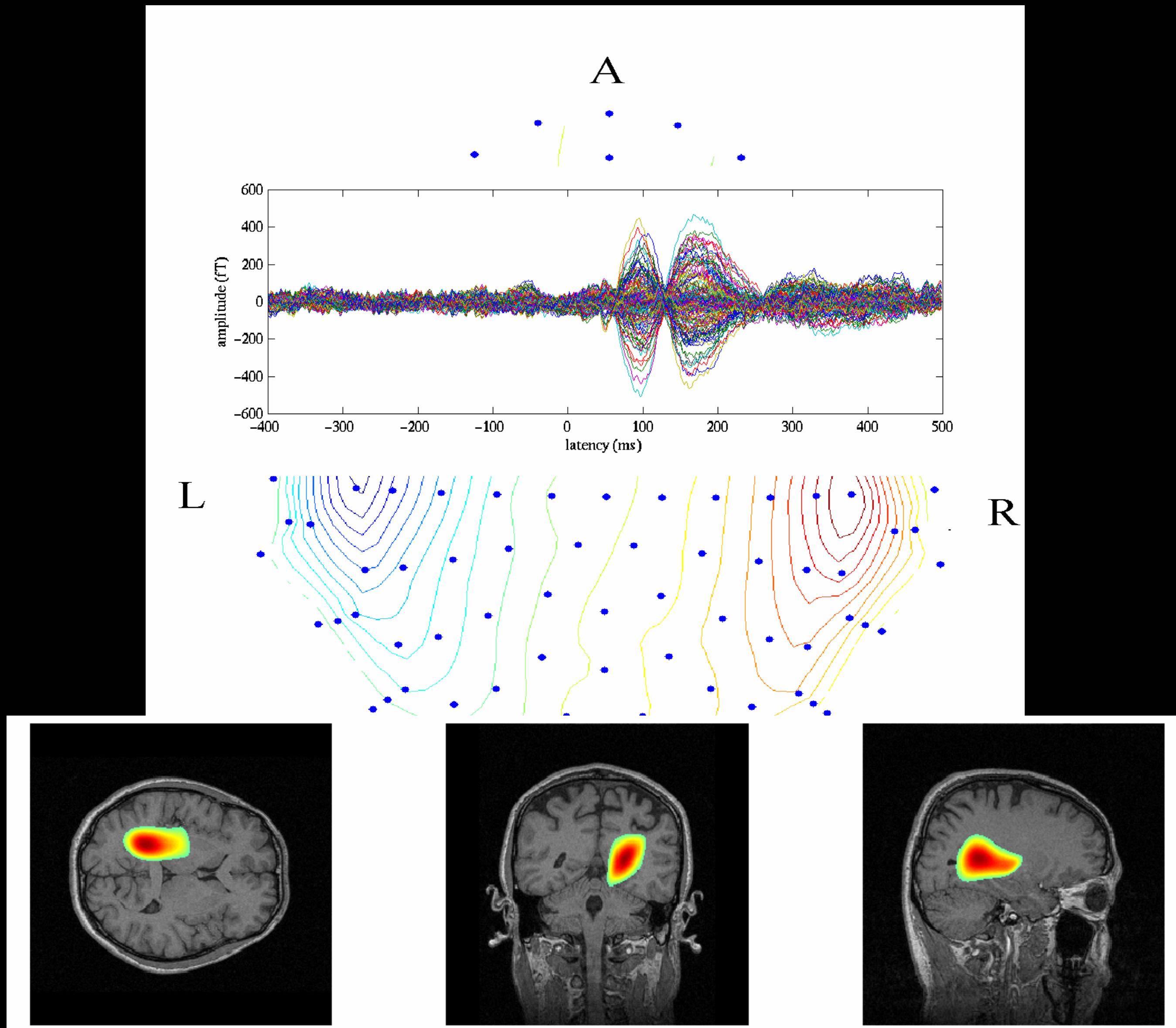

Reconstruction from right-hemisphere data only

Right auditory cortex activation




correlation coefficient: 0.97

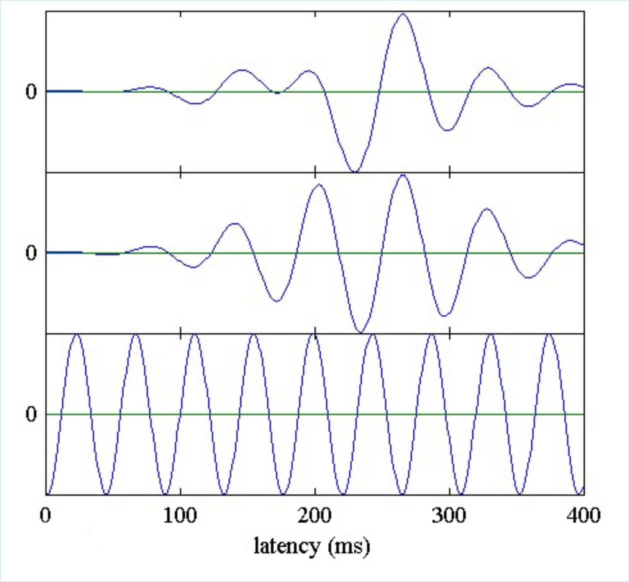
Left auditory cortex activation

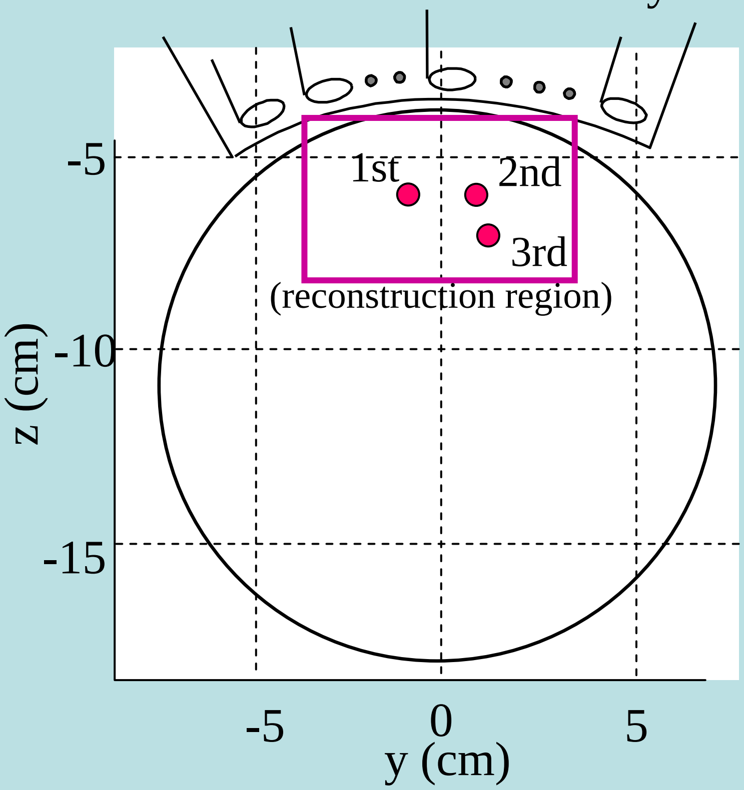


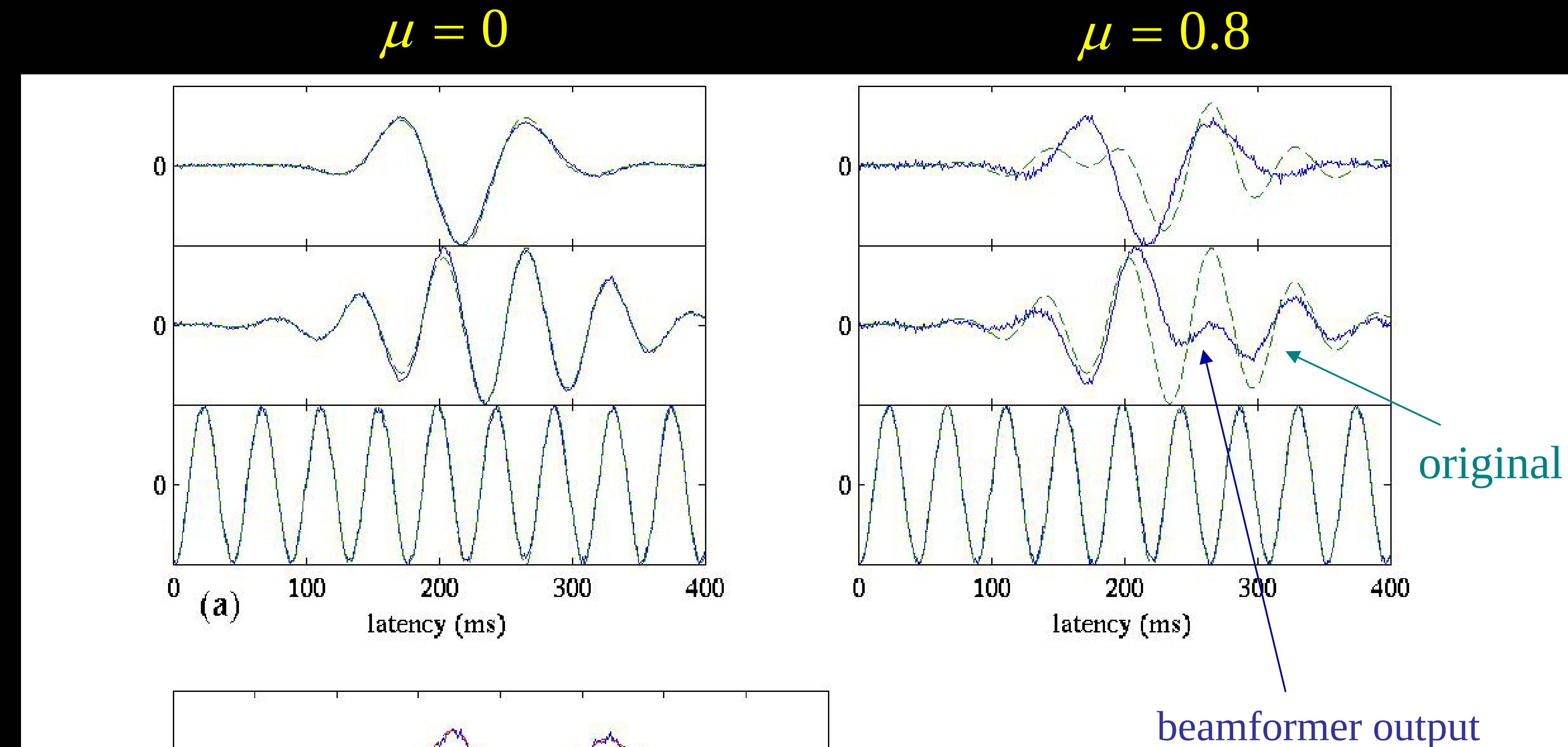

Reconstruction from all-channel data

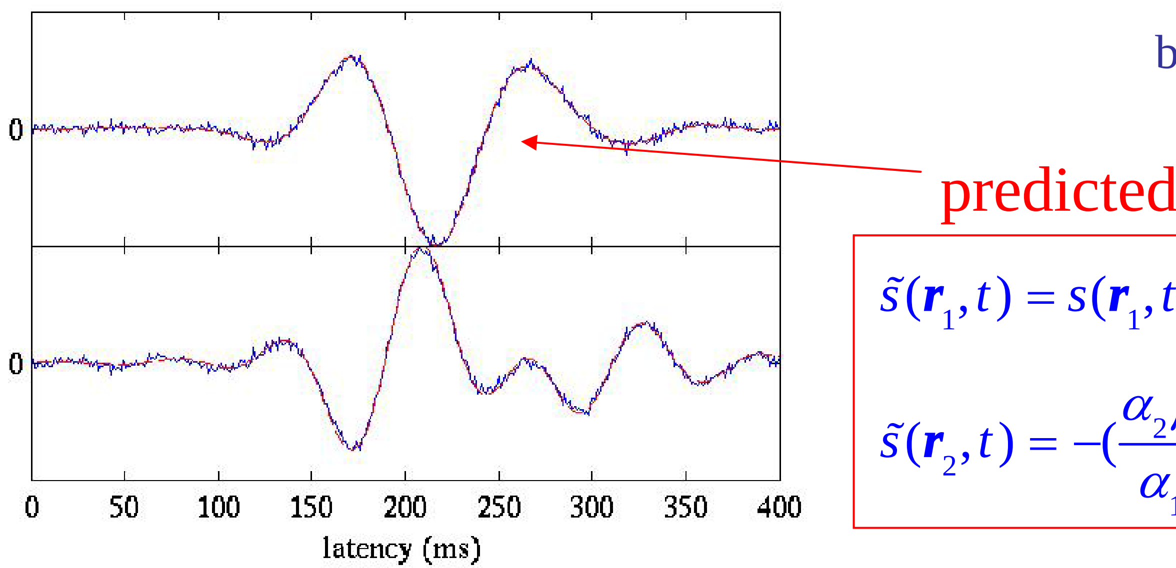
# Time course distortion

source correlation: 0.8






$$\tilde{s}(\boldsymbol{r}_1, t) = s(\boldsymbol{r}_1, t) - \left(\frac{\alpha_1 \mu}{\alpha_2}\right) s(\boldsymbol{r}_2, t)$$

$$\tilde{s}(\boldsymbol{r}_2, t) = -\left(\frac{\alpha_2 \mu}{\alpha_1}\right) s(\boldsymbol{r}_1, t) + s(\boldsymbol{r}_2, t)$$

# Time course retrieval

Two-source correlation cases

$$\begin{bmatrix} \tilde{s}(\boldsymbol{r}_1, t) \\ \tilde{s}(\boldsymbol{r}_2, t) \end{bmatrix} = \begin{bmatrix} 1 & -(\alpha_1 / \alpha_2)\mu \\ -(\alpha_2 / \alpha_1)\mu & 1 \end{bmatrix} \begin{bmatrix} s(\boldsymbol{r}_1, t) \\ s(\boldsymbol{r}_2, t) \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \hat{s}(\boldsymbol{r}_1, t) \\ \hat{s}(\boldsymbol{r}_2, t) \end{bmatrix} = \begin{bmatrix} 1 & -(\hat{\alpha}_1 / \hat{\alpha}_2)\hat{\mu} \\ -(\hat{\alpha}_2 / \hat{\alpha}_1)\hat{\mu} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \tilde{s}(\boldsymbol{r}_1, t) \\ \tilde{s}(\boldsymbol{r}_2, t) \end{bmatrix}$$

estimated time courses

output time courses

How can we estimate $(\hat{\alpha}_1 / \hat{\alpha}_2)\hat{\mu}$ and $(\hat{\alpha}_2 / \hat{\alpha}_1)\hat{\mu}$?

# Interesting results

$$\hat{\mu} = \left| \frac{\left\langle \tilde{s}(\boldsymbol{r}_1, t)\tilde{s}(\boldsymbol{r}_2, t) \right\rangle}{\sqrt{\left\langle \tilde{s}(\boldsymbol{r}_1, t)^2 \right\rangle \left\langle \tilde{s}(\boldsymbol{r}_2, t)^2 \right\rangle}} \right|$$

$$\Downarrow$$

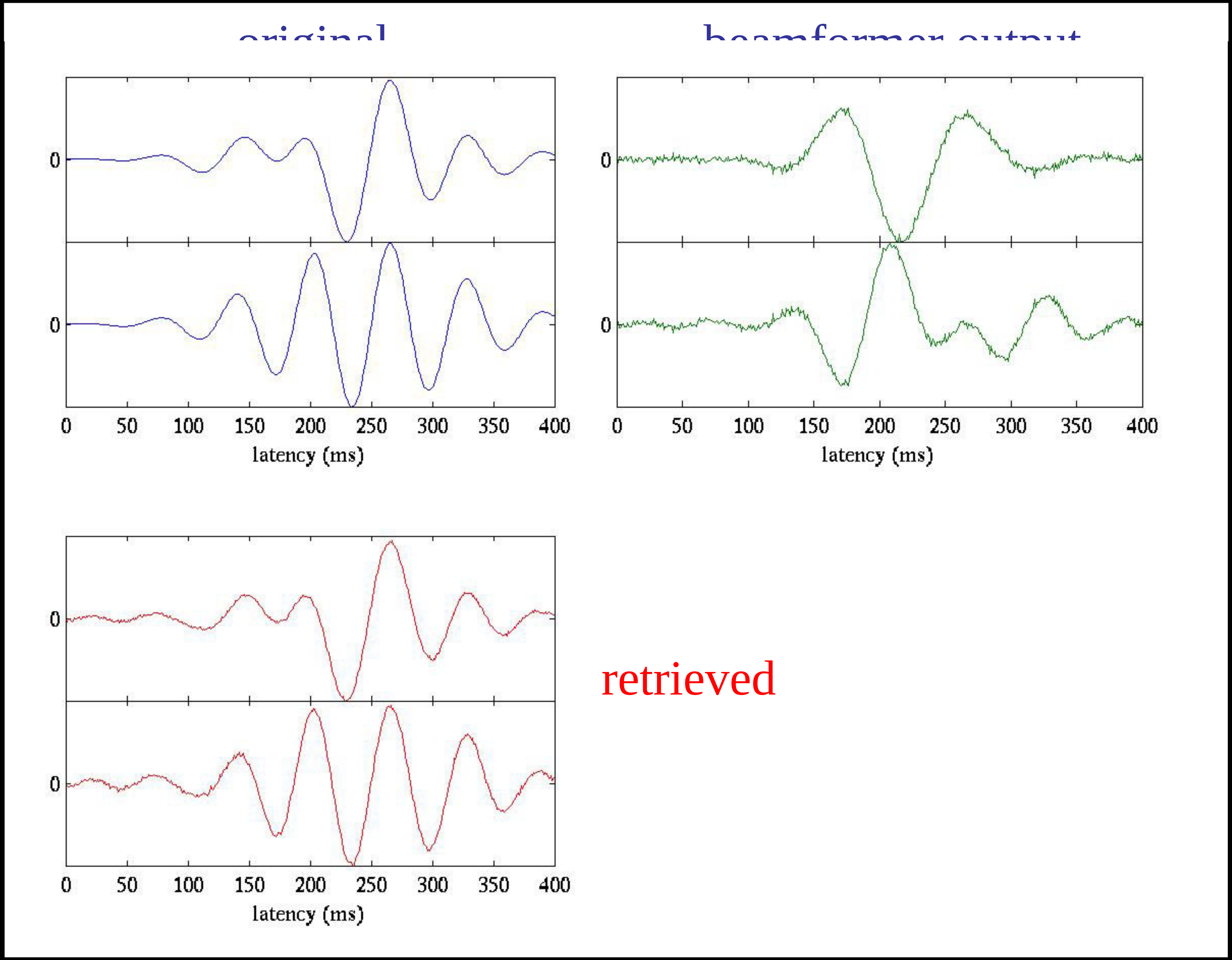
$$\hat{\mu} = \frac{\left| \alpha_1 \alpha_2 (\mu^3 - \mu) \right|}{\sqrt{\alpha_1^2 (1 - \mu^2) \alpha_2^2 (1 - \mu^2)}} = |\mu|$$

Magnitude correlation coefficient can be calculated directly using the beamformer outputs.

Then, with using $(\hat{\alpha}_1 / \hat{\alpha}_2) = \sqrt{\left\langle \tilde{s}(\boldsymbol{r}_1, t)^2 \right\rangle / \left\langle \tilde{s}(\boldsymbol{r}_1, t)^2 \right\rangle}$

we can estimate $\begin{bmatrix} 1 & -(\hat{\alpha}_1 / \hat{\alpha}_2)\hat{\mu} \\ -(\hat{\alpha}_2 / \hat{\alpha}_1)\hat{\mu} & 1 \end{bmatrix}^{-1}$

# Time course retrieval experiments for two correlated sources



# Summary for influence of source correlation

- The signal cancellation is not significant when the source correlation is not very high ($\mu < 0.7$). However, when the correlation is very high, the results may be very erroneous.
- The beamformer time-course output may be erroneous even for medium degree of source correlation.
- A method is developed for retrieving the original time courses when the number of major correlated sources is two.

# Influence of Various Types of Noise

## Noise in measurements

signal source of interest

sensor noise

$$\boldsymbol{b}(t) = \sum_{q=1}^{Q} \boldsymbol{L}(\boldsymbol{r}_q)\boldsymbol{s}(\boldsymbol{r}_q, t) + \sum_{k=1}^{K} \boldsymbol{L}(\boldsymbol{r}_k)\boldsymbol{\xi}(\boldsymbol{r}_k, t) + \boldsymbol{d}(t) + \boldsymbol{n}(t)$$

neurophysiological noise

external disturbances

# Sensor noise

- can be modeled by white Gaussian noise
- uncorrelated among sensor channels

sensor noise

$$\boldsymbol{b}(t) = \sum_{q=1}^{Q} \boldsymbol{L}(\boldsymbol{r}_q)\boldsymbol{s}(\boldsymbol{r}_q, t) + \boldsymbol{n}(t)$$

Sensor noise causes the spatial resolution degradation.

Resolution kernel: $\hat{s}(\boldsymbol{r}) = \int \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}')s(\boldsymbol{r}')d\boldsymbol{r}'$

When a single source exists at $\boldsymbol{r}_1$ ,

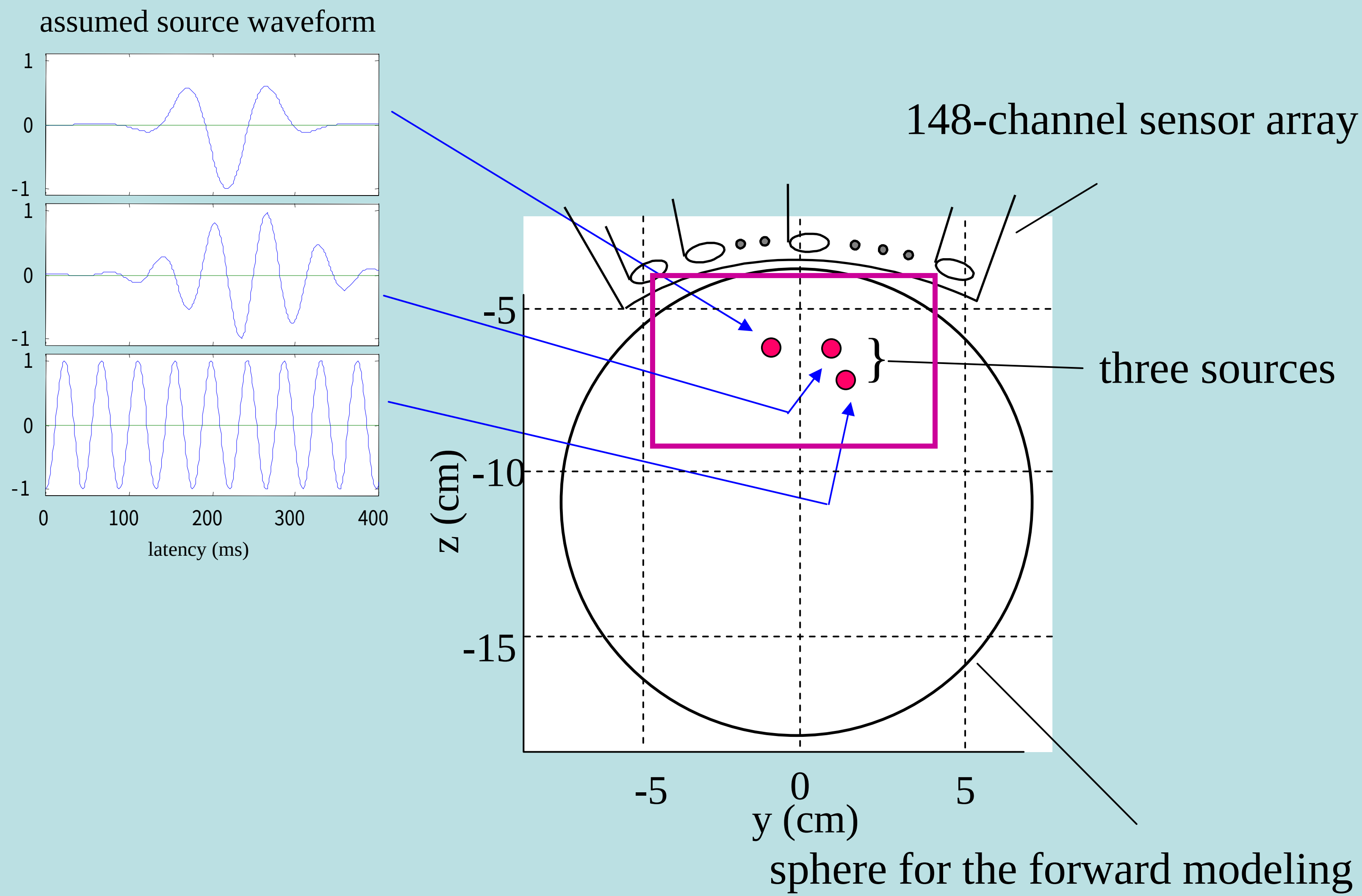
$$\mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1) = \frac{\|\boldsymbol{l}(\boldsymbol{r}_1)\|}{\|\boldsymbol{l}(\boldsymbol{r})\|} \frac{\cos[\boldsymbol{l}(\boldsymbol{r}), \boldsymbol{l}(\boldsymbol{r}_1)]}{[1 + (\mathrm{SNR})\sin^2[\boldsymbol{l}(\boldsymbol{r}), \boldsymbol{l}(\boldsymbol{r}_1)]]},$$

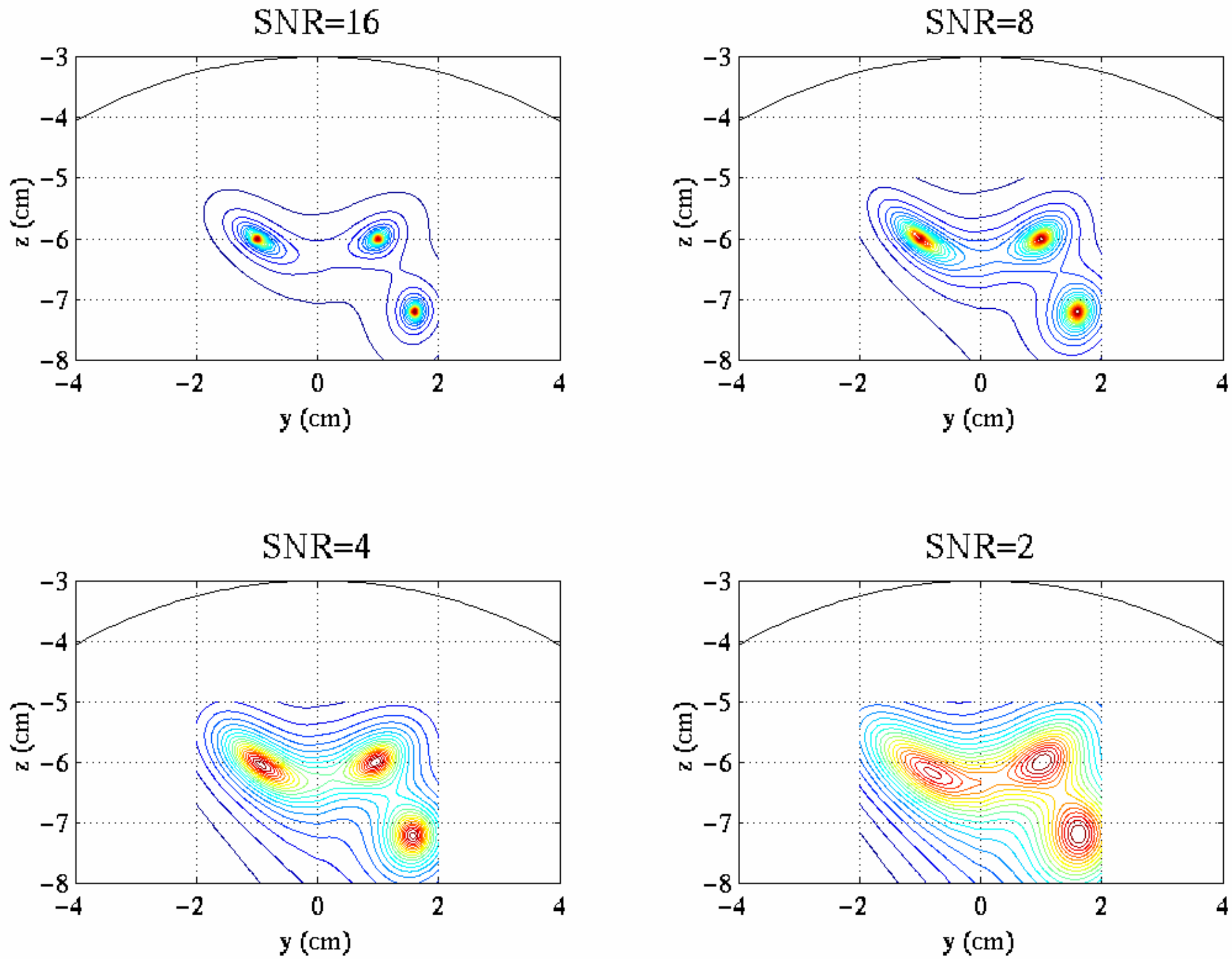
$\Uparrow$

Input SNR

where $\cos^2(\boldsymbol{a}, \boldsymbol{b}) = \left|\boldsymbol{a}^T \boldsymbol{b}\right|^2 \Big/ [(\boldsymbol{a}^T \boldsymbol{a})(\boldsymbol{b}^T \boldsymbol{b})]$,

$$\sin^2(\boldsymbol{a}, \boldsymbol{b}) = 1 - \cos^2(\boldsymbol{a}, \boldsymbol{b})$$


assumed source waveform
1
0
-1
1
0
-1
1
0
-1
0
100
200
300
400
latency (ms)
148-channel sensor array
three sources
-5
-10
-15
z (cm)
-5
0
5
y (cm)
sphere for the forward modeling


SNR=16
SNR=8
SNR=4
SNR=2
z (cm)
y (cm)
−3
−4
−5
−6
−7
−8
−4
−2
0
2
4

# External disturbances

$$\boldsymbol{b}(t) = \sum_{q=1}^{Q} \boldsymbol{L}(\boldsymbol{r}_q)\boldsymbol{s}(\boldsymbol{r}_q, t) + \boldsymbol{d}(t) + \boldsymbol{n}(t)$$

$\boldsymbol{d}(t)$ may includes:

- power-line interferences
- Base-line drift
- Artifacts from electrical appliances

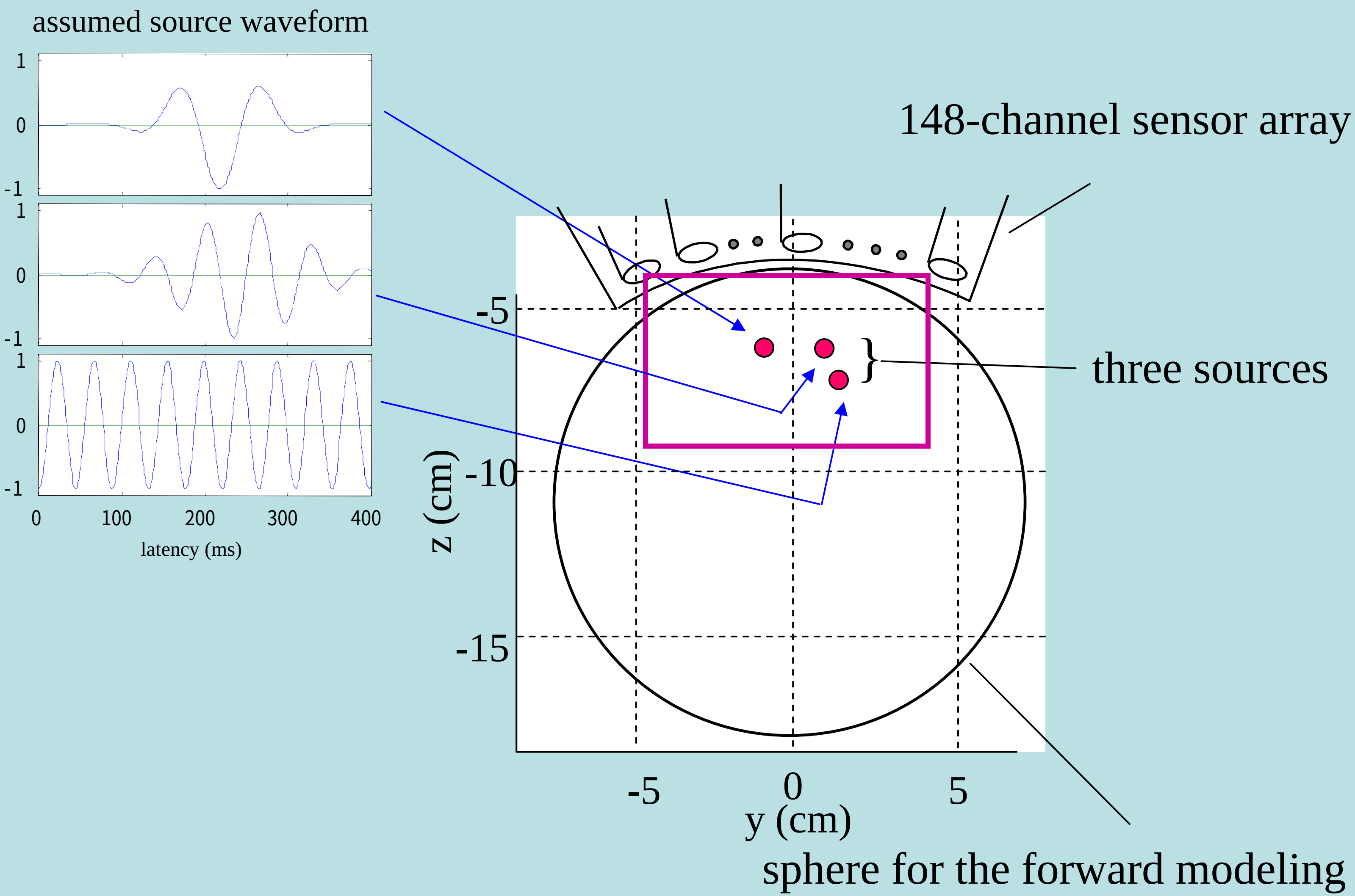
$\Downarrow$

Low rank

Their spatio-temporal activities have small number of significantly large eigenvalues.

# Simulated disturbances

- Case1: Recordings from right hemisphere channels (total 60 channels) contain the same periodic noise.
- Case2: All channel recordings have uniform linear trends
- Case3: Each channel has its own linear trend different to each other



assumed source waveform
1
0
-1
1
0
-1
1
0
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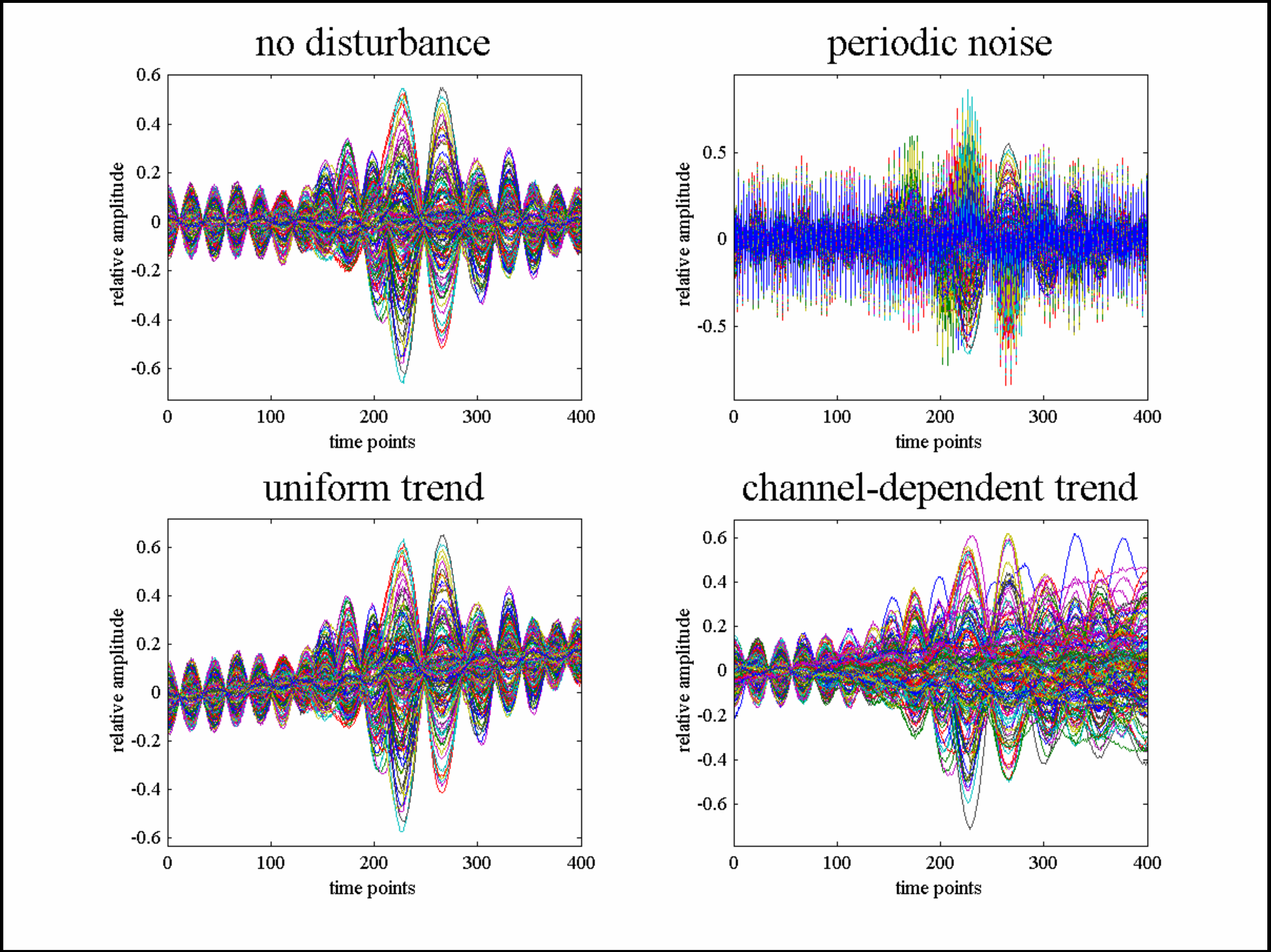
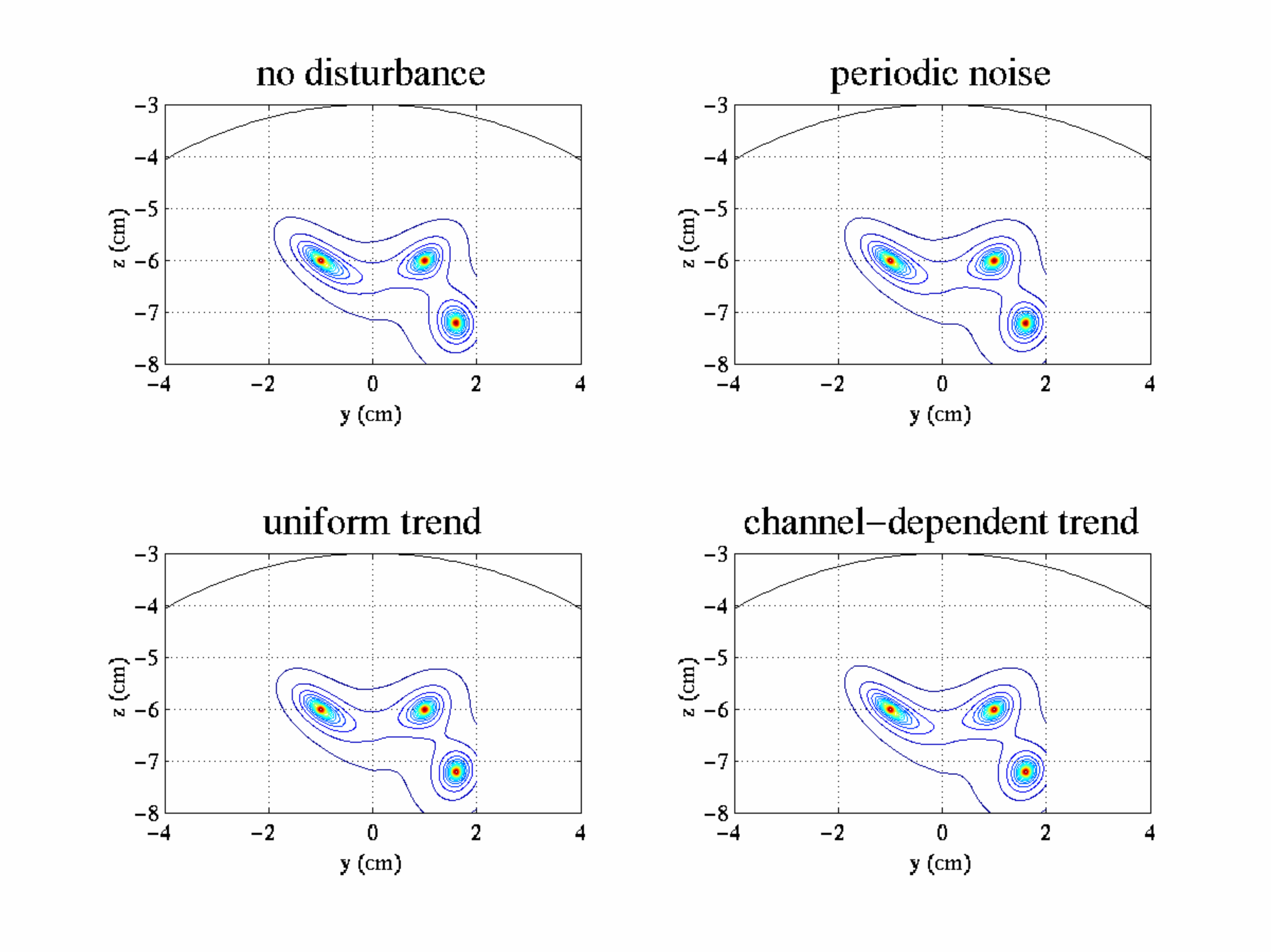
0
100
200
300
400
latency (ms)
148-channel sensor array
three sources
-5
-10
-15
z (cm)
-5
0
5
y (cm)
sphere for the forward modeling

# Simulated recordings
# Signal to sensor noise ratio: 16



# Minimum-variance spatial filter reconstruction results
# Signal to sensor noise ratio: 16



# Low-rank disturbance

signal source activities　　disturbance

$$\boldsymbol{b}(t) = \sum_{q=1}^{Q} \boldsymbol{L}(\boldsymbol{r}_q)\boldsymbol{s}(\boldsymbol{r}_q, t) + \boldsymbol{n}(t) + \boldsymbol{d}(t)$$

Assume no correlation between $\boldsymbol{s}(\boldsymbol{r}_q, t)$ and $\boldsymbol{d}(t)$

## Covariance matrices

from $\sum_{q=1}^{Q} \boldsymbol{L}(\boldsymbol{r}_q)\boldsymbol{s}(\boldsymbol{r}_q, t) + \boldsymbol{n}(t)$

$$\boldsymbol{R} = \boldsymbol{R}_B + \boldsymbol{R}_D$$

from $\boldsymbol{b}(t)$

from $(t)$ : $\ _D = \left\langle \ (t) \ ^T(t) \right\rangle$

When $\boldsymbol{R}_D$ is a rank one matrix,

$$\boldsymbol{R}_D = \lambda \boldsymbol{u}\boldsymbol{u}^T$$

$$\Downarrow$$

We can derive,

$$\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}_B^{-1}\boldsymbol{l}(\boldsymbol{r})\left[1 - \cos^2(\boldsymbol{l}, \boldsymbol{u} \mid \boldsymbol{R}_B^{-1})\right]$$

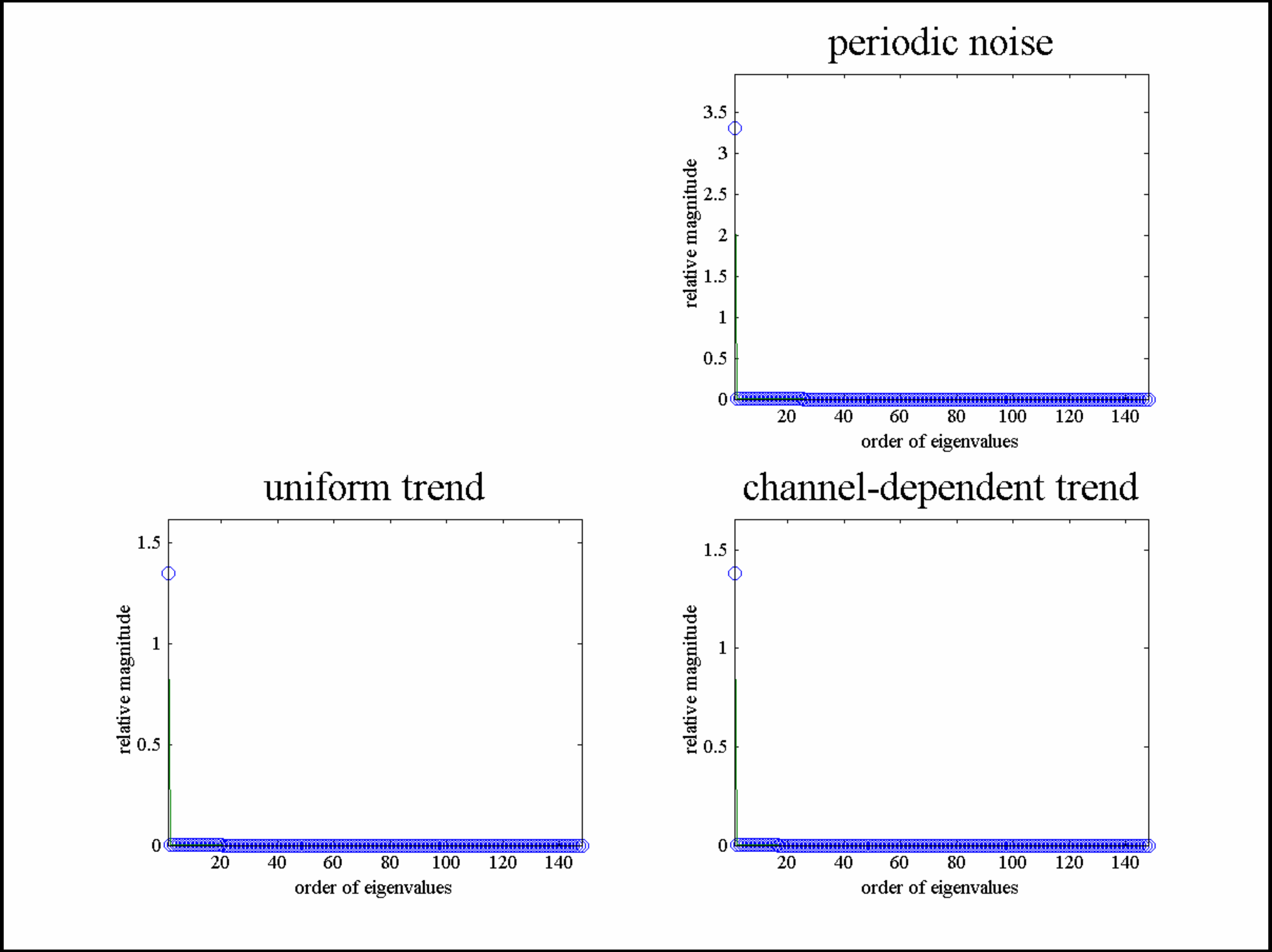
$$\Uparrow$$

generalized cosine between $\boldsymbol{l}$ and $\boldsymbol{u}$ with the metric $\boldsymbol{R}_B^{-1}$

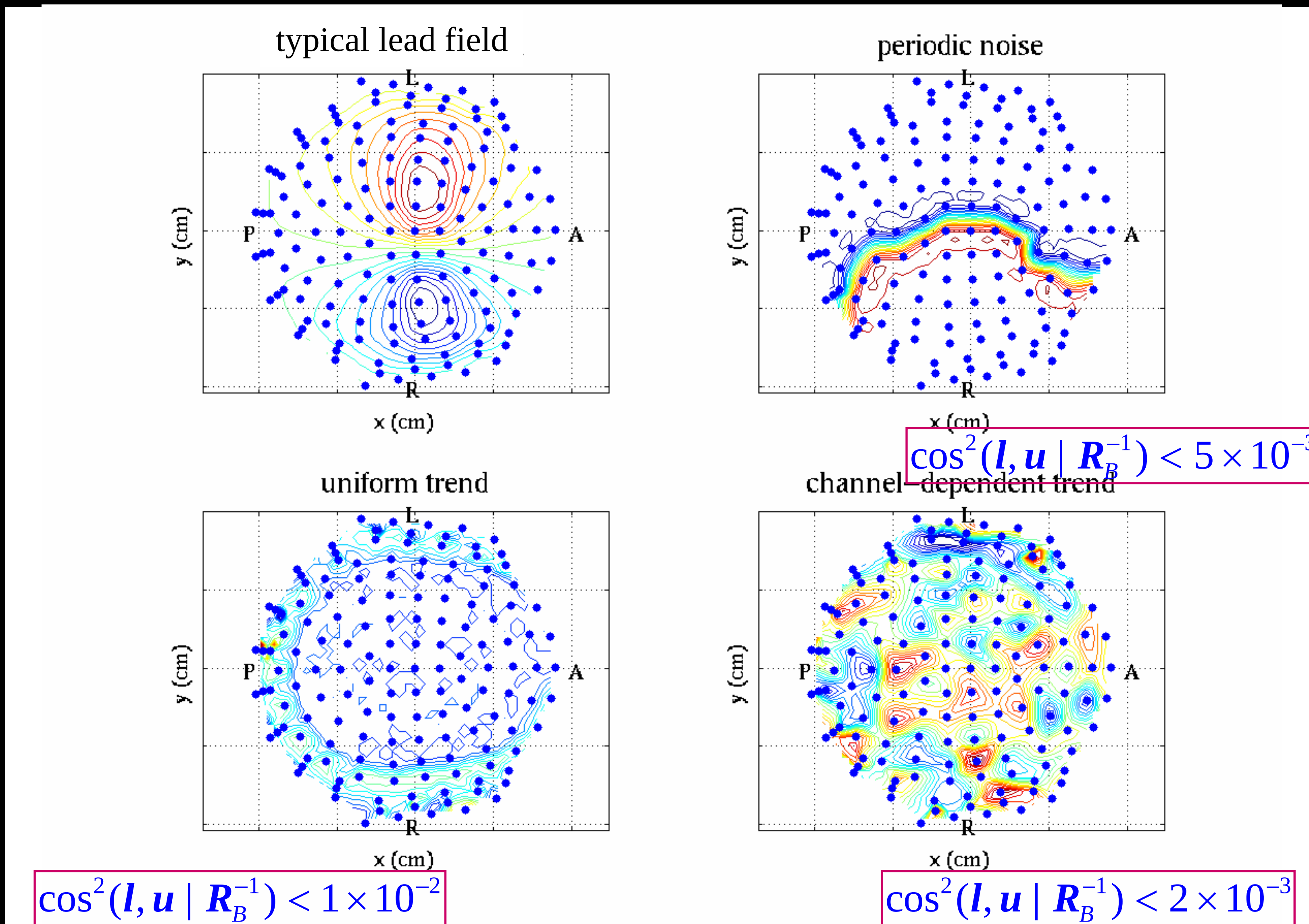
When $\boldsymbol{l}$ and $\boldsymbol{u}$ are very different, $\cos^2(\boldsymbol{l}, \boldsymbol{u} \mid \boldsymbol{R}_B^{-1}) \ll 1$, and

$$\boxed{\left\langle s(\boldsymbol{r})^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})} \approx \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}_B^{-1}\boldsymbol{l}(\boldsymbol{r})}}$$

# Eigenspectrum of $\boldsymbol{R}_D$



# Visualization of the first eigenvector of the disturbances




$\cos^2(\boldsymbol{l}, \boldsymbol{u} \mid \boldsymbol{R}_B^{-1}) < 5 \times 10^{-3}$

$\cos^2(\boldsymbol{l}, \boldsymbol{u} \mid \boldsymbol{R}_B^{-1}) < 1 \times 10^{-2}$

$\cos^2(\boldsymbol{l}, \boldsymbol{u} \mid \boldsymbol{R}_B^{-1}) < 2 \times 10^{-3}$

# Neurophysiological noise

$$\boldsymbol{b}(t) = \sum_{q=1}^{Q} \boldsymbol{L}(\boldsymbol{r}_q)\boldsymbol{s}(\boldsymbol{r}_q, t) + \sum_{k=1}^{K} \boldsymbol{L}(\boldsymbol{r}_k)\boldsymbol{\xi}(\boldsymbol{r}_k, t) + \boldsymbol{n}(t)$$

closely related to the resting state of the brain or the default mode of brain activities.

Neurophysiological noise can be modeled by randomly distributed incoherent dipoles.


•de Munck et al., IEEE Trans. Biomed. Eng., 39, 791-804, 1992.
•Valdes et al., Brain Topography, 4, 309-319, 1992.
•Lutkenhoner, J. Appl. Phys., 75, 7204-7210, 1994.

This type of noise may invalidate the low-rank signal assumption;

Number of sensors $M$ > Number of sources $P$

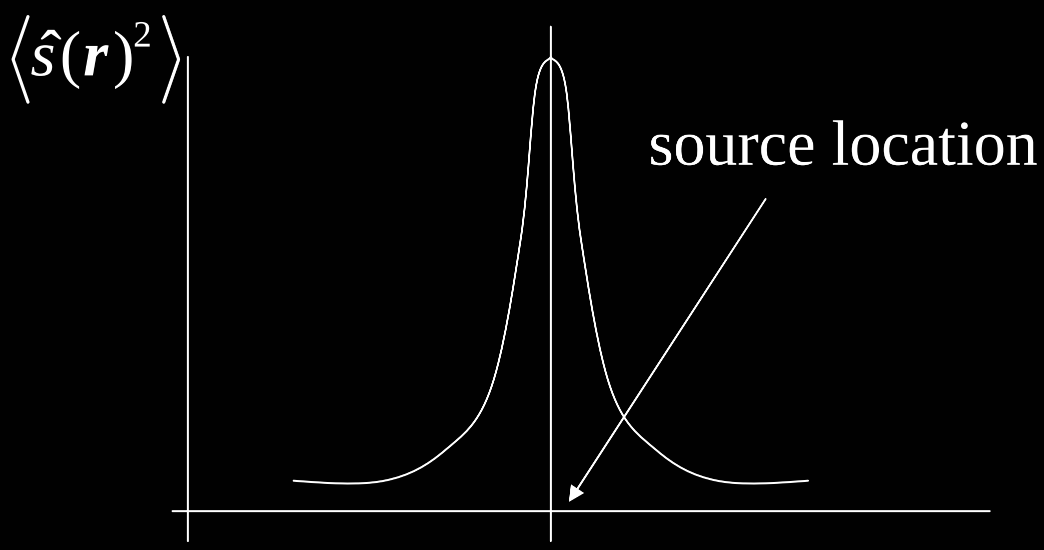
Even when the low-rank signal assumption is satisfied, the size of noise subspace affects the spatial resolution of the reconstructed results.

# Recall the orthogonality principle

$$\boxed{\boldsymbol{E}_N^T \boldsymbol{l}(\boldsymbol{r}_q) = \boldsymbol{\Gamma}_N^{-1} \boldsymbol{l}(\boldsymbol{r}_q) = 0 \text{ at any source location } \boldsymbol{r}_q}$$

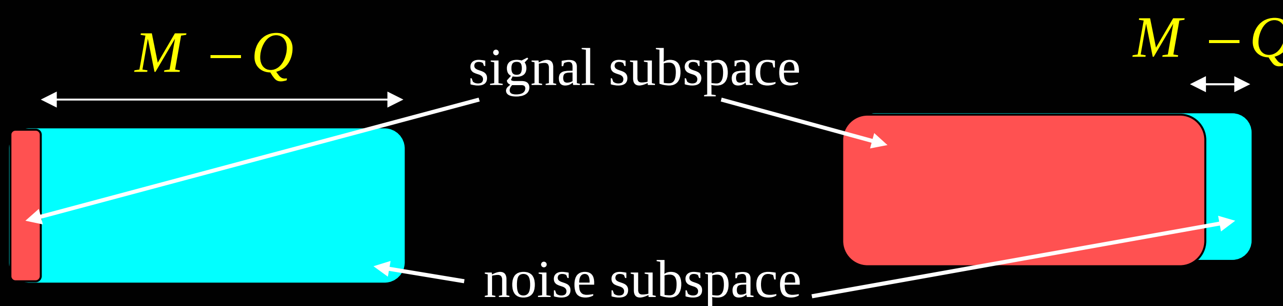
Minimum-variance spatial filter output:

$$\left\langle \hat{s}(\boldsymbol{r})^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})} = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S^{-1}\boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r})}$$




small $\leftarrow \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r}) \rightarrow$ large

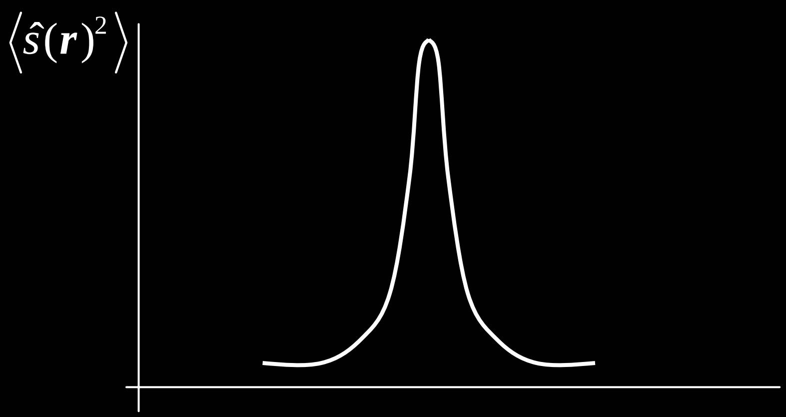
$$\boxed{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r}) \approx \sum_{j=Q+1}^{M} \left\| \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{e}_j \right\|^2 / \lambda_j^2}$$



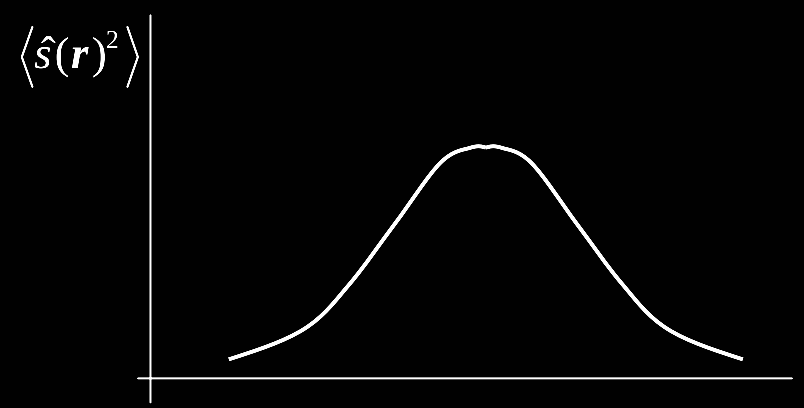

When $\boldsymbol{r} \neq$ source location,

$$\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r}) \qquad > \qquad \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r})$$

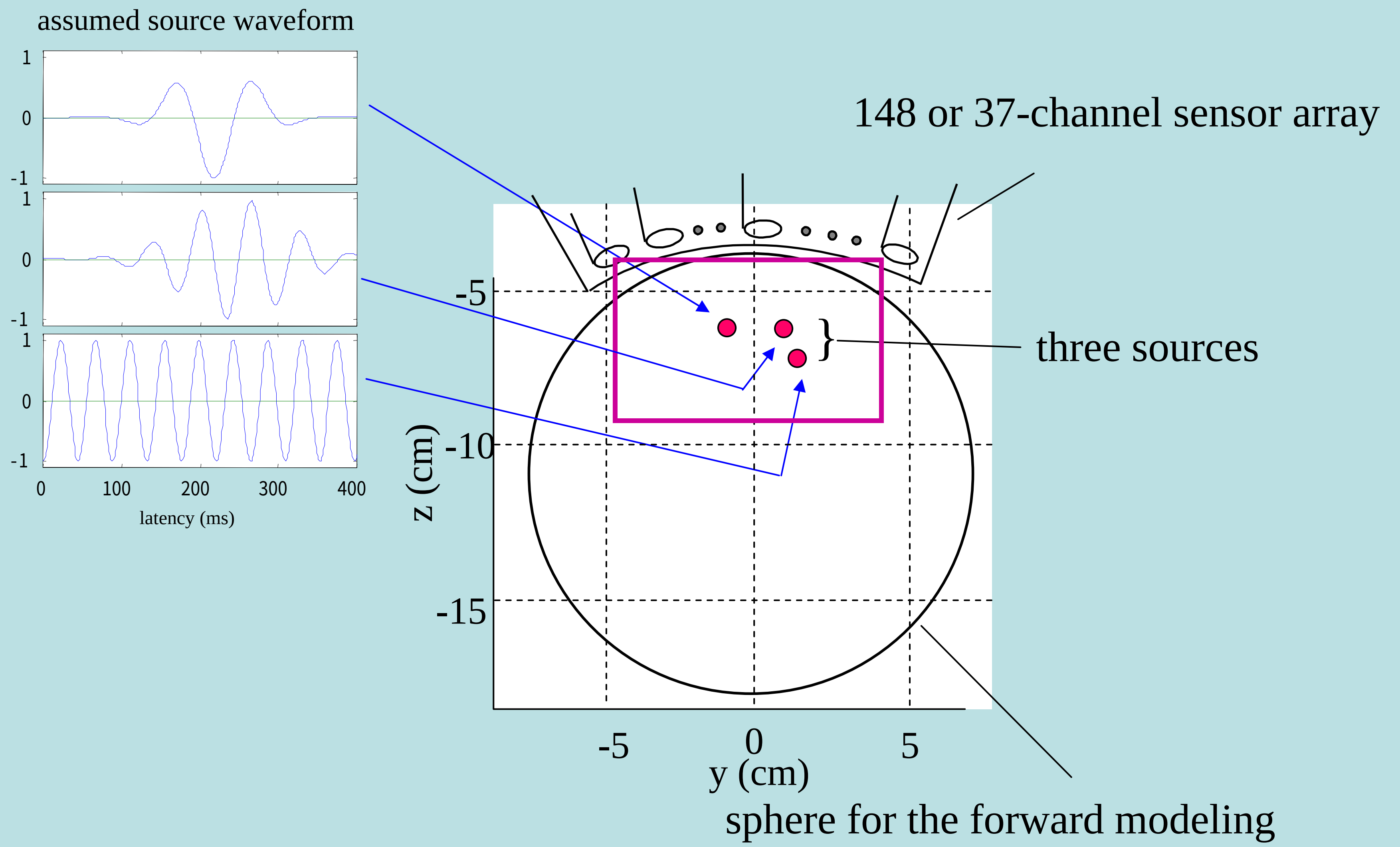
high spatial resolution




low spatial resolution




The spatial resolution depends on the size of the noise subspace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latency (ms)
148 or 37-channel sensor array
three sources
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-15
z (cm)
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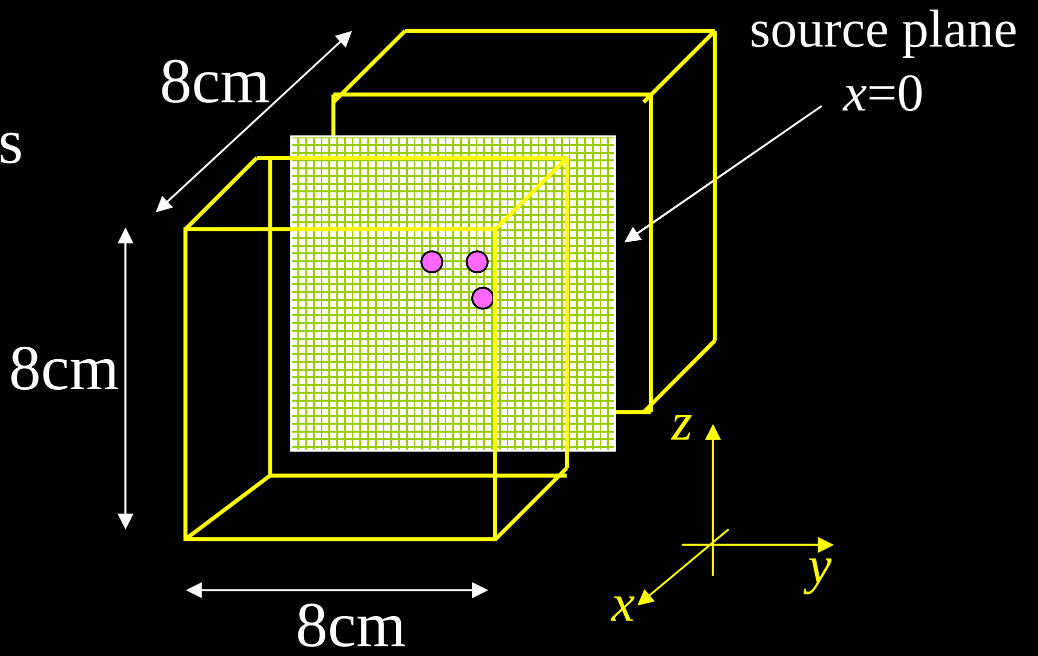
y (cm)
sphere for the forward modeling

Generate many random dipoles in a volume:

$$-4 < x < -1, \quad 1 < x < 4$$
$$-4 < y < 4$$
$$-10 < z < -2$$



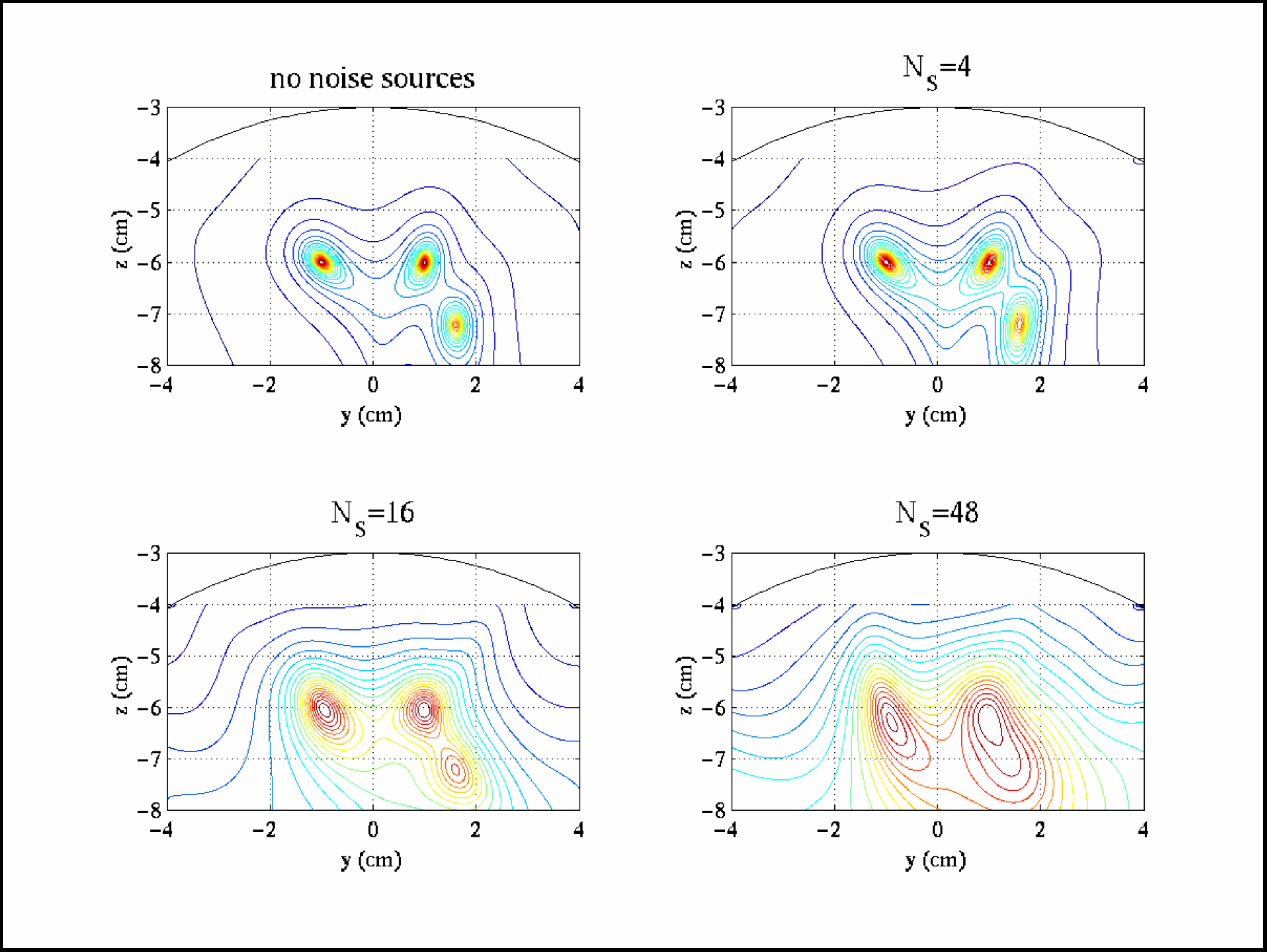

$N_S$ : Number of noise random dipoles

$P_N$ : The power of noise dipoles is fixed at

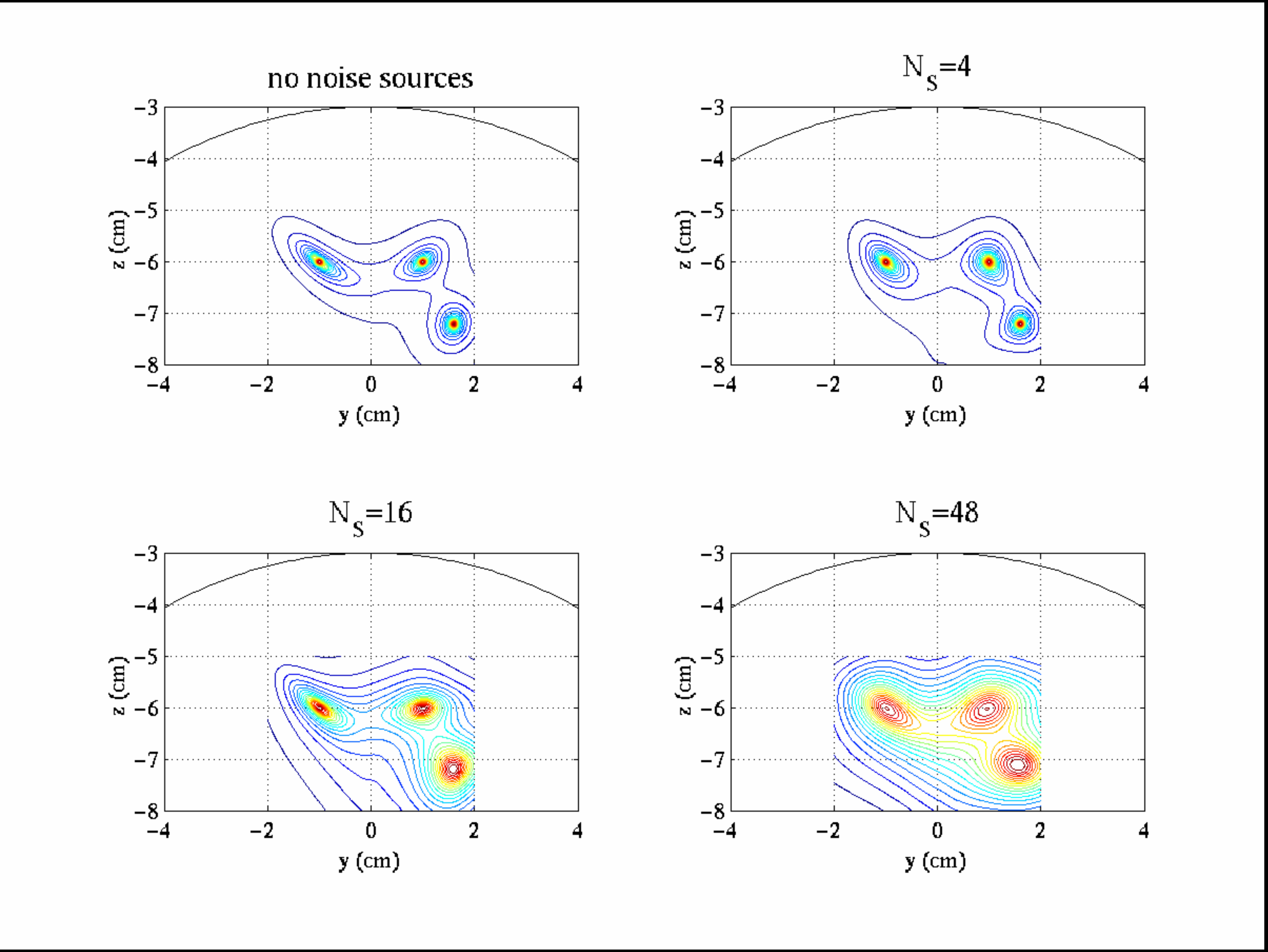
$P_N = 0.1P_3$ where $P_3$ is the power of the third source ($P_1 = P_2 = 1.2P_3$).

Time courses of the noise sources are incoherent to each other.

# 37-channel sensors used (*M*=37)

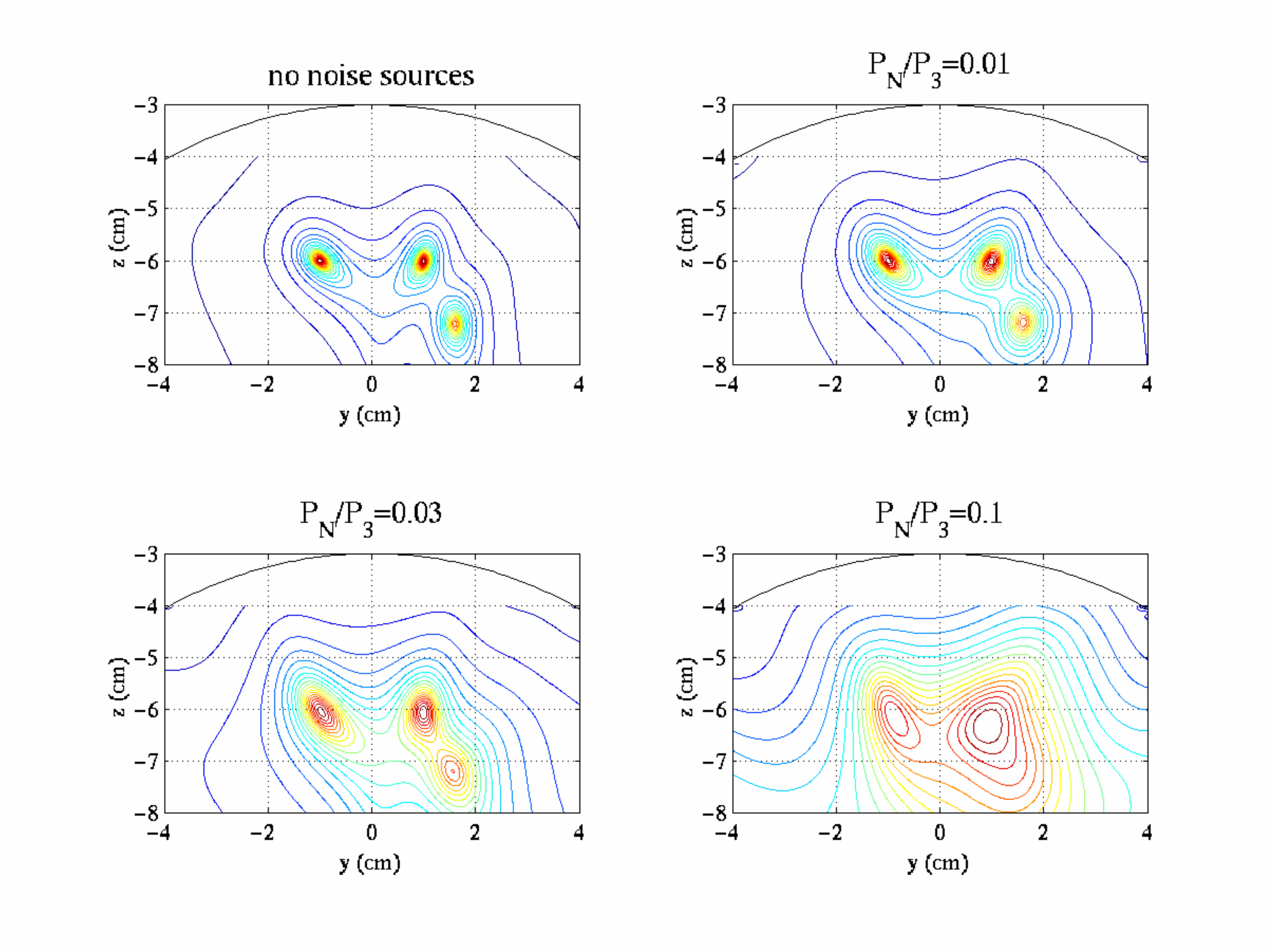


# 148-channel sensors used ($M$=148)



## Experiments for changing $P_N$ while $N_S$ is fixed at 48

$M$=37



# Open questions

A large number of randomly distributed dipoles
as a model of  spontaneous neural activity!!

Do such noise sources really exist?

If yes, how large is the power of each dipole?

# Summary for influence of various types of noise

## Sensor noise

The spatial resolution is affected by this type of noise.

## External disturbances

Their effects on the reconstruction is negligible, if their eigenvectors are very different from lead field vectors in the source space.

## Neurophysiology noise

It can seriously affect the quality of source reconstruction, if a large number of incoherent dipoles are an appropriate model for it.

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