Recent Advances in Biomagnetic Signal Processing and Source Localization
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Prewhitening Signal Covariance Estimation and Prewhitening Beamforming

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Data model

$$\boldsymbol{b}(t) = \boldsymbol{b}_S(t) + \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$$

Measured magnetic field

Signal magnetic field

Background spontaneous activity

Sensor noise

Definitions

Data covariance: $\boldsymbol{R} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

Signal covariance: $\boldsymbol{R}_S = \left\langle \boldsymbol{b}_S(t)\boldsymbol{b}_S^T(t) \right\rangle$

Interference plus noise covariance: $\boldsymbol{R}_{i+n} = \left\langle \left(\boldsymbol{b}_I(t) + \boldsymbol{n}(t)\right)\left(\boldsymbol{b}_I(t) + \boldsymbol{n}(t)\right)^T \right\rangle$

Two-condition experiments (ideal scenario)

Task: $\boldsymbol{b}(t) = \boldsymbol{b}_S(t) + \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

Control: $\boldsymbol{b}_C(t) = \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

↓

Covariance matrix relations

Task: $\boldsymbol{R} = \boldsymbol{R}_S + \boldsymbol{R}_{i+n}$

Control: $\boldsymbol{R}_C = \boldsymbol{R}_{i+n}$

Problem:

How to obtain interference-free source reconstruction

How to estimate $\boldsymbol{R}_S$ using $\boldsymbol{R}$ and $\boldsymbol{R}_C$

Existing approach: image-based subtraction (with *t* test)

Define $s(\boldsymbol{r})$: source reconstruction from $\boldsymbol{b}(t)$

$s_c(\boldsymbol{r})$: source reconstruction from $\boldsymbol{b}_c(t)$

Calculate $\Delta s(\boldsymbol{r}) = s(\boldsymbol{r}) - s_c(\boldsymbol{r})$

This approach works well when SIR is high but becomes less effective when large interference exists.

Naive approach: covariance-based subtraction

Estimate $\boldsymbol{R}_S$ using $\hat{\boldsymbol{R}}_S = \boldsymbol{R} - \boldsymbol{R}_C$

This approach has many problems, and generally does not work well

Prewhitening estimation of signal covariance

Calculate $\tilde{\boldsymbol{R}} = \boldsymbol{R}_C^{-1/2} \boldsymbol{R} \boldsymbol{R}_C^{-1/2}$

(Tilde is used to indicate the prewhitened version of a matrix)

$$\tilde{\boldsymbol{R}}_S = \sum_{j=1}^{Q} \gamma_j \boldsymbol{u}_j \boldsymbol{u}_j^T \Rightarrow \underline{\tilde{\boldsymbol{R}} = \sum_{j=1}^{Q} (\gamma_j + 1) \boldsymbol{u}_j \boldsymbol{u}_j^T} + \sum_{j=Q+1}^{M} \boldsymbol{u}_j \boldsymbol{u}_j^T .$$

$\Uparrow$

$\tilde{\boldsymbol{R}}$ has signal-level eigenvalues greater than 1,

and their eigenvectors are equal to those of $\tilde{\boldsymbol{R}}_S$

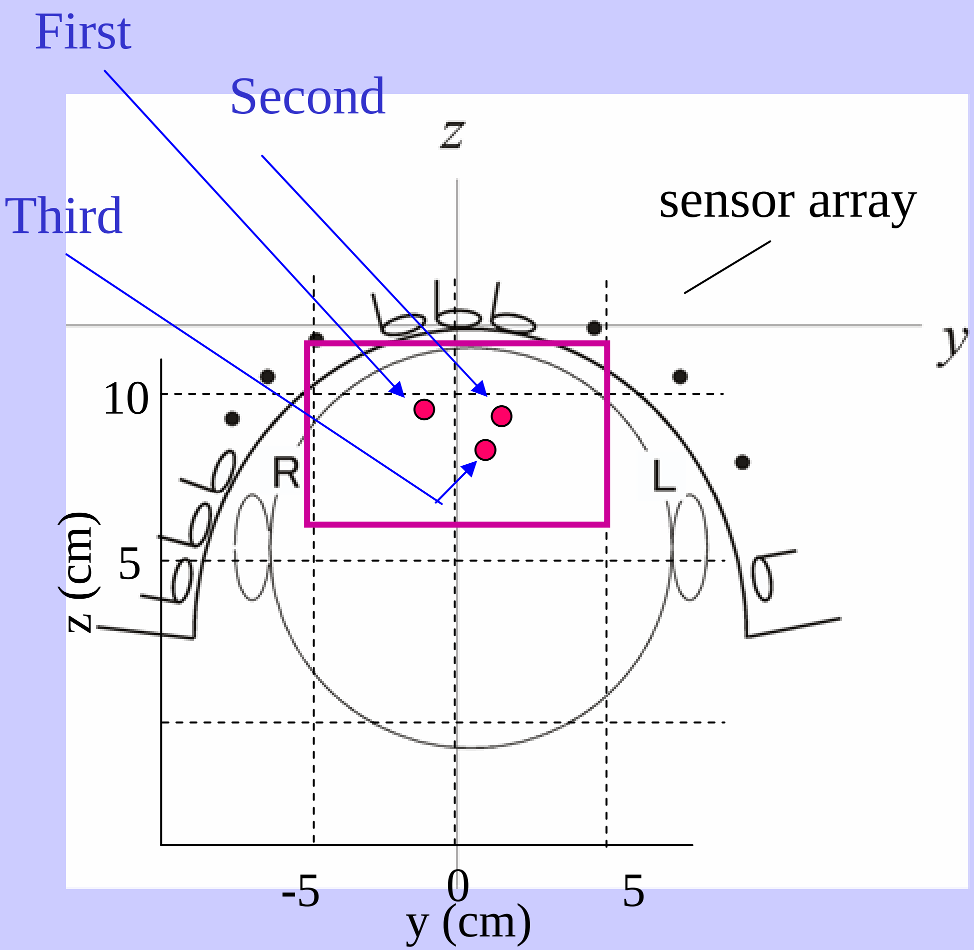
Signal covariance estimation

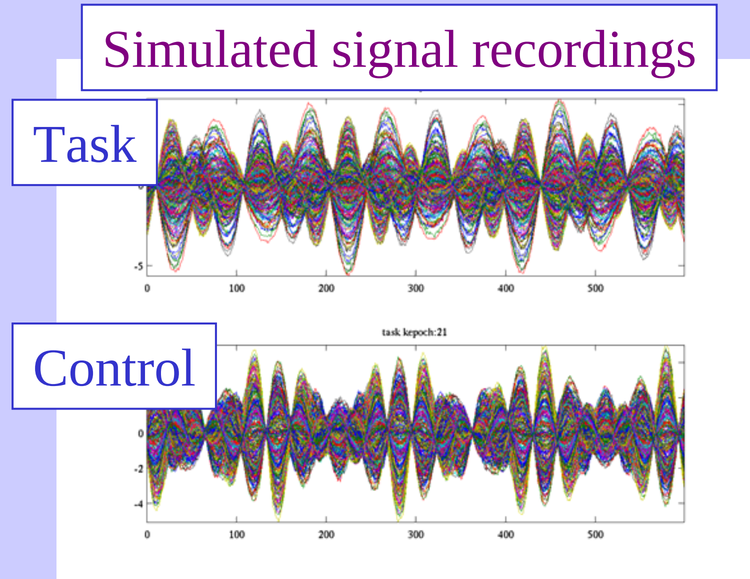
$$\hat{\boldsymbol{R}}_S = \boldsymbol{R}_C^{1/2} \left[\boldsymbol{U}_S \boldsymbol{U}_S^T (\tilde{\boldsymbol{R}} - \boldsymbol{I}) \right] \boldsymbol{R}_C^{1/2} = \boldsymbol{R}_C^{1/2} \left[\sum_{j=1}^{Q} (\gamma_j - 1) \boldsymbol{u}_j \right] \boldsymbol{R}_C^{1/2}$$

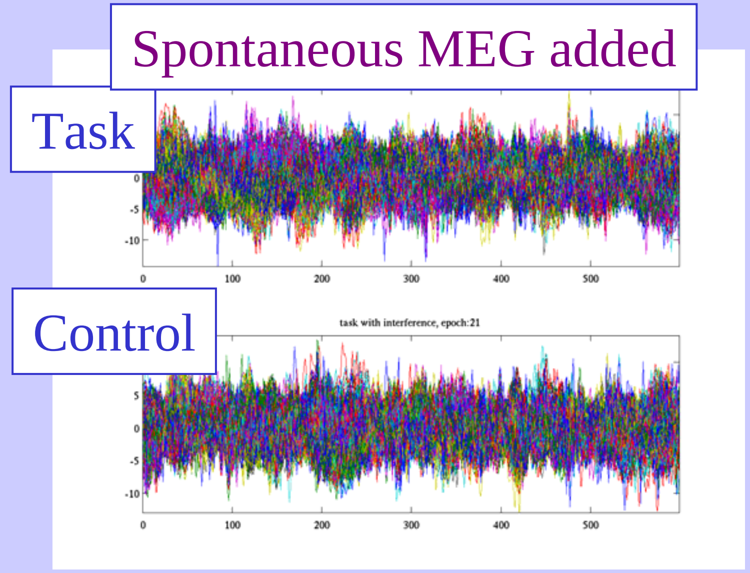
$\uparrow$

$$\boldsymbol{U}_S = [\boldsymbol{u}_1, \ldots, \boldsymbol{u}_Q]$$

Computer simulation on two-condition multi-epoch measurements

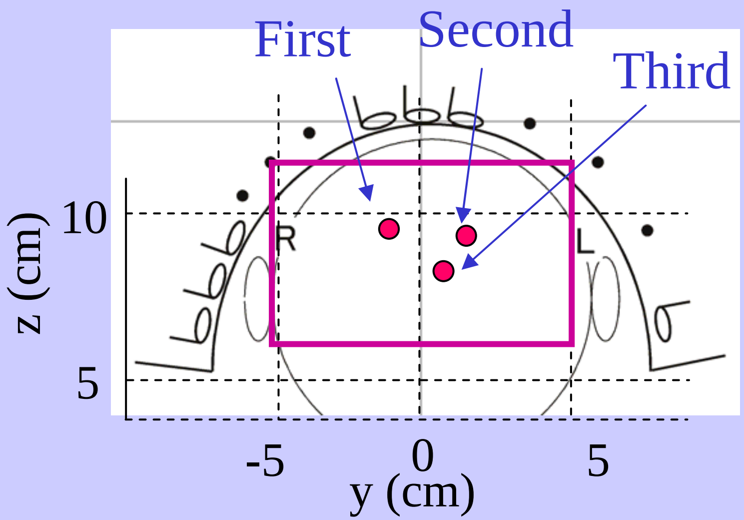






A total 96 of these paired data sets were generated, and used for calculating $\boldsymbol{R}$ and $\boldsymbol{R}_C$.

Two-condition measurements-Computer Simulation



Signal-to-Interference Ratio:0.5

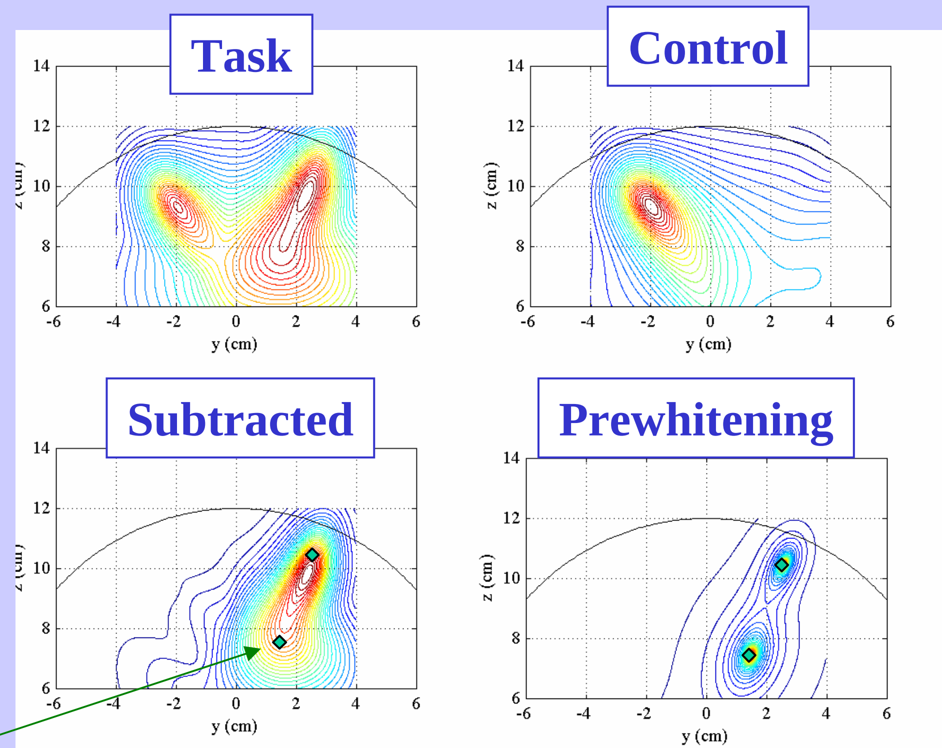
Relative source intensity

	control	task
First source	1	1
Second source	0	1
Third source	0	1

Minimum-variance source reconstruction

$$\hat{s}(\boldsymbol{r},t)=\frac{1}{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})]}$$

$$\hat{s}_{PW}(\boldsymbol{r},t)=\frac{1}{[\boldsymbol{l}^T(\boldsymbol{r})(\hat{\boldsymbol{R}}_S+\mu\boldsymbol{I})^{-1}\boldsymbol{l}(\boldsymbol{r})]}$$



source location

Two-condition experiments: more realistic scenario - Scenario I

Some sources exist only in the control condition

Task: $\boldsymbol{b}(t) = \boldsymbol{b}_S(t) + \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

Control: $\boldsymbol{b}_C(t) = \boldsymbol{b}_I(t) + \boldsymbol{n}(t) + \boldsymbol{b}_\Delta(t)$

$\Uparrow$ Signal from control-only sources

$\downarrow$

Task: $\boldsymbol{R} = \boldsymbol{R}_S + \boldsymbol{R}_{i+n}$

Control: $\boldsymbol{R}_C = \boldsymbol{R}_{i+n} + \boldsymbol{R}_\Delta$

covariance from control-only source: $\boldsymbol{R}_\Delta = \left\langle \boldsymbol{b}_\Delta(t)\boldsymbol{b}_\Delta^T(t) \right\rangle$

Scenario I-analysis

Define

$$\tilde{\boldsymbol{R}}_S = \sum_{j=1}^{Q} \gamma_j \boldsymbol{u}_j \boldsymbol{u}_j^T, \text{ and } \tilde{\boldsymbol{R}}_\Delta = \sum_{j=1}^{P} \beta_j \boldsymbol{v}_j \boldsymbol{v}_j^T$$

Then

$$\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{R}}_S + \boldsymbol{I} - \tilde{\boldsymbol{R}}_\Delta = \sum_{j=1}^{Q} \gamma_j \boldsymbol{u}_j \boldsymbol{u}_j^T + \sum_{j=P+1}^{M} \beta_j \boldsymbol{v}_j \boldsymbol{v}_j^T + \sum_{j=1}^{P} (1-\beta_j) \boldsymbol{v}_j \boldsymbol{v}_j^T$$

Thus

$$\tilde{\boldsymbol{R}} \boldsymbol{u}_j = \left(\sum_{j=1}^{Q} \gamma_j \boldsymbol{u}_j \boldsymbol{u}_j^T\right) \boldsymbol{u}_j + \left(\sum_{j=P+1}^{M} \beta_j \boldsymbol{v}_j \boldsymbol{v}_j^T\right) \boldsymbol{u}_j + \left(\sum_{j=1}^{P} (1-\beta_j) \boldsymbol{v}_j \boldsymbol{v}_j^T\right) \boldsymbol{u}_j$$

$$= \gamma_j \boldsymbol{u}_j + \sum_{j=1}^{P} (1-\beta_j)(\boldsymbol{v}_j^T \boldsymbol{u}_j) \boldsymbol{v}_j$$

Prewhitening estimation $\hat{\boldsymbol{R}}_S = \boldsymbol{R}_C^{1/2} \left[\boldsymbol{U}_S \boldsymbol{U}_S^T (\tilde{\boldsymbol{R}} - \boldsymbol{I}) \right] \boldsymbol{R}_C^{1/2}$ is still effective if $span(\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_Q) \perp span(\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_P)$ approximately holds.

Two-condition experiments: more realistic scenario - Scenario II

Target signal sources are active also in the control condition (Their intensities change between the two conditions)

Task: $\boldsymbol{b}(t) = \boldsymbol{b}_S(t) + \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

Control: $\boldsymbol{b}_C(t) = \boldsymbol{b}_S(t)' + \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

↓

Task: $\boldsymbol{R} = \boldsymbol{R}_S + \boldsymbol{R}_{i+n}$

Control: $\boldsymbol{R}_C = \boldsymbol{R}_S' + \boldsymbol{R}_{i+n}$

$$\boldsymbol{R}_S' = \left\langle \boldsymbol{b}_S(t)' \boldsymbol{b}_S^T(t)' \right\rangle$$

Scenario II-analysis

We finally have

$$\tilde{\boldsymbol{R}} = \tilde{\boldsymbol{D}}_P + \boldsymbol{I} - \tilde{\boldsymbol{D}}_N$$

where

$\boldsymbol{D}_P$: covariance matrix from signal sources stronger in the task than in the control state.

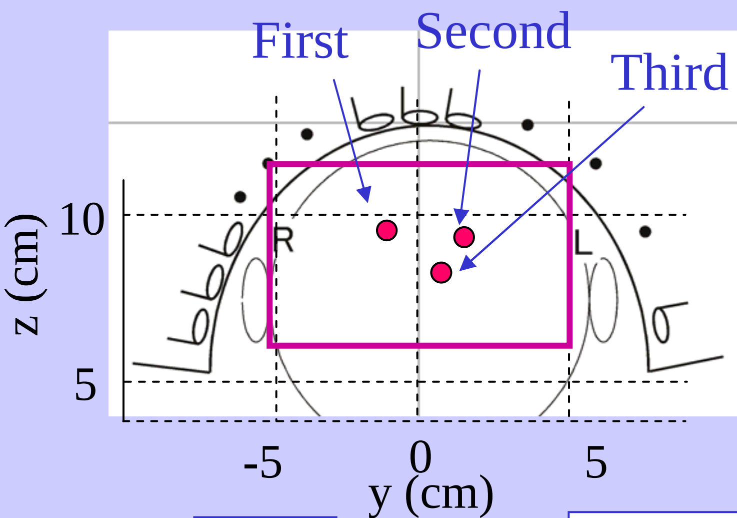
$\boldsymbol{D}_N$: covariance matrix from signal sources weaker in the task than in the control state.

The above covariance relationship is the same as that for Scenario I

We can still use $\hat{\boldsymbol{D}}_P = \boldsymbol{R}_C^{1/2}\left[\boldsymbol{U}_S\boldsymbol{U}_S^T(\tilde{\boldsymbol{R}} - \boldsymbol{I})\right]\boldsymbol{R}_C^{1/2}$

$\hat{\boldsymbol{D}}_N$ is obtained from flipped prewhitening where $\tilde{\boldsymbol{R}}_C = \boldsymbol{R}^{-1/2}\boldsymbol{R}_C\,\boldsymbol{R}^{-1/2}$ is used.

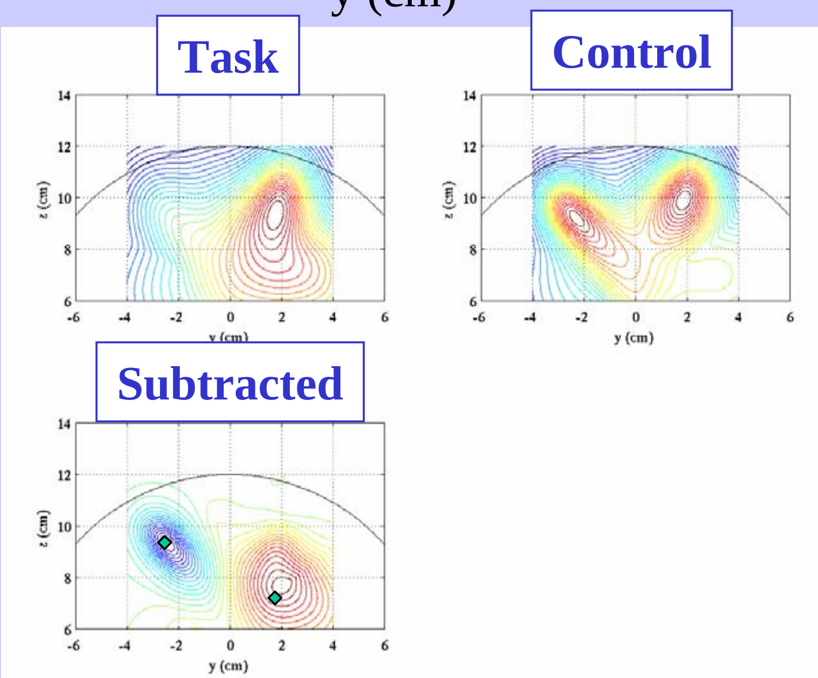
Scenario I and II--Numerical Experiments

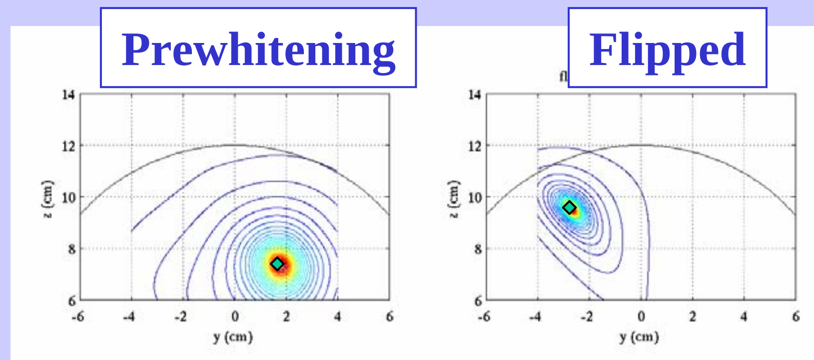


Signal-to-Interference Ratio:0.5

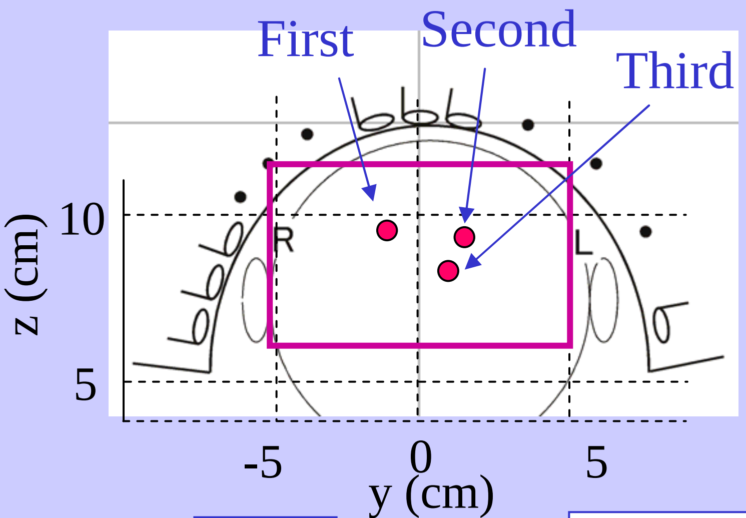
Relative source intensity

	control	task
First source	1	0.5
Second source	1	1
Third source	0.5	1





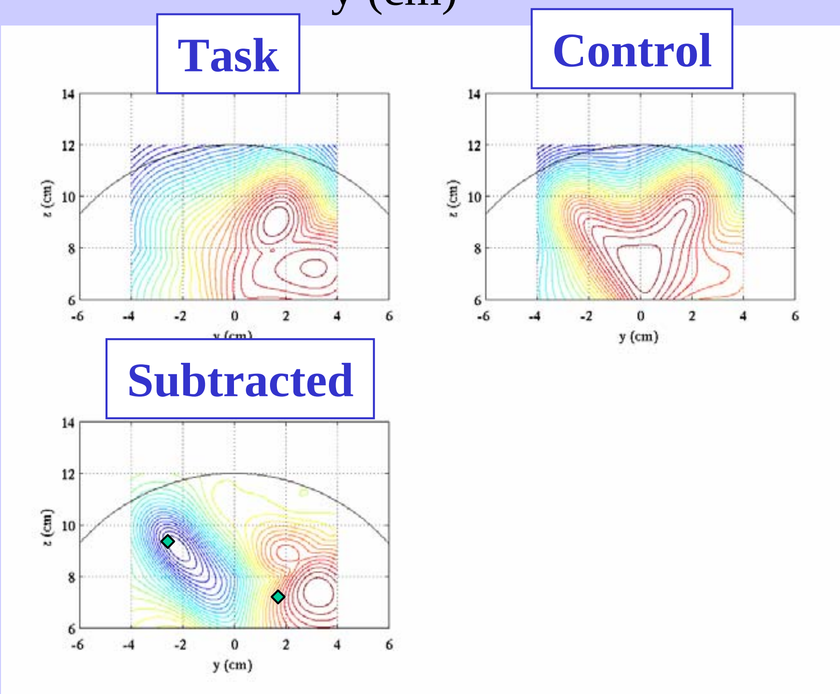
Scenario I and II--Numerical Experiments

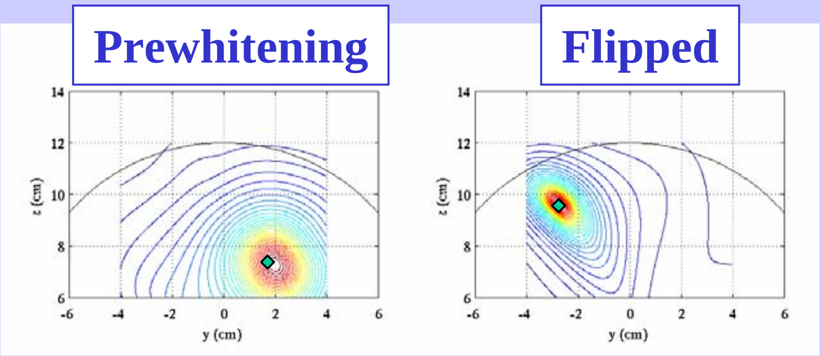


Signal-to-Interference Ratio:0.25

Relative source intensity

	control	task
First source	1	0.5
Second source	1	1
Third source	0.5	1

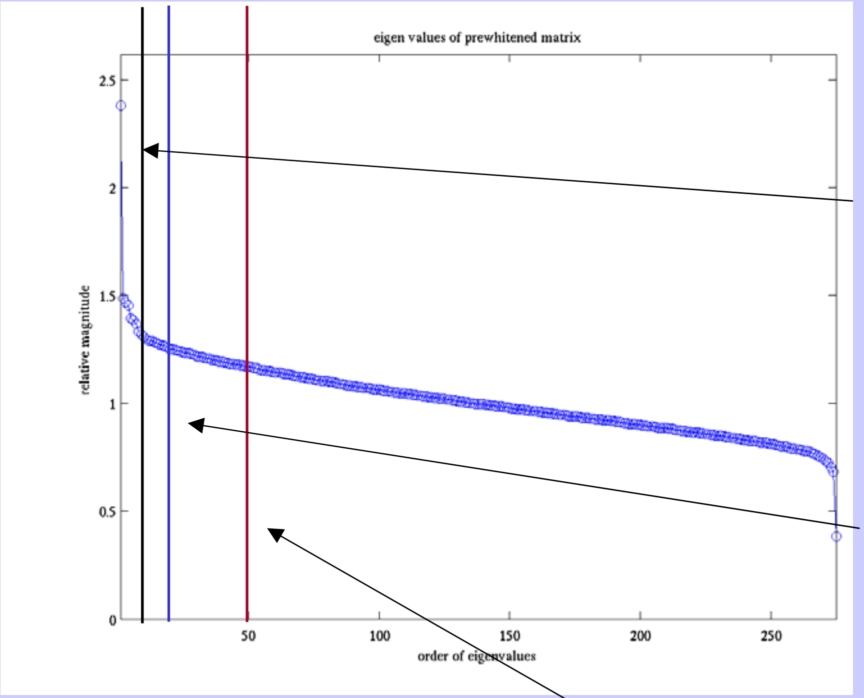




Robustness to overestimation of signal subspace dimension

Signal-to-Interference Ratio:0.5

Eigenvalue spectrum of $\tilde{\boldsymbol{R}}$

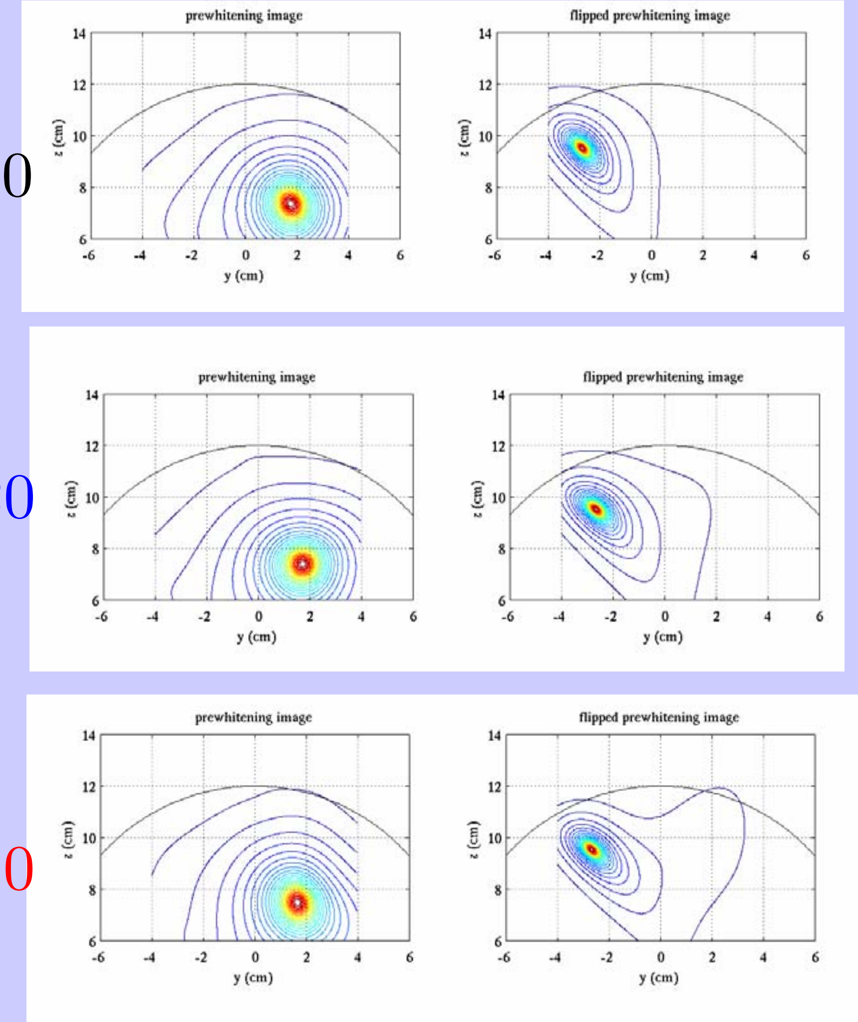


$Q = 10$

$Q = 20$

$Q = 50$

Prewhitening **Flipped**



Overestimation of signal-subspace dimension

$$\hat{\boldsymbol{R}}_S = \boldsymbol{R}_C^{1/2}\left[(\boldsymbol{U}_S\boldsymbol{U}_S^T + \boldsymbol{U}_\varepsilon\boldsymbol{U}_\varepsilon^T)(\tilde{\boldsymbol{R}} - \boldsymbol{I})\right]\boldsymbol{R}_C^{1/2} = \boldsymbol{R}_S + \Delta\boldsymbol{R}_S$$

Overestimated term

Error

$$\Delta\boldsymbol{R}_S = \underbrace{\sum_{j=1}^{Q}\Delta\lambda_j\boldsymbol{e}_j\boldsymbol{e}_j^T}_{\text{signal subspace component}} + \underbrace{\sum_{j=Q+1}^{M}\Delta\mu_j\boldsymbol{e}_j\boldsymbol{e}_j^T}_{\text{noise subspace component}}$$

$$\hat{\boldsymbol{R}}_{S+n} = \boldsymbol{R}_S + \Delta\boldsymbol{R}_S + \mu\boldsymbol{I} = \sum_{j=1}^{Q}(\lambda_j + \Delta\lambda_j)\boldsymbol{e}_j\boldsymbol{e}_j^T + \sum_{j=Q+1}^{M}(\mu + \Delta\mu_j)\boldsymbol{e}_j\boldsymbol{e}_j^T$$

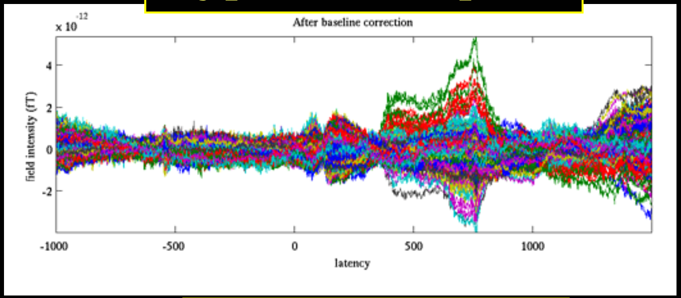
These terms modify source intensity

These terms modify regularization constant

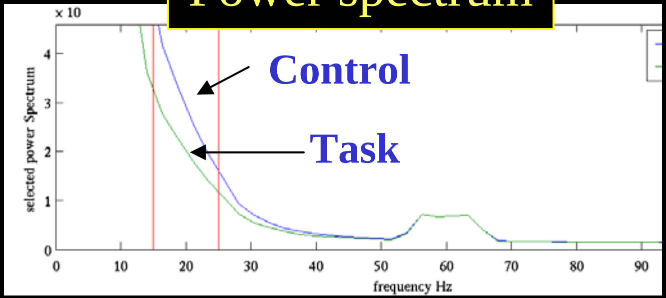
Hand-motor measurement

Voluntary right finger movement (every 5 sec)

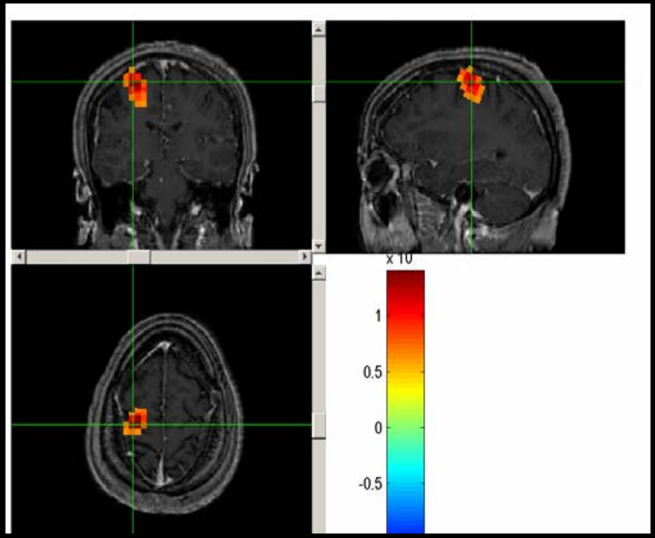
Typical raw epoch



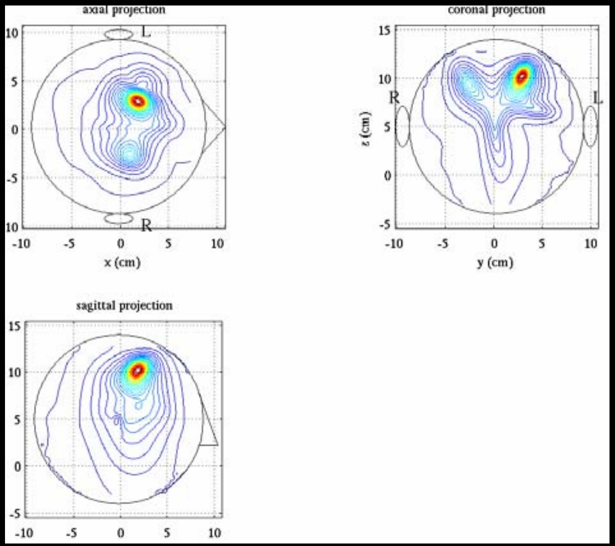
Power spectrum



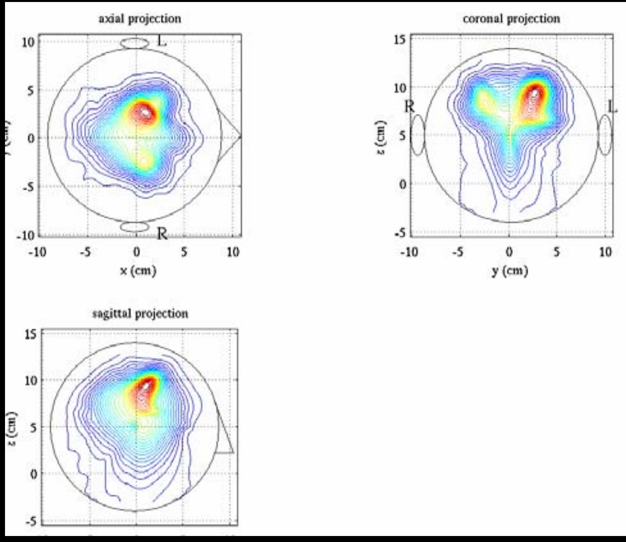
MRI overlay



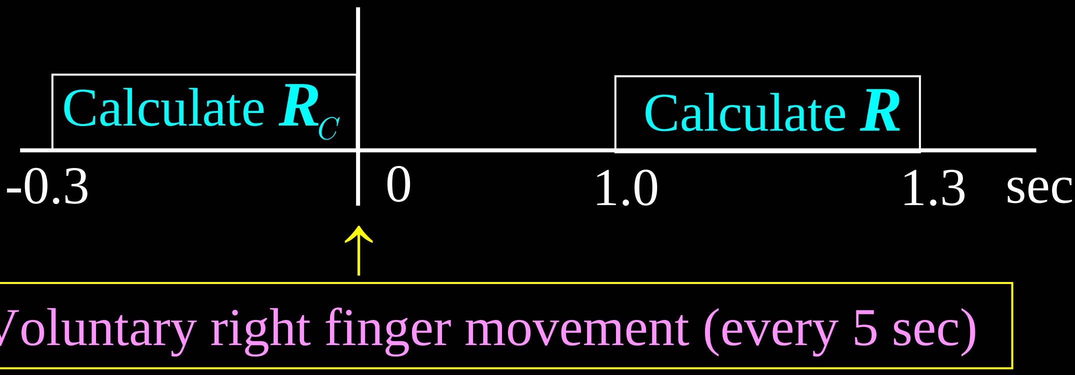
Prewhitening results



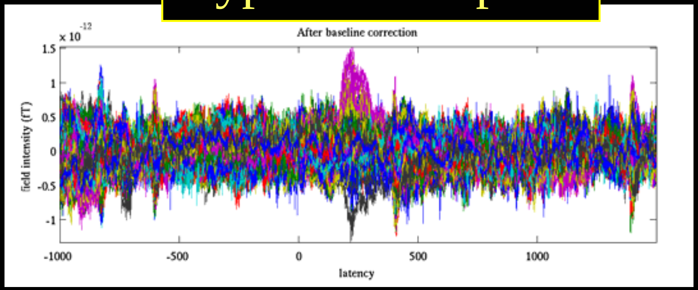
Subtraction results



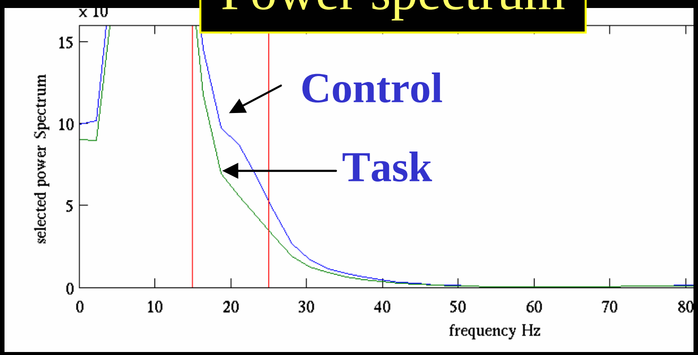
Hand-motor measurement



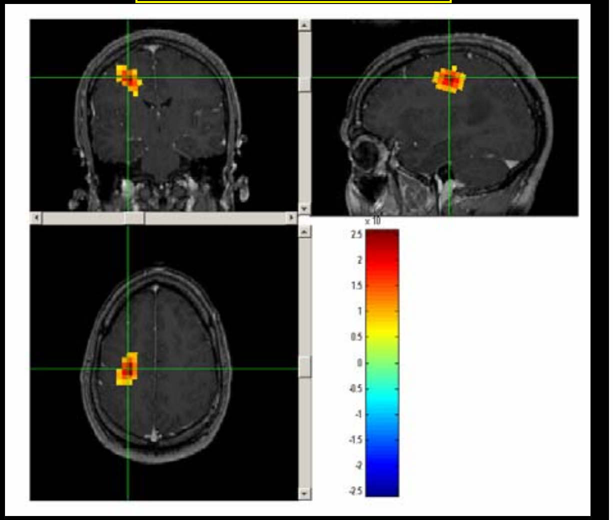
Typical raw epoch



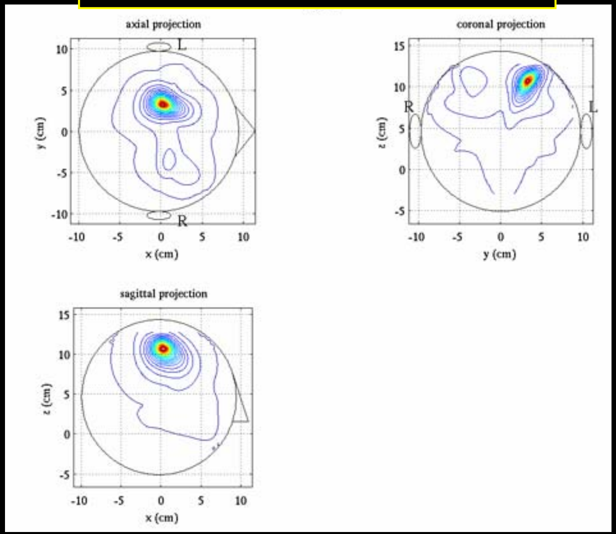
Power spectrum



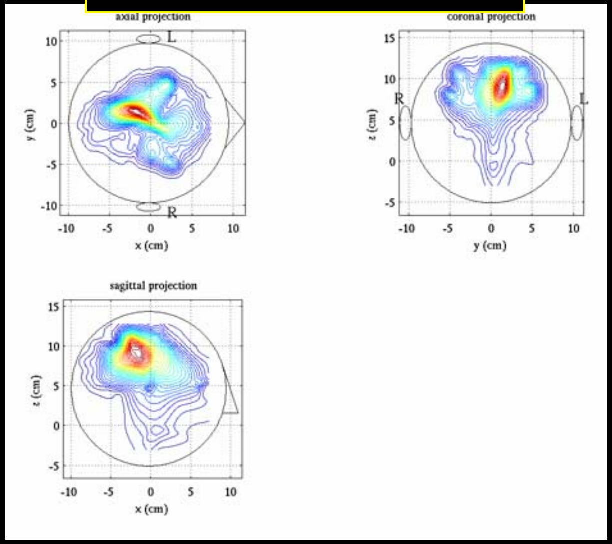
MRI overlay



Prewhitening results



Subtraction results



Summary

•We propose a novel prewhitening signal covariance estimation method.

•The proposed prewhitening method is shown to be effective in more realistic (non-ideal) scenarios of two-condition experiments.

•Prewhitening method gives significantly better source reconstruction (less bias and higher spatial resolution) than the existing subtraction-based method.

This investigation is presented at poster P-163, (session P4-1) 8/22 3:00—5:00PM

Collaborators

Kenneth E. Hilde
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Thank you for your attention

Visit http://www.tmit.ac.jp/~sekihara/