MEG adaptive beamformer source reconstruction technique in the presence of correlated sources

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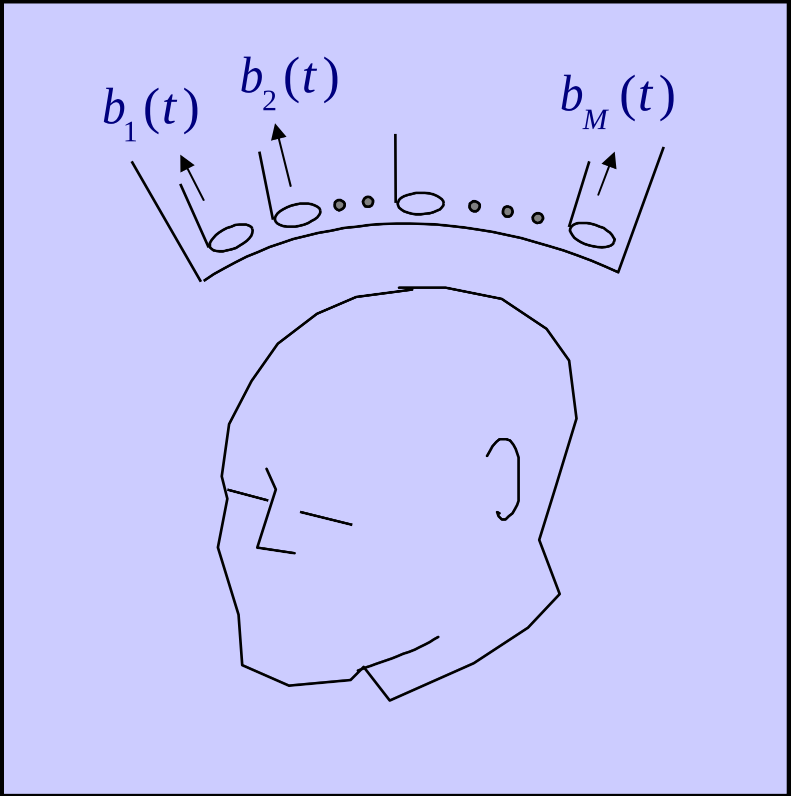
This talk presents results of the investigation that evaluates the performance of the MEG adaptive beamformer technique in the presence of correlated sources

Definitions

- data vector: $\boldsymbol{b}(\mathrm{t}) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$



$\left[b_j(t)\text{: the } j\text{th sensor recording at } t \right]$

- data covariance matrix: $\boldsymbol{D} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

$\langle \cdot \rangle$ represents time average

Source moment

- magnitude at $\boldsymbol{r} = [x, y, z]$

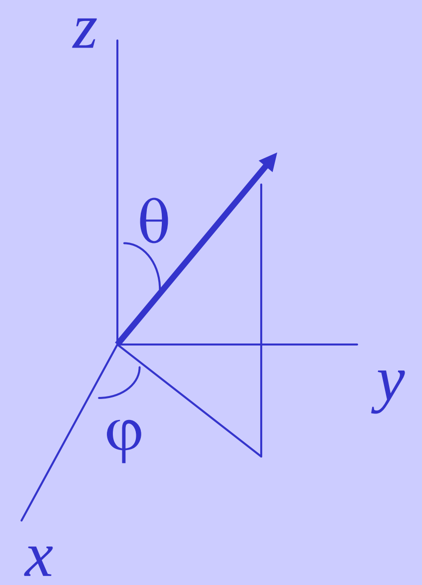
  and at t: $s(\boldsymbol{r}, t)$

- orientation:

$$\boldsymbol{\eta}(\boldsymbol{r}, t) = [\eta_x(\boldsymbol{r}, t), \eta_y(\boldsymbol{r}, t), \eta_z(\boldsymbol{r}, t)]$$

- source moment vector:

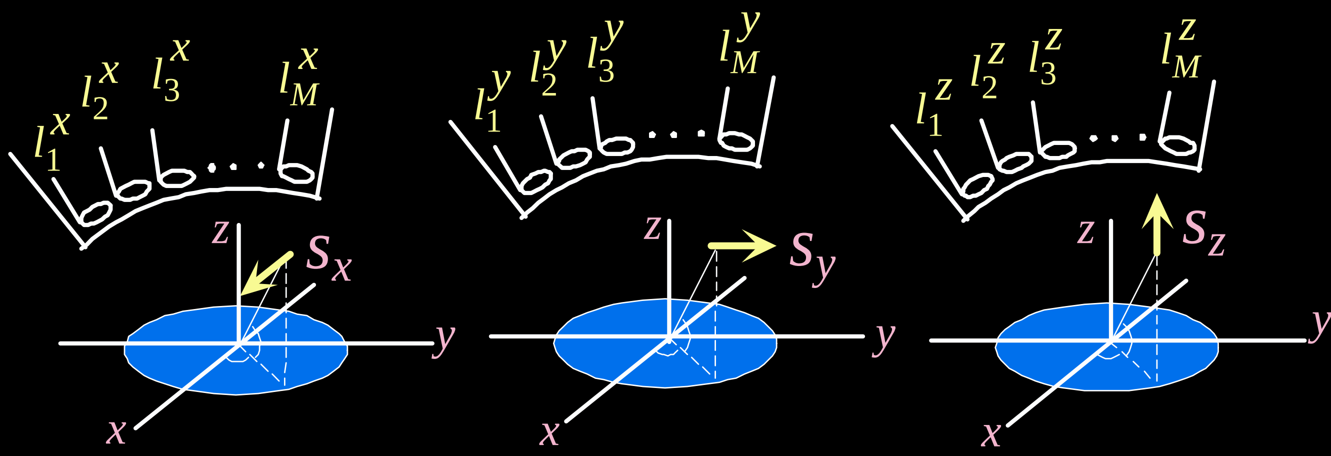
$$\boldsymbol{s}(\boldsymbol{r}, t) = s(\boldsymbol{r}, t) \begin{bmatrix} \eta_x(\boldsymbol{r}, t) \\ \eta_y(\boldsymbol{r}, t) \\ \eta_z(\boldsymbol{r}, t) \end{bmatrix} = \begin{bmatrix} s_x(\boldsymbol{r}, t) \\ s_y(\boldsymbol{r}, t) \\ s_z(\boldsymbol{r}, t) \end{bmatrix}$$



$$\eta_x = \sin\theta \cos\varphi$$

$$\eta_y = \sin\theta \sin\varphi$$

$$\eta_x = \cos\theta$$



l_1^x l_2^x l_3^x . . . l_M^x
z s_x y x
l_1^y l_2^y l_3^y . . . l_M^y
z s_y y x
l_1^z l_2^z l_3^z . . . l_M^z
z s_z y x

$$|s_x|=|s_y|=|s_z|=1$$

Lead field vector for the *j* th sensor

$$\boldsymbol{l}_j(\boldsymbol{r}) = [l_j^x(\boldsymbol{r}), l_j^y(\boldsymbol{r}), l_j^z(\boldsymbol{r})]$$

Lead field matrix for the whole sensor array

$$\boldsymbol{L}(\boldsymbol{r}) = \begin{bmatrix} \boldsymbol{l}_1(\boldsymbol{r}) \\ \boldsymbol{l}_2(\boldsymbol{r}) \\ \vdots \\ \boldsymbol{l}_M(\boldsymbol{r}) \end{bmatrix} = \begin{bmatrix} l_1^x(\boldsymbol{r}) & l_1^y(\boldsymbol{r}) & l_1^z(\boldsymbol{r}) \\ l_2^x(\boldsymbol{r}) & l_2^y(\boldsymbol{r}) & l_2^z(\boldsymbol{r}) \\ \vdots & \vdots & \vdots \\ l_M^x(\boldsymbol{r}) & l_M^y(\boldsymbol{r}) & l_M^z(\boldsymbol{r}) \end{bmatrix} = \left[\boldsymbol{l}_x(\boldsymbol{r}), \boldsymbol{l}_y(\boldsymbol{r}), \boldsymbol{l}_z(\boldsymbol{r})\right]$$

Basic relationship

$$b_j(t) = \int \boldsymbol{l}_j(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r},t)d\boldsymbol{r}$$

or

$$\boldsymbol{b}(t) = \int \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r},t)d\boldsymbol{r}$$

Problem of source localization:

Estimate $\boldsymbol{s}(\boldsymbol{r},t)$ from the measurement $\boldsymbol{b}(t)$

What is adaptive beamformer?

Spatial filter

$$\hat{\boldsymbol{s}}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = [w_1(\boldsymbol{r}),\ldots,w_M(\boldsymbol{r})]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t)$$

$\Uparrow$ estimate of $\boldsymbol{s}(\boldsymbol{r},t)$

$\Uparrow$ weight vector

Non-adaptive weight

$\boldsymbol{w}(\boldsymbol{r})$ is data independent

Adaptive weight

$\boldsymbol{w}(\boldsymbol{r})$ is data dependent

Non-adaptive weight

minimum-norm estimate (Hamalainen *et al*.)

The weight $\boldsymbol{w}(\boldsymbol{r})$ is obtained by

$$\boldsymbol{w}^T(\boldsymbol{r}) = \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{G}^{-1}, \text{ where } G_{i,j} = \int \boldsymbol{l}_i(\boldsymbol{r})\boldsymbol{l}_j^T(\boldsymbol{r})d\boldsymbol{r}$$

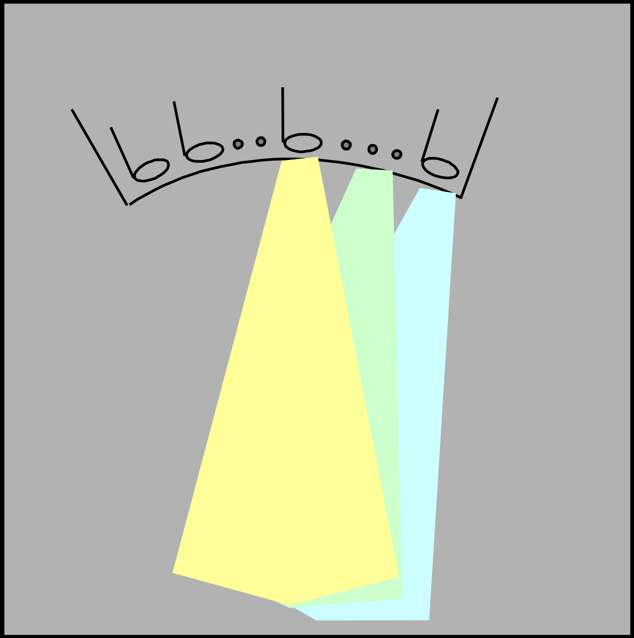
Inverse solution: $\hat{\boldsymbol{s}}(\boldsymbol{r}) = \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{G}^{-1}\boldsymbol{b}$

$\Uparrow$

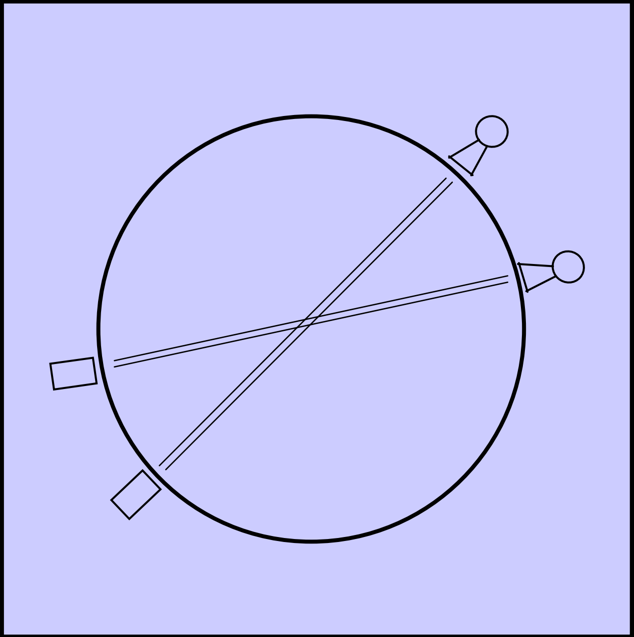
This is erroneous

Property of *G* matrix

$$G_{i,j} = \int \boldsymbol{l}_i(\boldsymbol{r})\boldsymbol{l}_j(\boldsymbol{r})d\boldsymbol{r}$$



Biomagnetic instruments



X-ray computed tomography

Overlaps of sensor lead fields is large

G is poorly conditioned

G ≈ unit matrix

$\boldsymbol{G}$ is poorly conditioned

- Apply regularization when calculating $\boldsymbol{G}$

  use $(\boldsymbol{G} + \gamma \boldsymbol{I})^{-1}$, instead of $\boldsymbol{G}^{-1}$

  Bayesian methods

- Do not use $\boldsymbol{G}$

  $\longrightarrow$ Adaptive beamforming technique

Adaptive beamformer

minimum-variance beamformer

beamformer pointing orientation

$$\boxed{\min_{\boldsymbol{w}} \boldsymbol{w}^T \boldsymbol{D} \boldsymbol{w} \text{ subject to } \boldsymbol{w}^T \boldsymbol{l}(\boldsymbol{r}, \eta) = \boldsymbol{L}(\boldsymbol{r})\eta = 1}$$

$$\boldsymbol{w}^T(\boldsymbol{r}) = \frac{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{D}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{D}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

and

$$\hat{s}(\boldsymbol{r}, t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = \frac{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{D}^{-1}\boldsymbol{b}(t)}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{D}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

use $\boldsymbol{l}(\boldsymbol{r})$ instead of $\boldsymbol{l}(\boldsymbol{r}, \eta)$ for simplicity

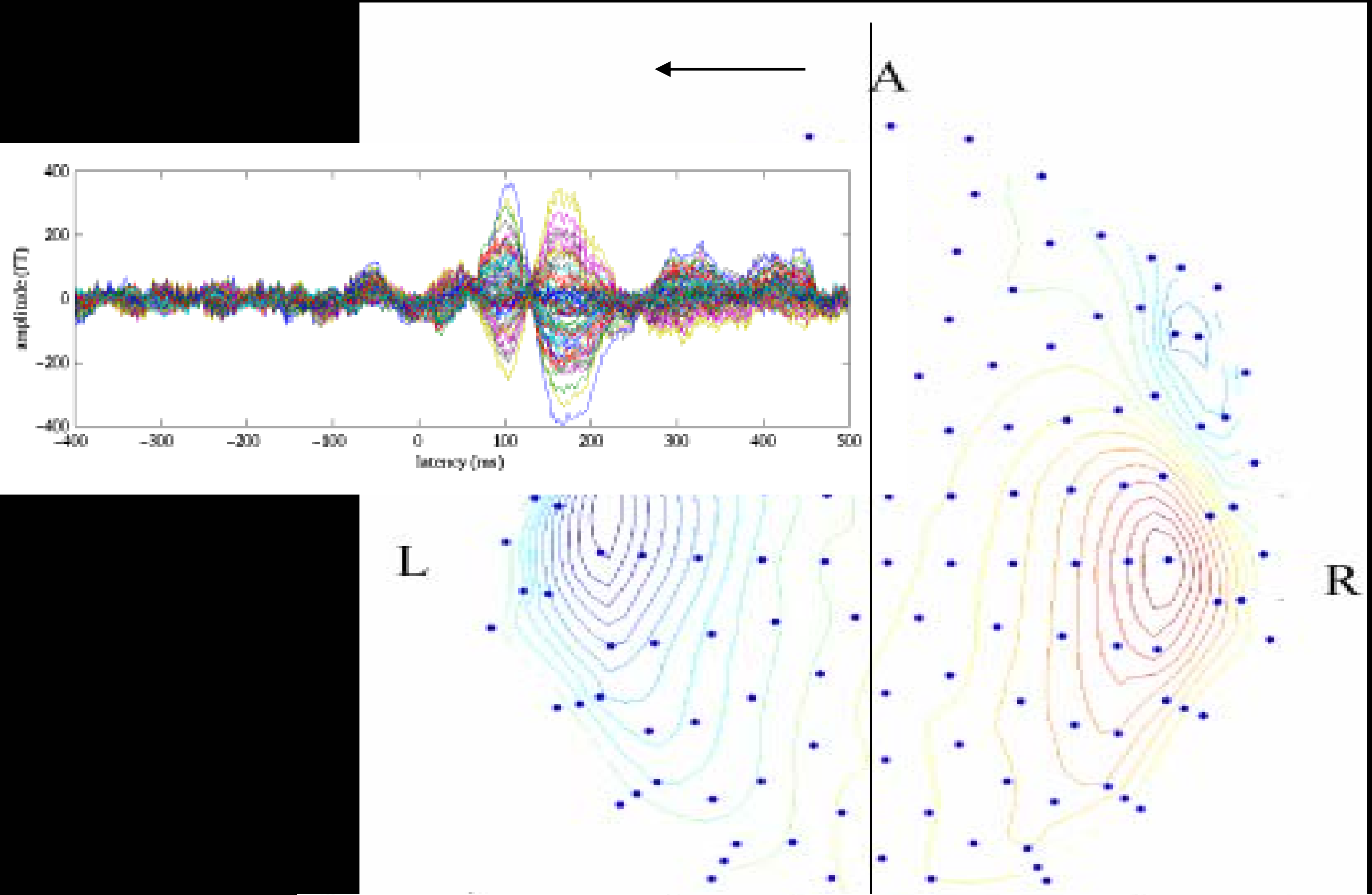
Adaptive beamformer

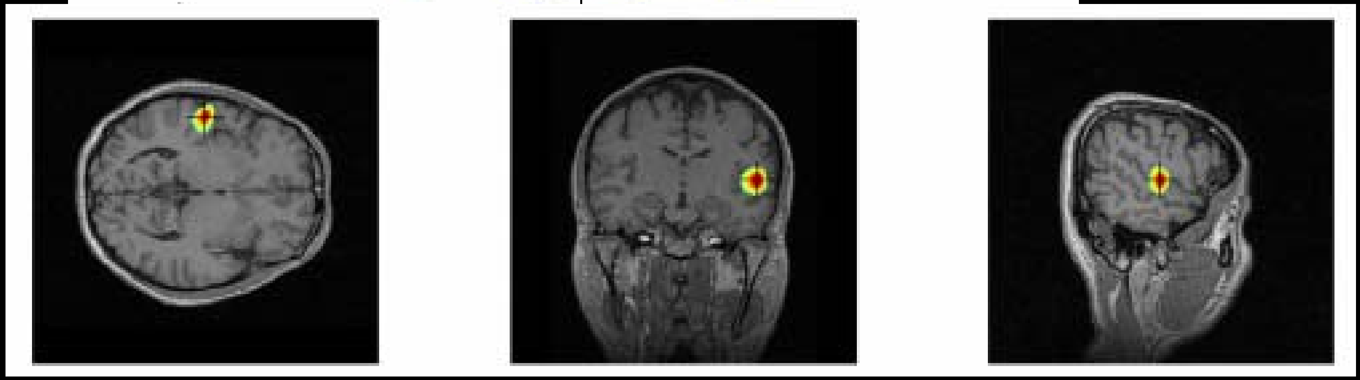
- Spatial resolution can exceed the limit imposed by the sensor-array configuration.

$$\Downarrow$$

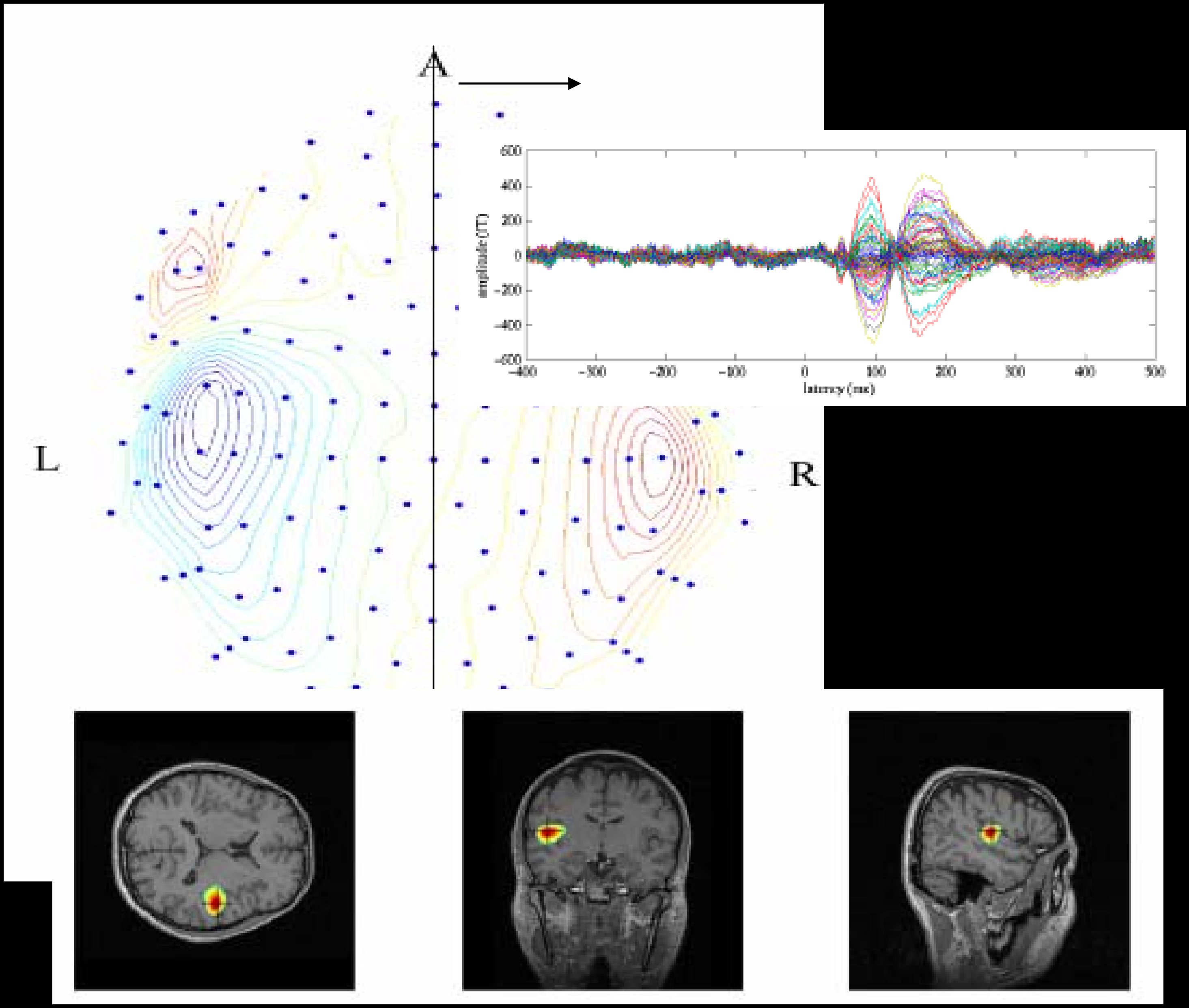
Possibility of providing high spatial resolution

- Strong temporal correlation among source activities degrades the quality of final reconstruction results.



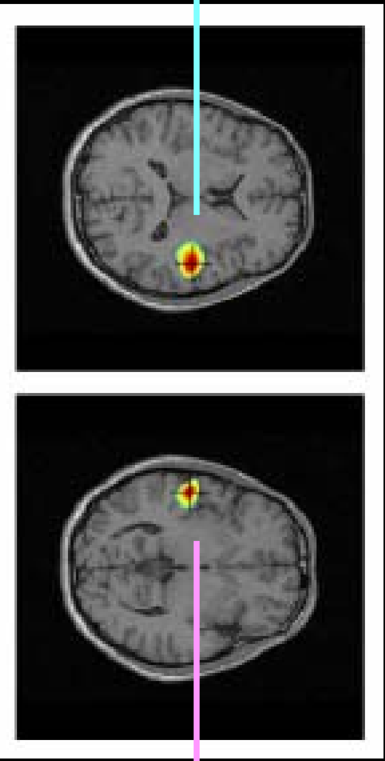


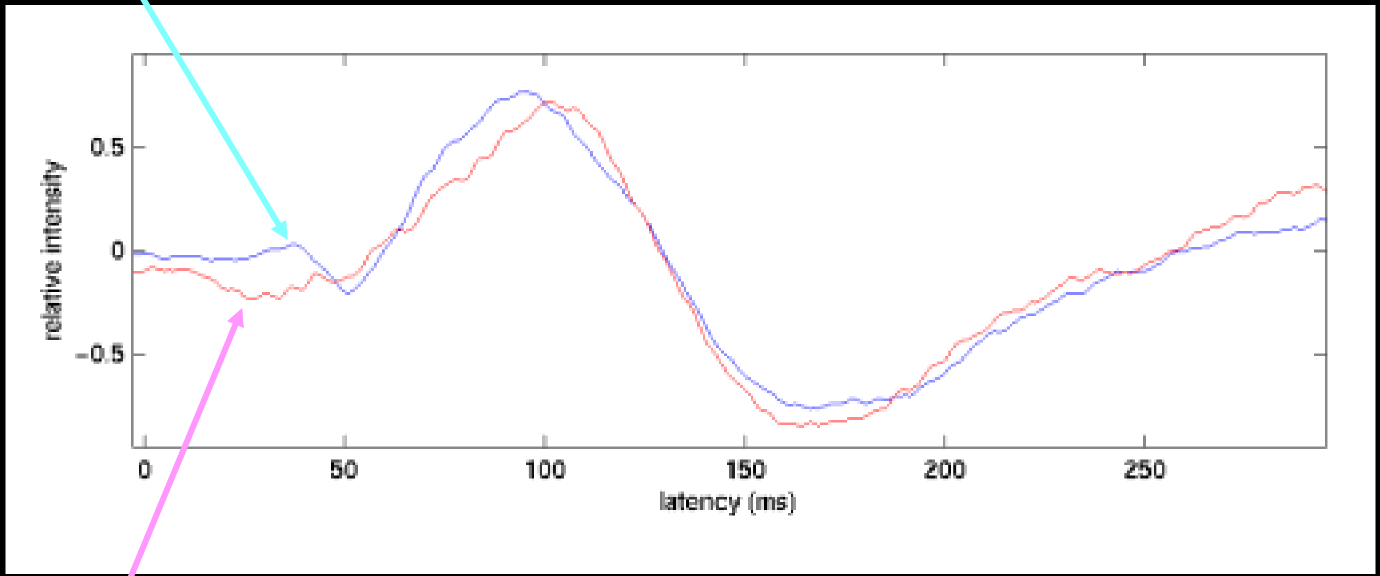
Reconstruction from left-hemisphere data only



Reconstruction from right-hemisphere data only

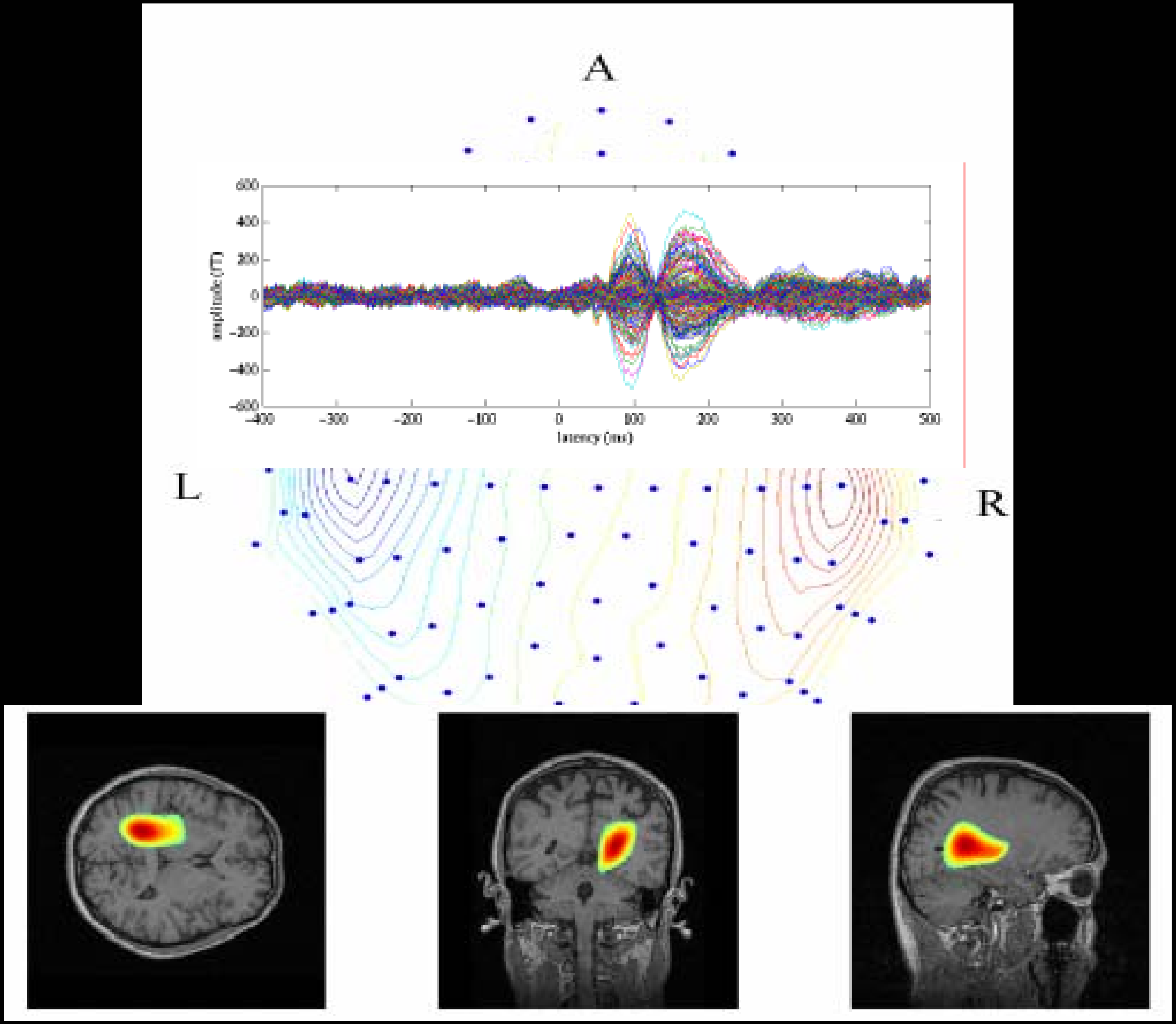
Right auditory cortex activation





correlation coefficient: 0.97

Left auditory cortex activation



Reconstruction from all-channel data

How does the adaptive beamformer technique perform when sources are moderately correlated ?

Influence of the source correlation

Adaptive beamformer cannot perfectly block the signal from correlated sources.

$\Downarrow$

Signal cancellation: intensity of reconstructed source monent decreases

Erroneous time course estimate: reconstructed source time course becomes a mixture of time courses of correlated source .

Spatial blur: spatial resolution is degraded due to the source correlation.

Basic relationship:

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}}$$

M. D. Zoltowski, IEEE Trans. Signal Process. Vol.36, pp.945-947, 1988

Assume that Q sources are correlated,

$$\tilde{s}(\boldsymbol{r}_p, t) = s(\boldsymbol{r}_p, t) + \sum_{q=1}^{Q} \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}} s(\boldsymbol{r}_q, t)$$

$\boldsymbol{R}_S$: source covariance matrix, $[\boldsymbol{R}_S^{-1}]_{pq}$:the (p,q) element of $\boldsymbol{R}_S^{-1}$

When two sources are correlated

$$\bar{\boldsymbol{R}}_S^{-1} = \frac{1}{\alpha_1^2\alpha_2^2(1-\mu^2)}\begin{bmatrix} \alpha_2^2 & -\mu\alpha_1\alpha_2 \\ -\mu\alpha_1\alpha_2 & \alpha_1^2 \end{bmatrix}$$

$\Uparrow$

submatrix relating to the correlated two sources

Then

$$\tilde{s}(\boldsymbol{r}_1, t) = s(\boldsymbol{r}_1, t) - (\frac{\alpha_1\mu}{\alpha_2})s(\boldsymbol{r}_2, t)$$

$$\tilde{s}(\boldsymbol{r}_2, t) = -(\frac{\alpha_2\mu}{\alpha_1})s(\boldsymbol{r}_1, t) + s(\boldsymbol{r}_2, t)$$

α_j^2 : the jth source power defined by $\alpha_j^2 = \left\langle s(\boldsymbol{r}_j, t)^2 \right\rangle$,

μ : correlation between the two sources defined by $\mu = \dfrac{\left\langle s(\boldsymbol{r}_1, t)s(\boldsymbol{r}_2, t) \right\rangle}{\sqrt{\left\langle s(\boldsymbol{r}_1, t)^2 \right\rangle \left\langle s(\boldsymbol{r}_2, t)^2 \right\rangle}}$

Interesting results

$$\tilde{\mu} = \left| \frac{\langle \tilde{s}(\boldsymbol{r}_1, t)\tilde{s}(\boldsymbol{r}_2, t) \rangle}{\sqrt{\langle \tilde{s}(\boldsymbol{r}_1, t)^2 \rangle \langle \tilde{s}(\boldsymbol{r}_2, t)^2 \rangle}} \right|$$

$$\Downarrow$$

$$\tilde{\mu} = \frac{\left| \alpha_1 \alpha_2 (\mu^3 - \mu) \right|}{\sqrt{\alpha_1^2 (1 - \mu^2) \alpha_2^2 (1 - \mu^2)}} = |\mu|$$

Magnitude correlation coefficient calculated using the beamformer outputs is equal to the true magnitude correlation coefficient.

Signal cancellation (when two sources are correlated)

$$\tilde{s}(\boldsymbol{r}_1,t) = s(\boldsymbol{r}_1,t) - (\frac{\alpha_1\mu}{\alpha_2})s(\boldsymbol{r}_2,t)$$

$$\tilde{s}(\boldsymbol{r}_2,t) = -(\frac{\alpha_2\mu}{\alpha_1})s(\boldsymbol{r}_1,t) + s(\boldsymbol{r}_2,t)$$

$$\Downarrow$$

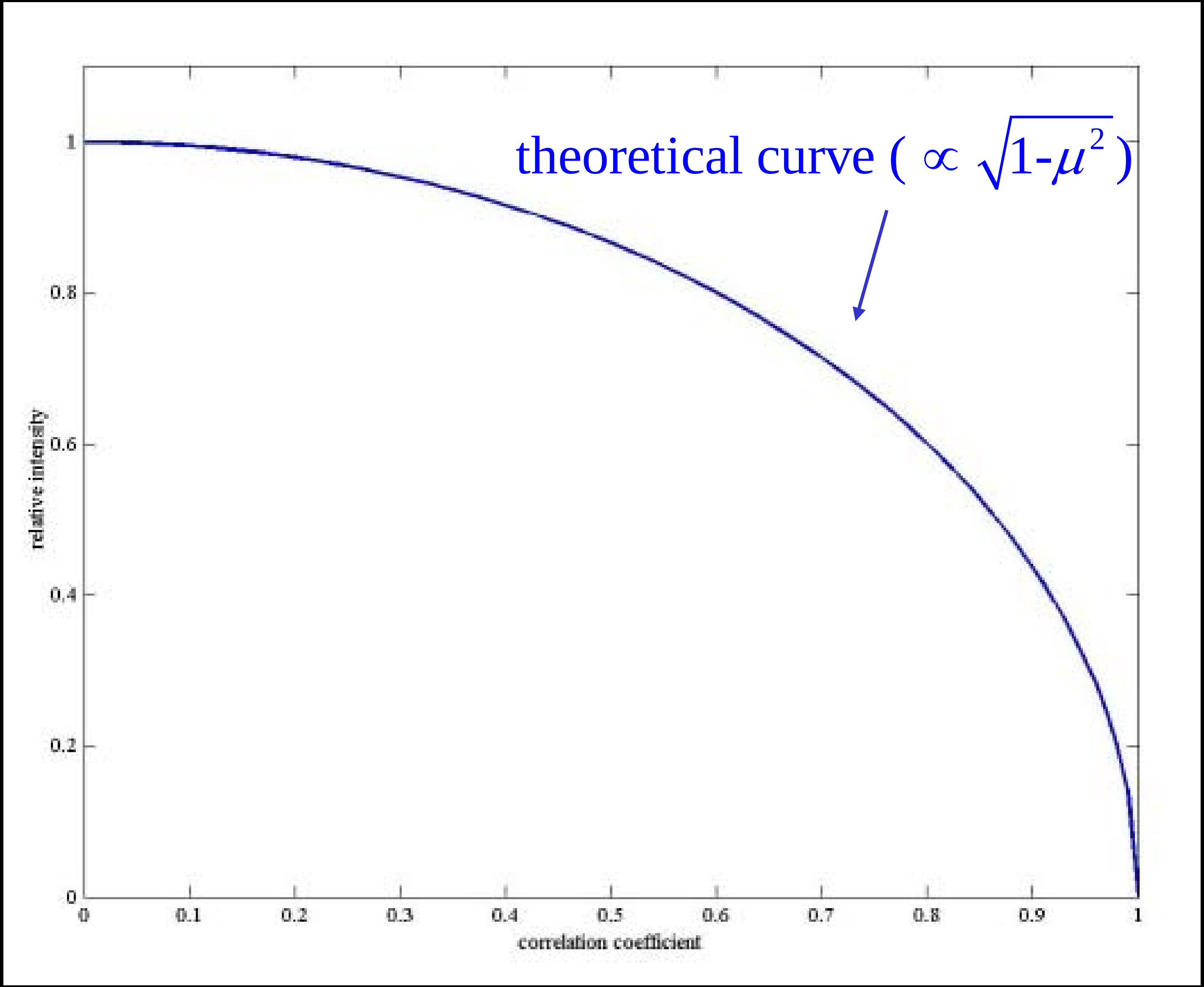
$$\left\langle \tilde{s}(\boldsymbol{r}_1,t)^2 \right\rangle = \alpha_1^2(1-\mu^2) = (1-\mu^2)\left\langle s(\boldsymbol{r}_1,t)^2 \right\rangle$$

$$\left\langle \tilde{s}(\boldsymbol{r}_2,t)^2 \right\rangle = \alpha_2^2(1-\mu^2) = (1-\mu^2)\left\langle s(\boldsymbol{r}_2,t)^2 \right\rangle$$

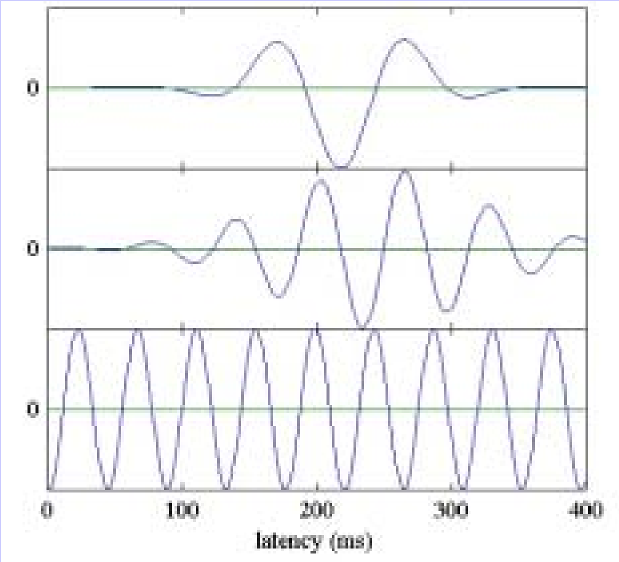
$$\Uparrow$$

Source intensity decreases by a factor of $(1-\mu^2)$

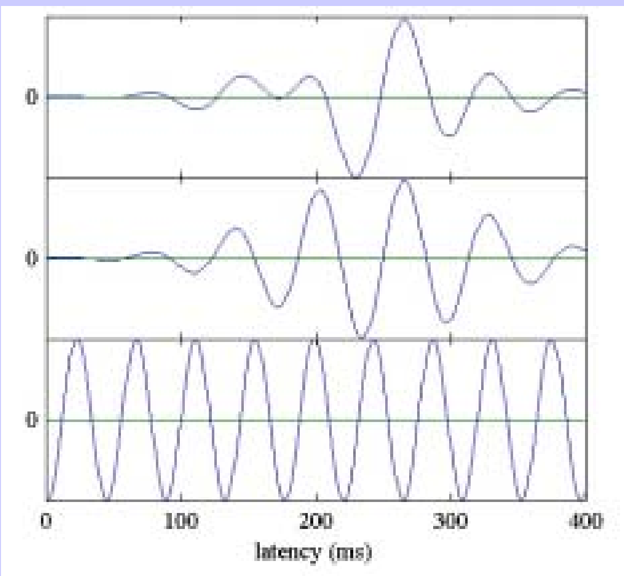
Intensity vs. correlation

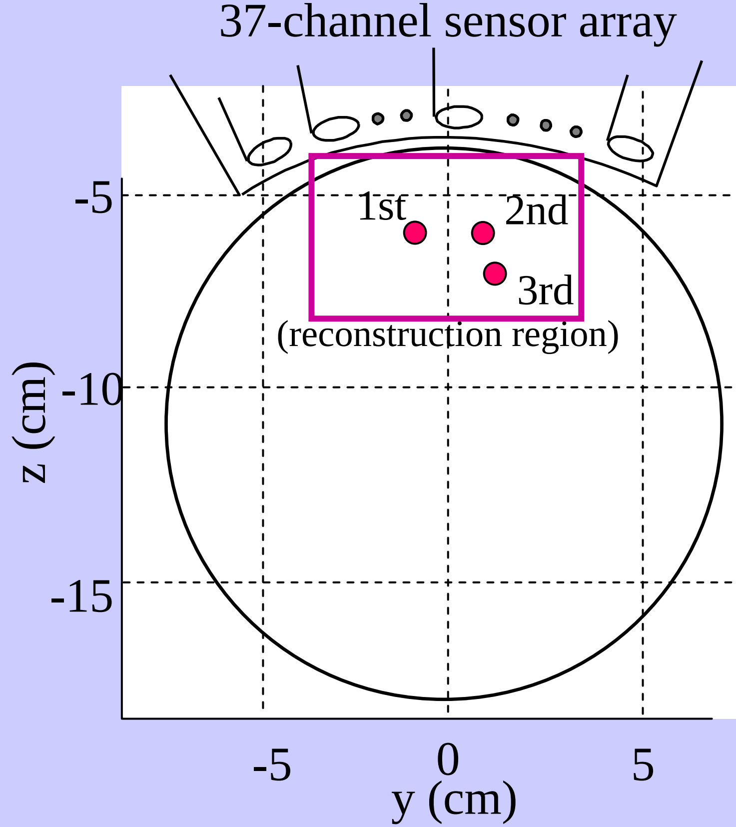


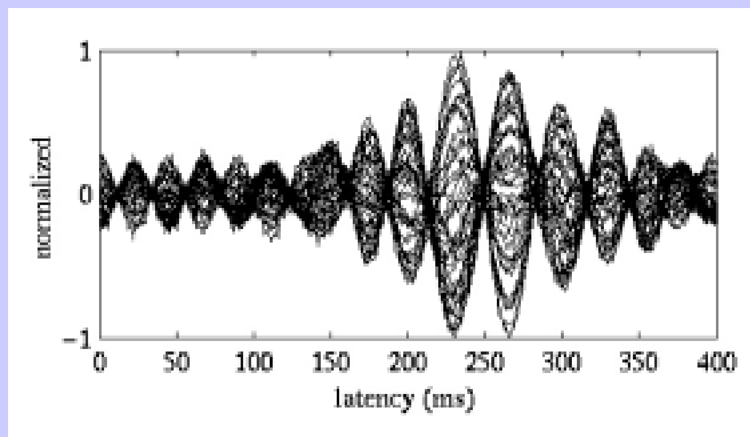
source correlation: zero



source correlation: 0.8





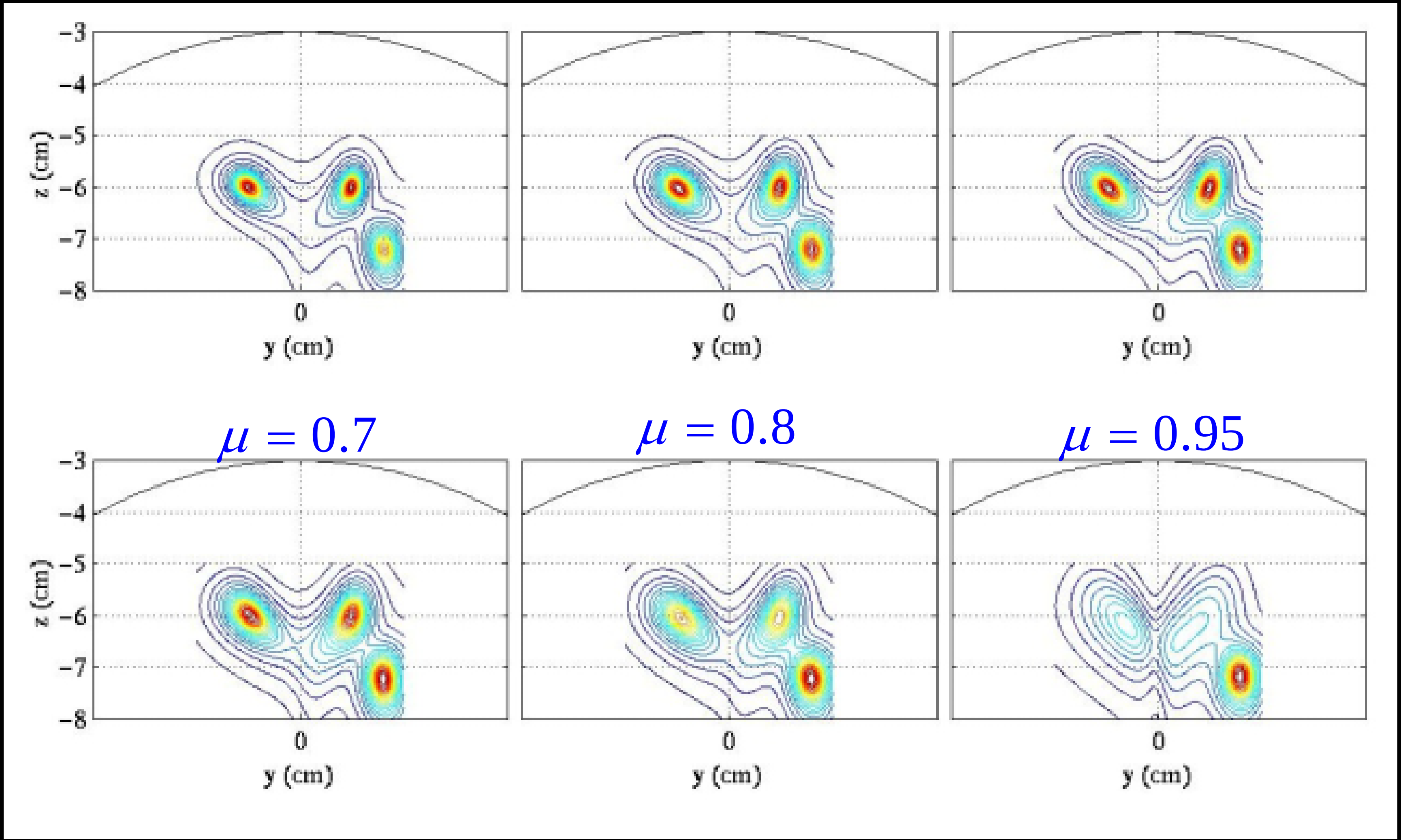


SNR=8

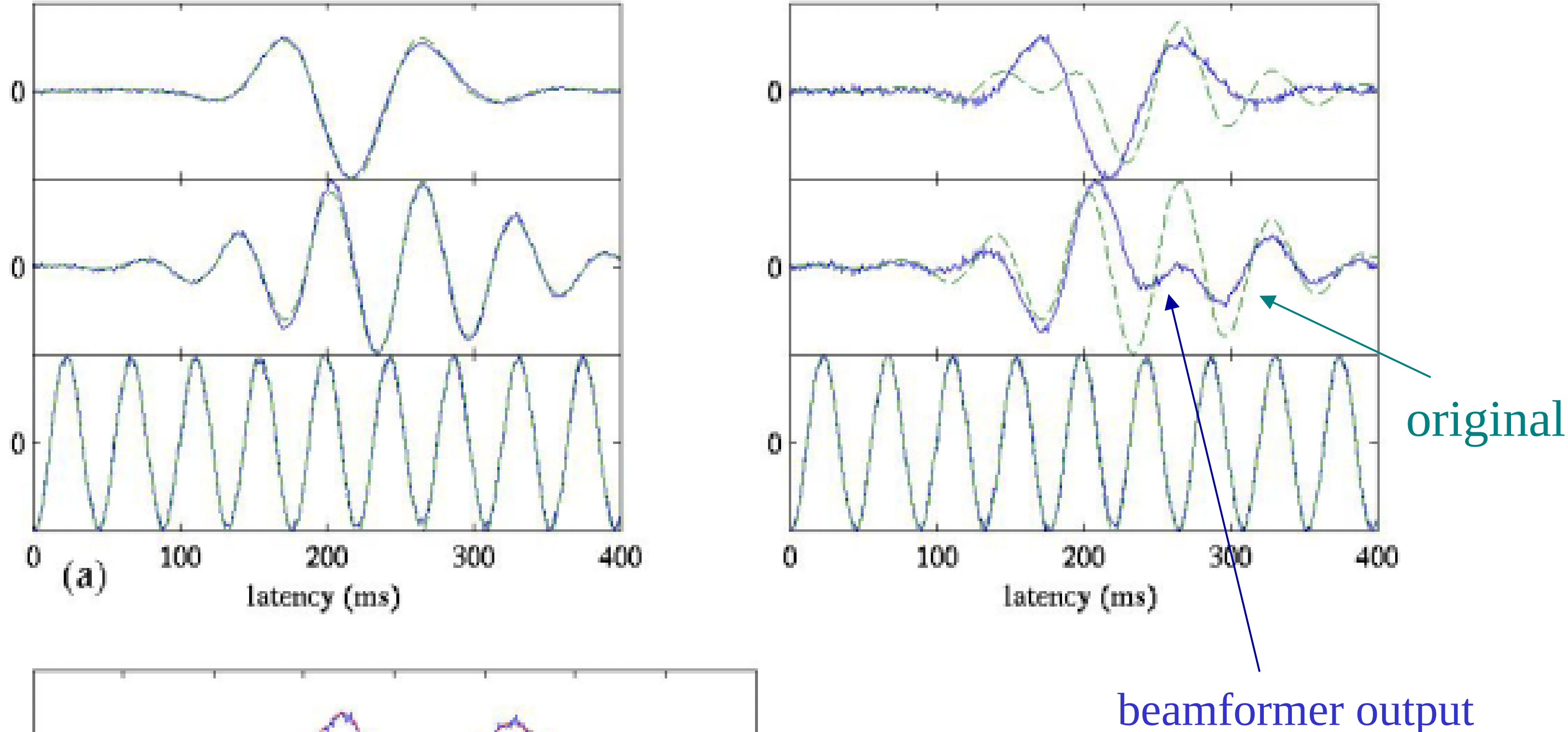
Reconstruction results

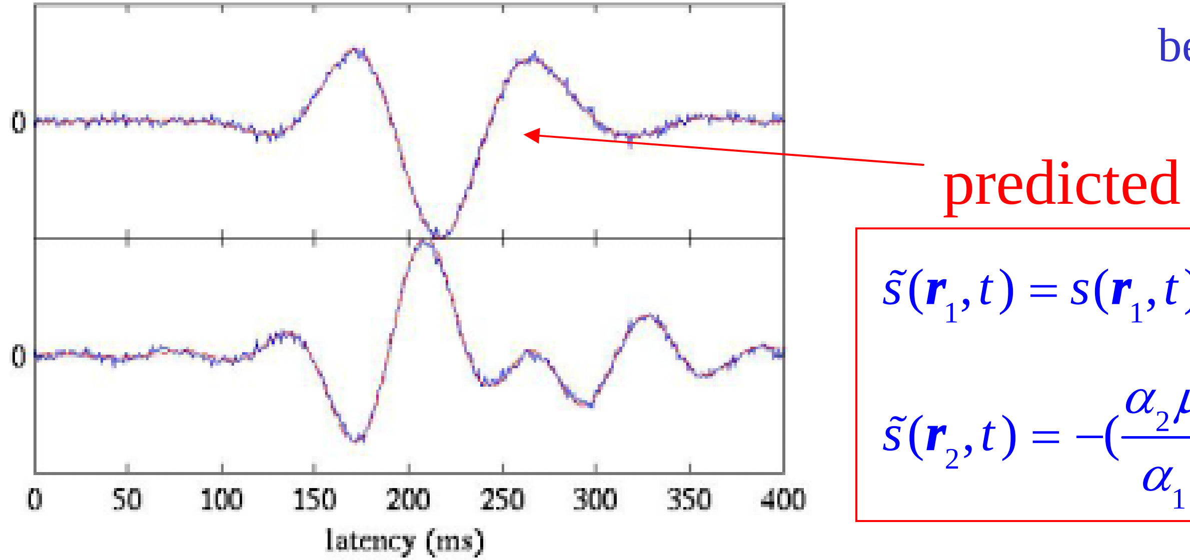
$\mu = 0$ $\mu = 0.5$ $\mu = 0.6$

$\mu = 0.7$ $\mu = 0.8$ $\mu = 0.95$



$\mu = 0$ $\mu = 0.8$





$$\tilde{s}(\boldsymbol{r}_1, t) = s(\boldsymbol{r}_1, t) - \left(\frac{\alpha_1 \mu}{\alpha_2}\right) s(\boldsymbol{r}_2, t)$$

$$\tilde{s}(\boldsymbol{r}_2, t) = -\left(\frac{\alpha_2 \mu}{\alpha_1}\right) s(\boldsymbol{r}_1, t) + s(\boldsymbol{r}_2, t)$$

Time course retrieval

Two-source correlation cases

$$\begin{bmatrix} \tilde{s}(\boldsymbol{r}_1,t) \\ \tilde{s}(\boldsymbol{r}_2,t) \end{bmatrix} = \begin{bmatrix} 1 & -(\alpha_1/\alpha_2)\mu \\ -(\alpha_2/\alpha_1)\mu & 1 \end{bmatrix} \begin{bmatrix} s(\boldsymbol{r}_1,t) \\ s(\boldsymbol{r}_2,t) \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \hat{s}(\boldsymbol{r}_1,t) \\ \hat{s}(\boldsymbol{r}_2,t) \end{bmatrix} = \begin{bmatrix} 1 & -(\alpha_1/\alpha_2)\mu \\ -(\alpha_2/\alpha_1)\mu & 1 \end{bmatrix}^{-1} \begin{bmatrix} \tilde{s}(\boldsymbol{r}_1,t) \\ \tilde{s}(\boldsymbol{r}_2,t) \end{bmatrix}$$

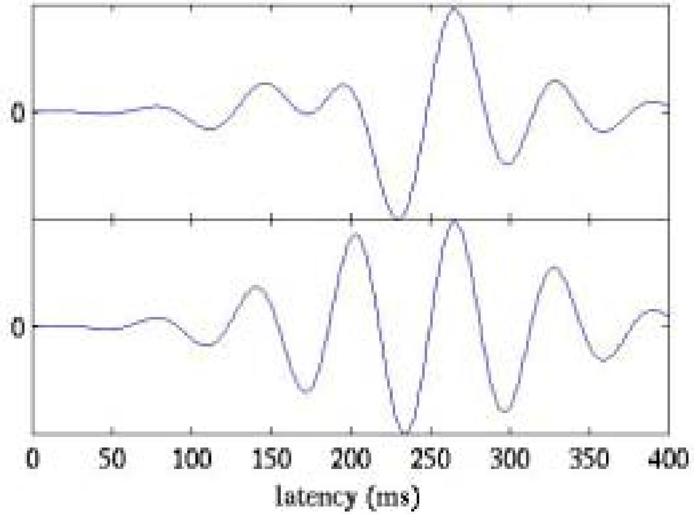
estimated time courses

measured time courses

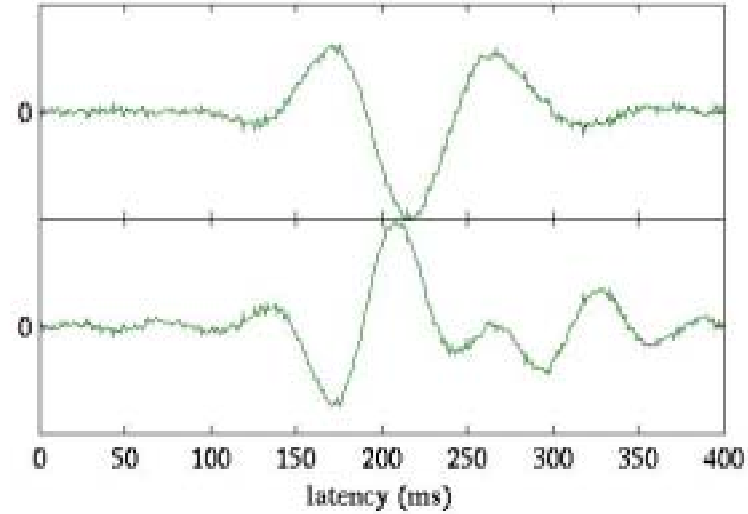
$$\boxed{\begin{aligned} \hat{\mu} &= \tilde{\mu} \\ \hat{\alpha}_j^2 &= \alpha_j^2 / (1-\tilde{\mu}^2) \end{aligned}}$$

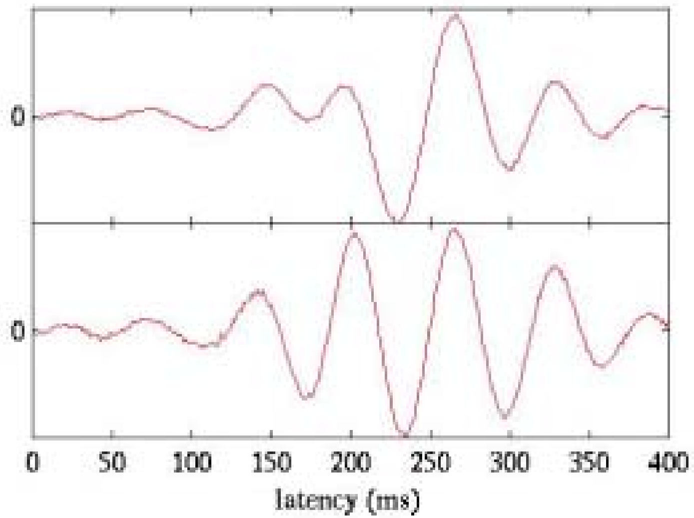
Time course retrieval experiments for two correlated sources

original



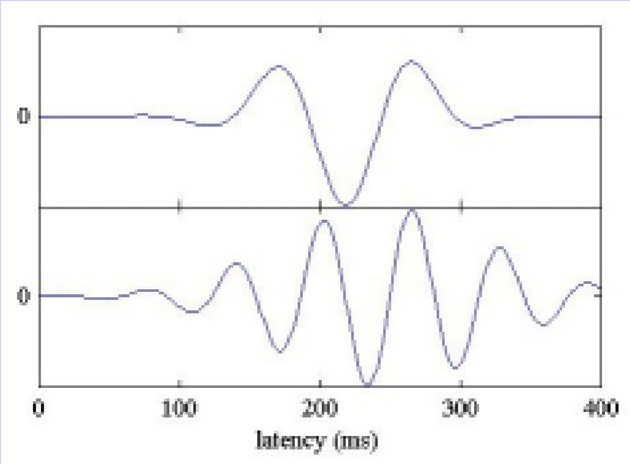
beamformer output





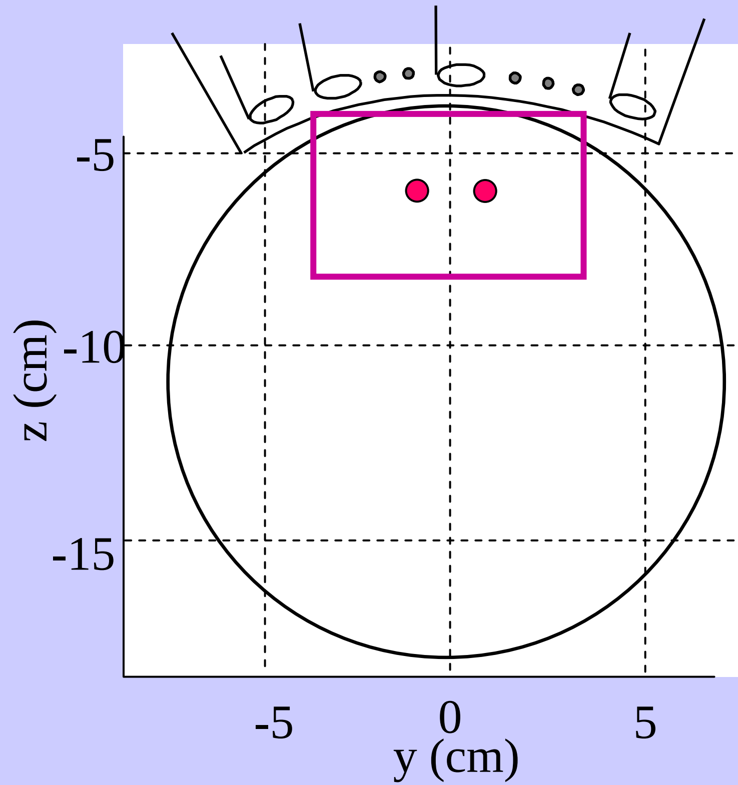
retrieved

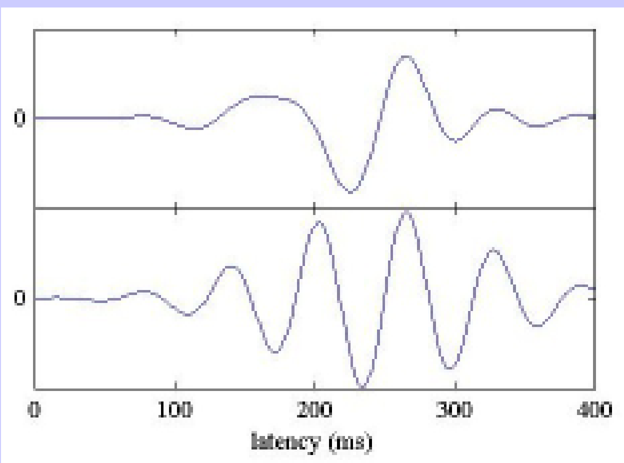
Influence of the source correlation on the spatial resolution



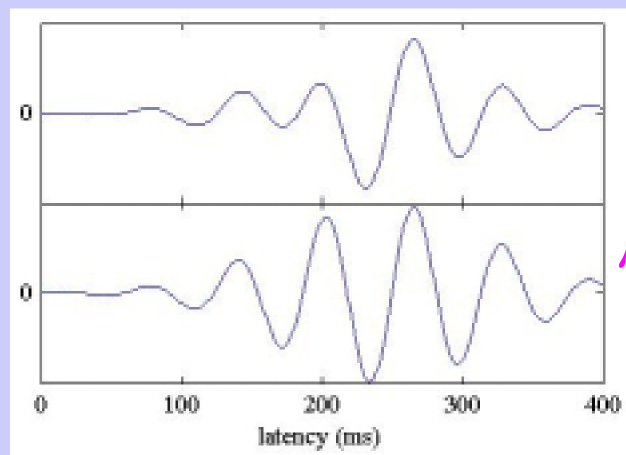
$\mu = 0.1$

37-channel sensor array





$\mu = 0.5$



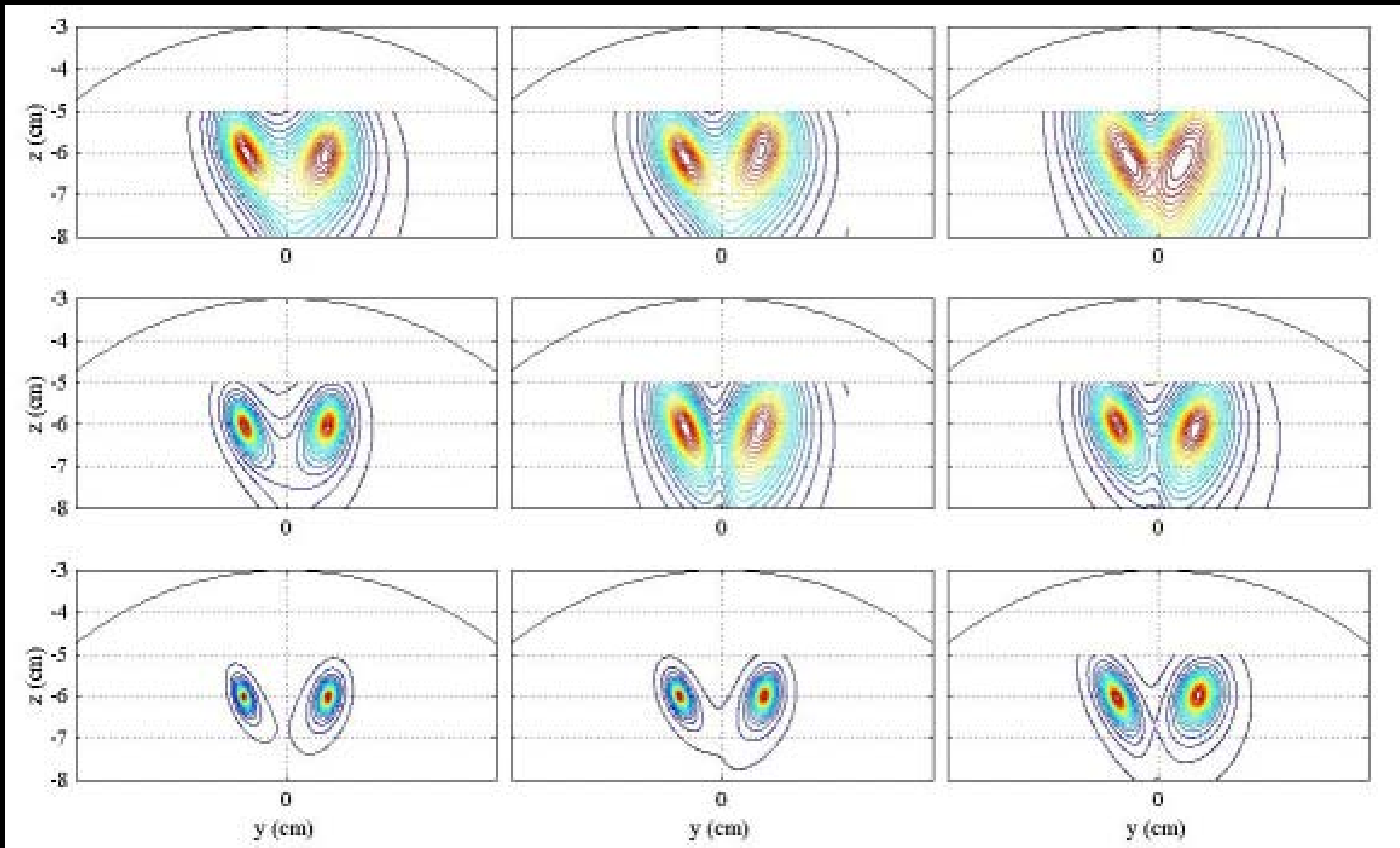
$\mu = 0.8$

$\mu = 0.1$ $\mu = 0.5$ $\mu = 0.8$

SNR=2

SNR=4

SNR=8



Assume two sources with identical power

$\boldsymbol{r}_1,\ \boldsymbol{r}_2$: source locations, $\boldsymbol{\eta}_1,\ \boldsymbol{\eta}_2$: source orientations

α^2 : power of each source, μ: correlation coefficient

Define $\boldsymbol{l}_1 = \boldsymbol{L}(\boldsymbol{r}_1)\eta_1$ and $\boldsymbol{l}_2 = \boldsymbol{L}(\boldsymbol{r}_2)\eta_2$

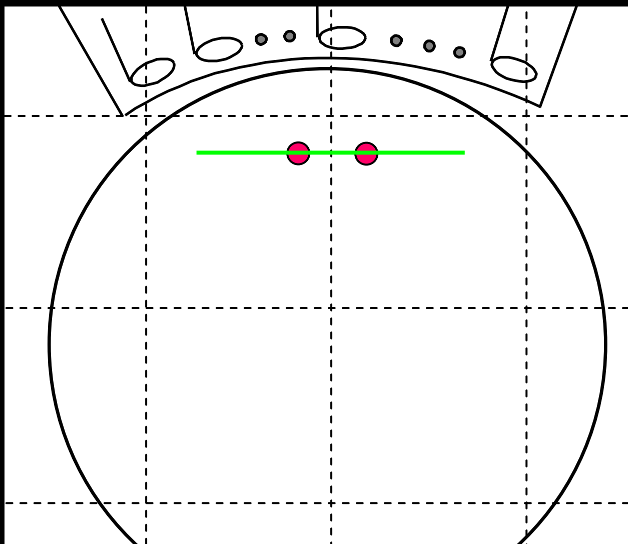
Then, $$\boldsymbol{D} = \sigma^2 \boldsymbol{I} + \alpha^2 \left[\boldsymbol{l}_1\ \boldsymbol{l}_2\right] \begin{bmatrix} 1 & \mu \\ \mu & 1 \end{bmatrix} \left[\boldsymbol{l}_1\ \boldsymbol{l}_2\right]^T$$

$$= \sigma^2 \left(\boldsymbol{I} + \tfrac{\alpha^2}{\sigma^2}\left[\boldsymbol{l}_1 \boldsymbol{l}_1^T + \boldsymbol{l}_2 \boldsymbol{l}_2^T + \mu\left(\boldsymbol{l}_1 \boldsymbol{l}_2^T + \boldsymbol{l}_2 \boldsymbol{l}_1^T\right)\right]\right)$$

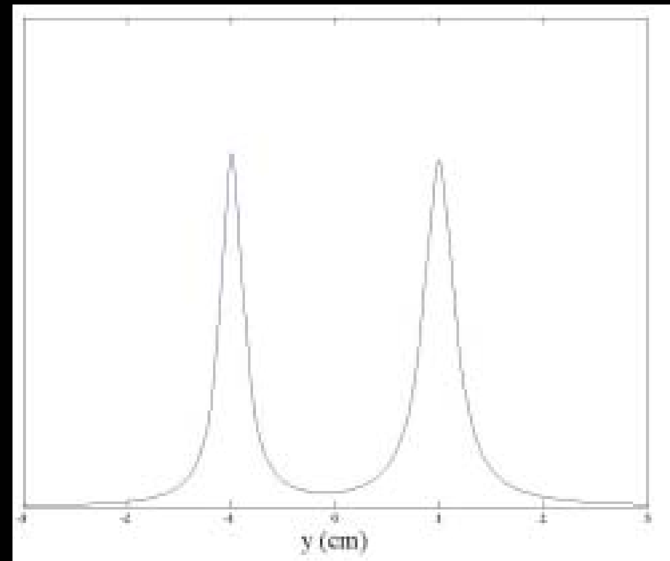
beamformer response at $\boldsymbol{r}$

$$p(\boldsymbol{r}) = \sum_{j=x,y,z} \boldsymbol{w}_j^T(\boldsymbol{r}) \boldsymbol{D} \boldsymbol{w}_j(\boldsymbol{r})$$

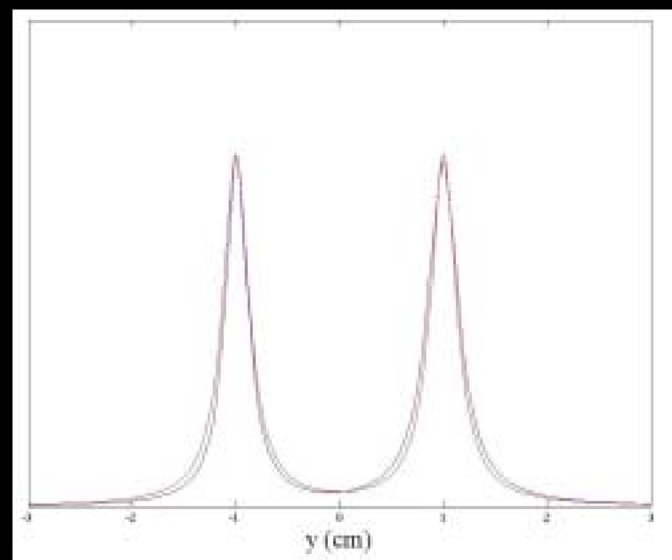
Lorenzean curve fitting



plot $p(\boldsymbol{r})$



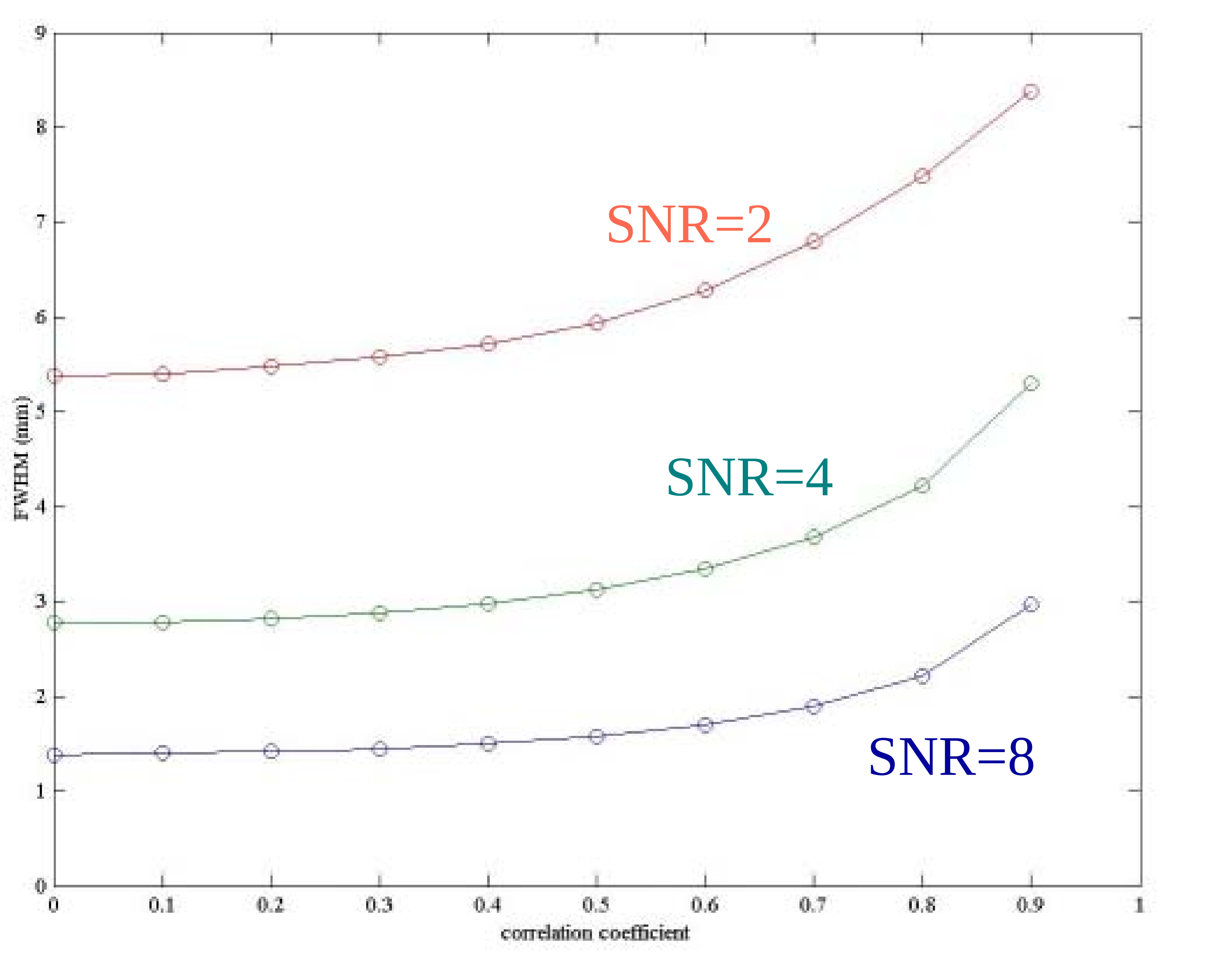
curve fitting



Lorenzean curve

$$f(y) = \frac{1}{1 + [(y - y_j)/\Delta]^2}$$

Δ : FWHM of the curve



SNR=2
SNR=4
SNR=8
FWHM (mm)
correlation coefficient
0
1
2
3
4
5
6
7
8
9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Summary

The performance degradation of the adaptive beamformer techniques in the presence of correlated sources was analyzed when two correlated sources exist:

- The performance is generally not significantly degraded in the presence of moderately correlated sources ($\mu < 0.7$).

- The time course estimate may be erroneous even for such moderate degree of source correlation.

- A method is developed for retrieving the original time courses, when the number of correlated sources are two or three and this number is known.