IEEE Asian Pacific Conference on Biomedical Engineering 2003
Keihanna, Japan, November, 2003

Neuromagnetic Source Reconstruction and Inverse Modeling

Application of adaptive spatial filter techniques

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This talk:

- formulates the neuromagnetic source reconstruction problem using spatial filter.
- introduces adaptive spatial filter (adaptive beamformer) techniques.
- clarifies its implicit assumptions on the source configuration.
- discuss a problem of SNR degradation caused by the errors in the forward modeling.

Magnetoencephalography
(Neuromagnetic measurements)

• can provide a high temporal resolution.

• <u>cannot provide (adequate) information on the source spatial configuration.</u>

$\Downarrow$

Efficient numerical algorithms for estimating source configuration are need to be developed.

(Source localization problem)

Source localization problem

- Dipole modeling approach

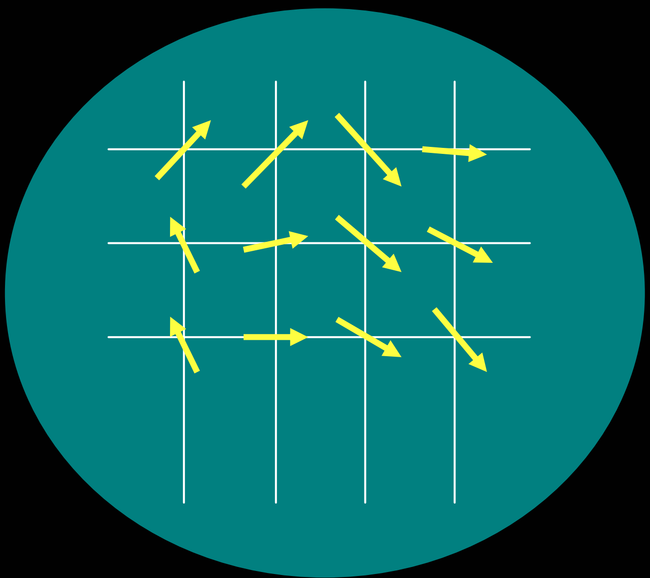
- Image reconstruction approach
  - Tomographic reconstruction methods
  - Spatial filter

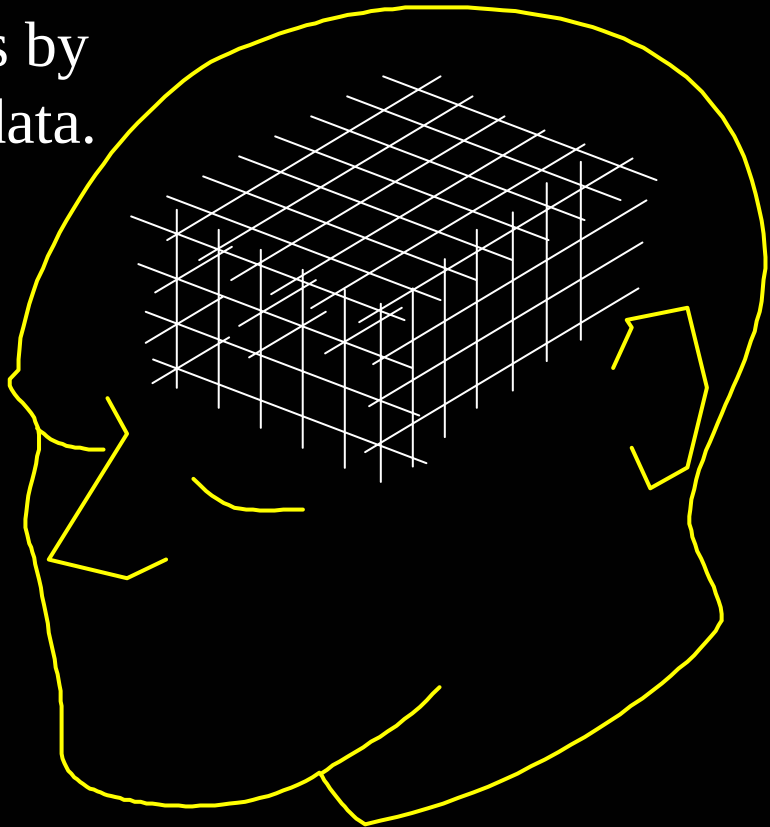
Tomographic reconstruction

- Assume pixel grids in the region of interest.

- Assume a source at each grid.

- Estimate the moment of these sources by least-squares fitting to the measured data.

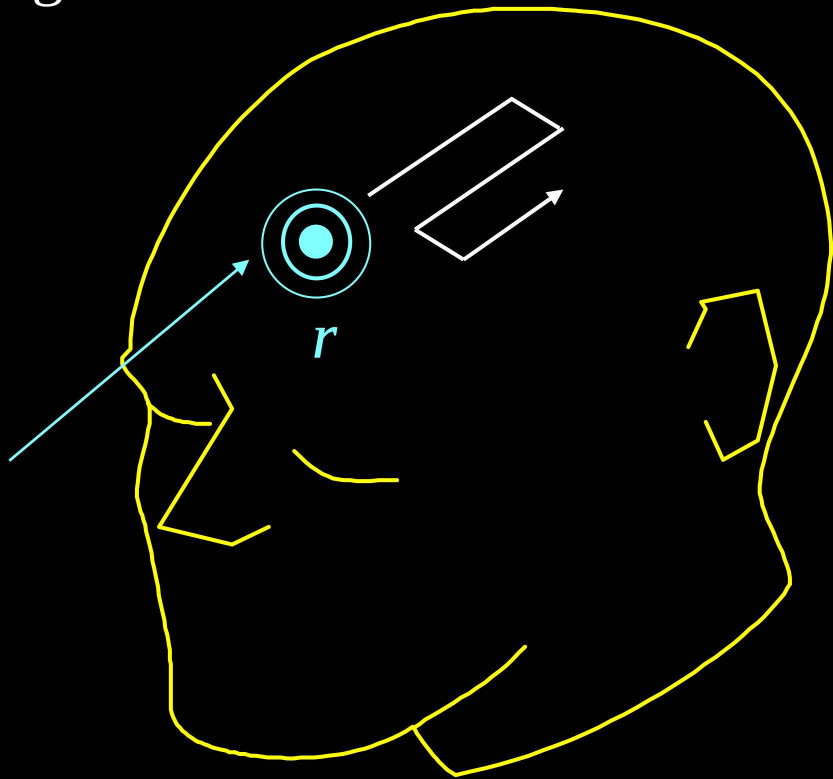




Spatial filter technique

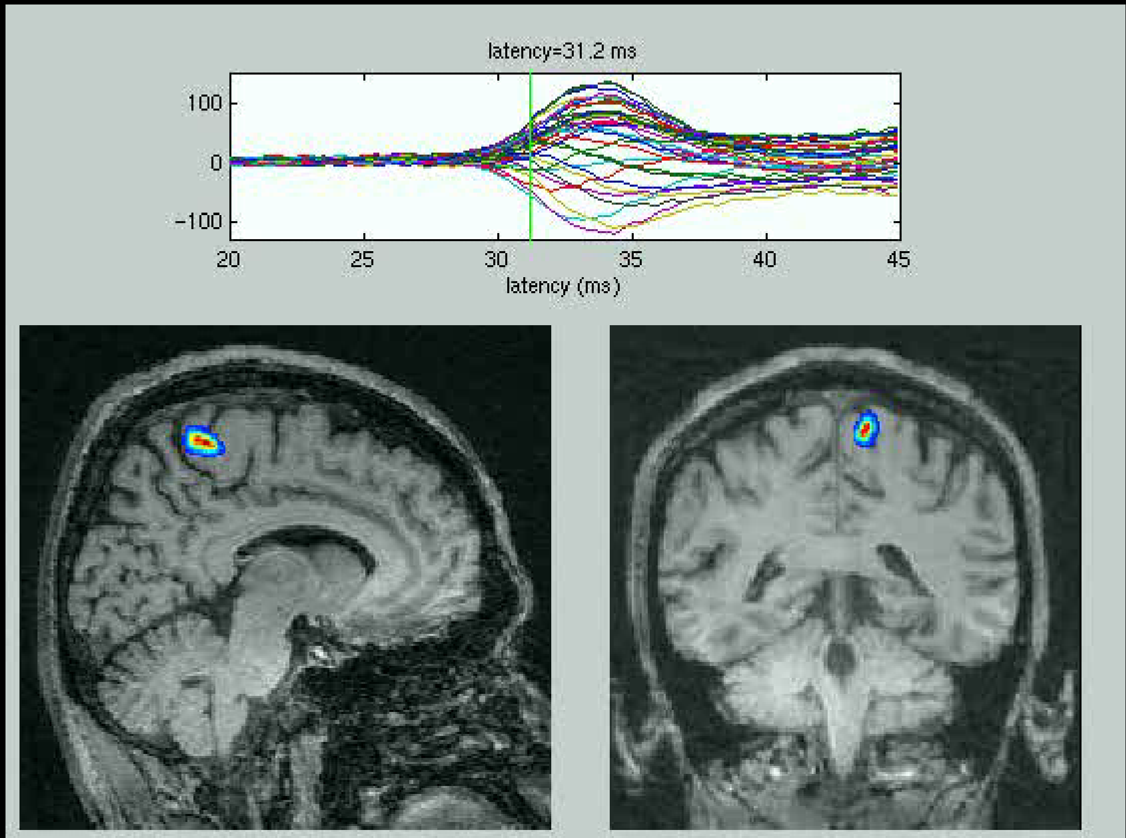
•Form spatial filter weight $w(r)$ that focuses the sensitivity of the sensor array at a small area at $\boldsymbol{r}$.

•Scan this focused area over the region of interest to obtain source reconstruction.



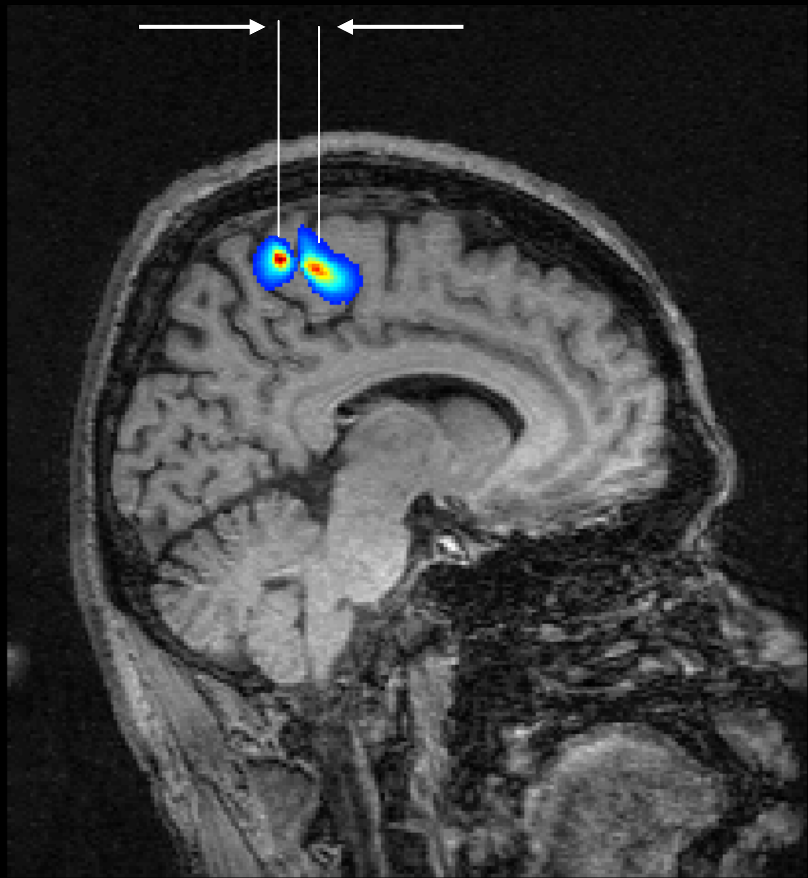
Right posterior tibial nerve stimulation

measured by a 37-channel sensor array



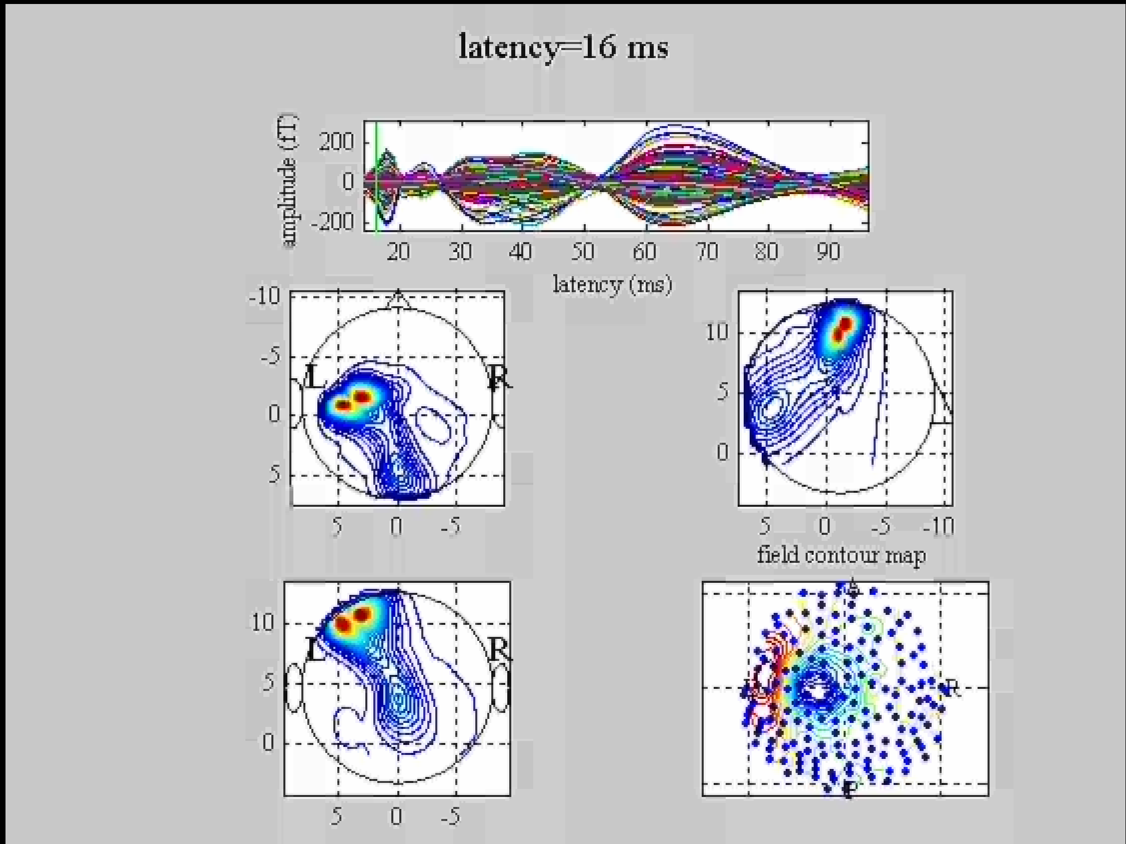
Hashimoto et al., NeuroReport 2001

7 mm

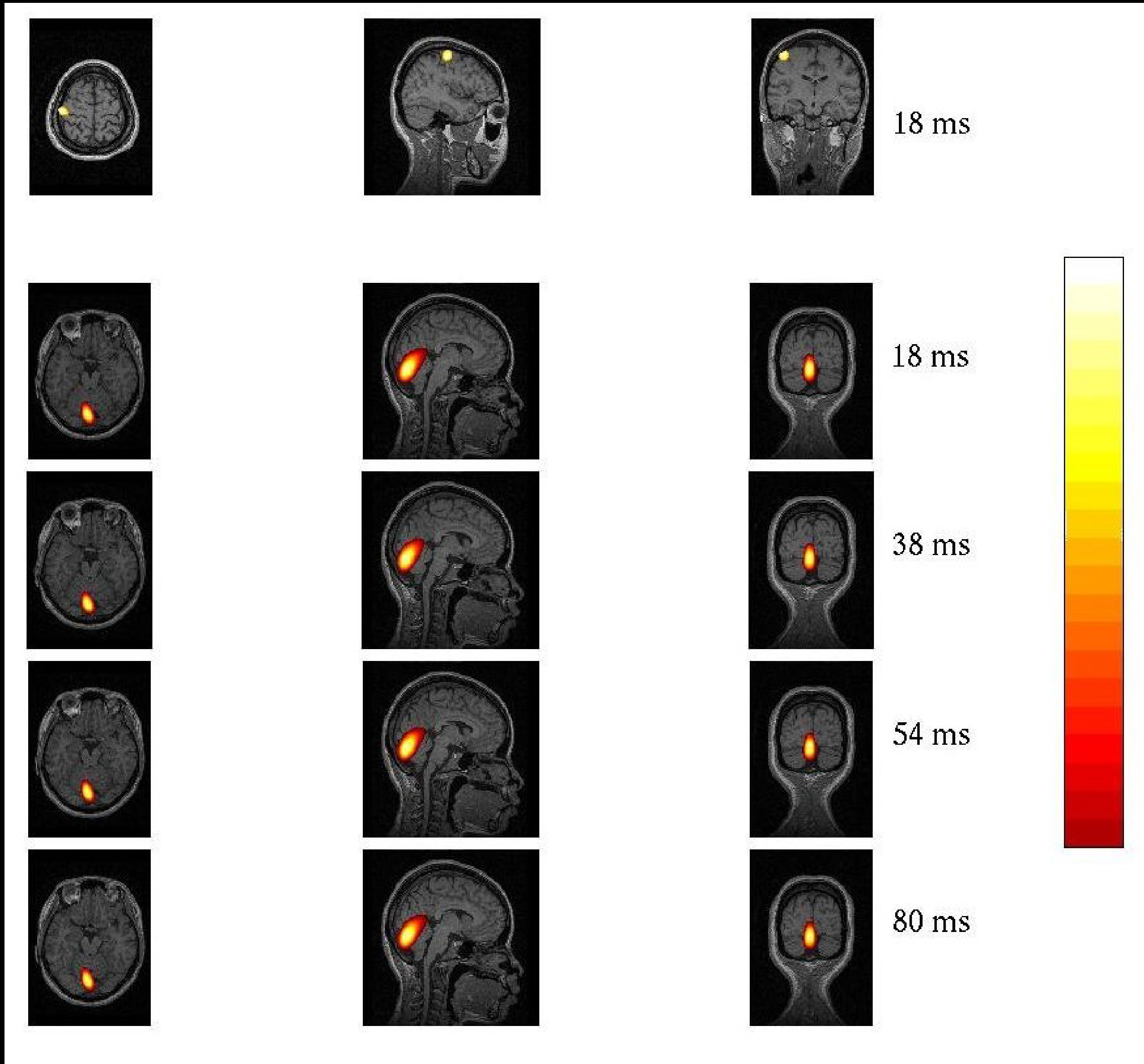


Right median nerve stimulation

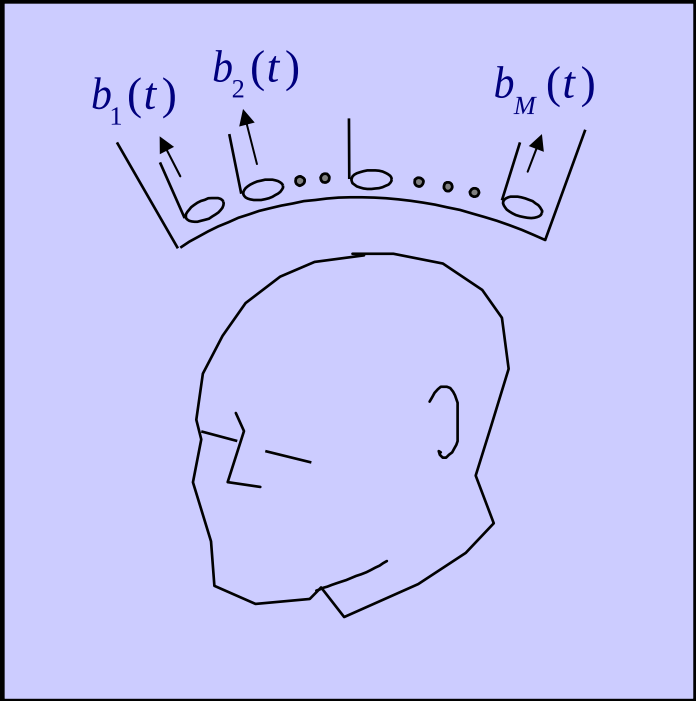
measured by a 160-channel whole-head sensor array



Hashimoto et al., J. Clinical Neurophysiology submitted for publication


18 ms
18 ms
38 ms
54 ms
80 ms

Definitions

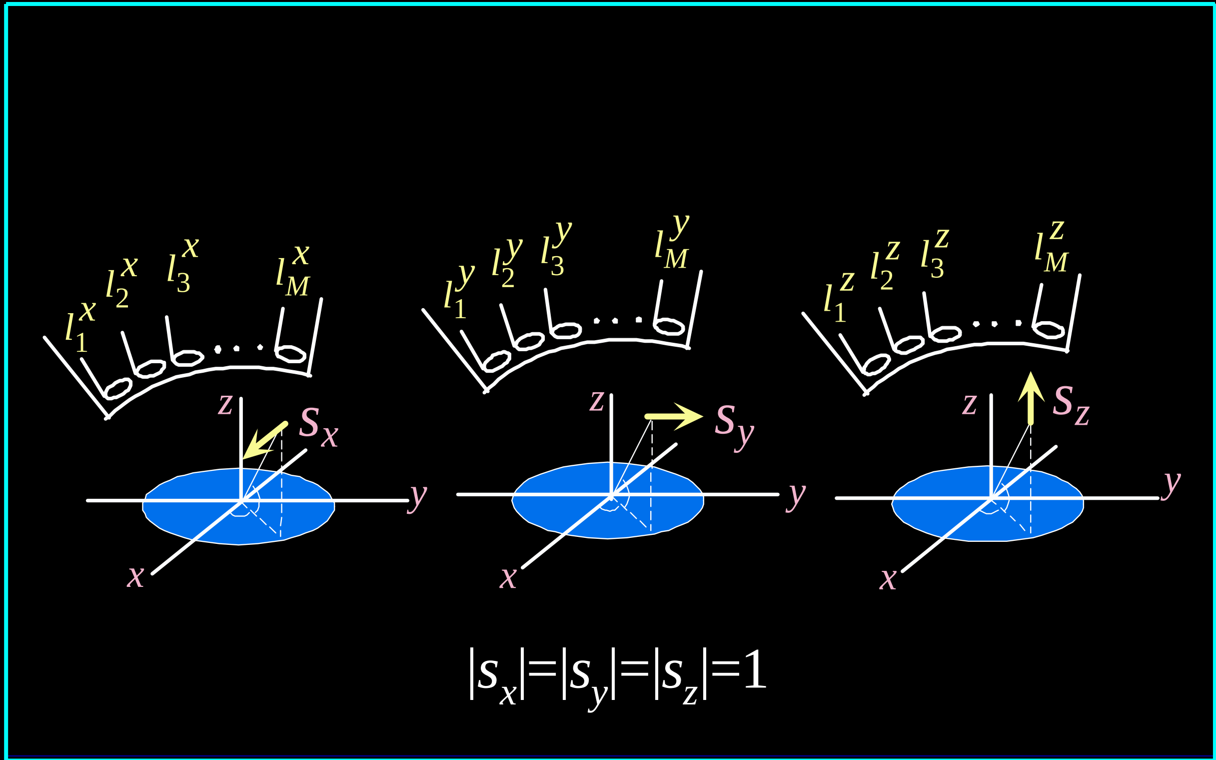


- data vector: $\boldsymbol{b}(\mathrm{t}) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$

- data covariance matrix: $\boldsymbol{R} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

- source magnitude: $s(\boldsymbol{r}, t)$

- source orientation: $\boldsymbol{\eta}(\boldsymbol{r}, t) = [\eta_x(\boldsymbol{r}, t), \eta_y(\boldsymbol{r}, t), \eta_z(\boldsymbol{r}, t)]^T$



Lead field vector for the source orientation $\boldsymbol{\eta}(\boldsymbol{r})$

$$\boldsymbol{L}(\boldsymbol{r}) = \begin{bmatrix} l_1^x(\boldsymbol{r}) & l_1^y(\boldsymbol{r}) & l_1^z(\boldsymbol{r}) \\ l_2^x(\boldsymbol{r}) & l_2^y(\boldsymbol{r}) & l_2^z(\boldsymbol{r}) \\ \vdots & \vdots & \vdots \\ l_M^x(\boldsymbol{r}) & l_M^y(\boldsymbol{r}) & l_M^z(\boldsymbol{r}) \end{bmatrix}, \quad \boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{L}(\boldsymbol{r}) \begin{bmatrix} \eta_x(\boldsymbol{r}) \\ \eta_y(\boldsymbol{r}) \\ \eta_z(\boldsymbol{r}) \end{bmatrix}$$

Basic relationship

$$b_j(t) = \int l_j(r)s(r,t)dr$$

or

$$b(t) = \int L(r)s(r,t)dr$$

Problem of source localization:

<u>Estimate $s(r,t)$ from the measurement $b(t)$</u>

Spatial filter

$$\hat{s}(r,t) = w^T(r)b(t) = [w_1(r),\ldots,w_M(r)]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t)$$

$\Uparrow$ estimate of $s(r,t)$

$\Uparrow$ weight vector

Non-adaptive weight: $w(r)$ is data independent

Adaptive weight: $w(r)$ is data dependent

Adaptive spatial filter

Minimum-variance beamformer

$$\hat{s}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = [w_1(\boldsymbol{r}),\ldots,w_M(\boldsymbol{r})]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t)$$

$$\min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T\boldsymbol{l}(\boldsymbol{r}) = 1 \quad \Rightarrow \quad \boldsymbol{w}^T(\boldsymbol{r}) = \frac{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

$$\left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

Assumption that source activities are uncorrelated

With constraint: $\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_p) = 1,$

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r}_p) = \left\langle s(\boldsymbol{r}_p,t)^2 \right\rangle + \sum_{q \neq p} \left\langle s(\boldsymbol{r}_q,t)^2 \right\rangle \left\| \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) \right\|$$

$$\Uparrow$$

$$\left\langle s(\boldsymbol{r}_p,t) s(\boldsymbol{r}_q,t) \right\rangle = 0 \quad \text{when} \quad p \neq q$$

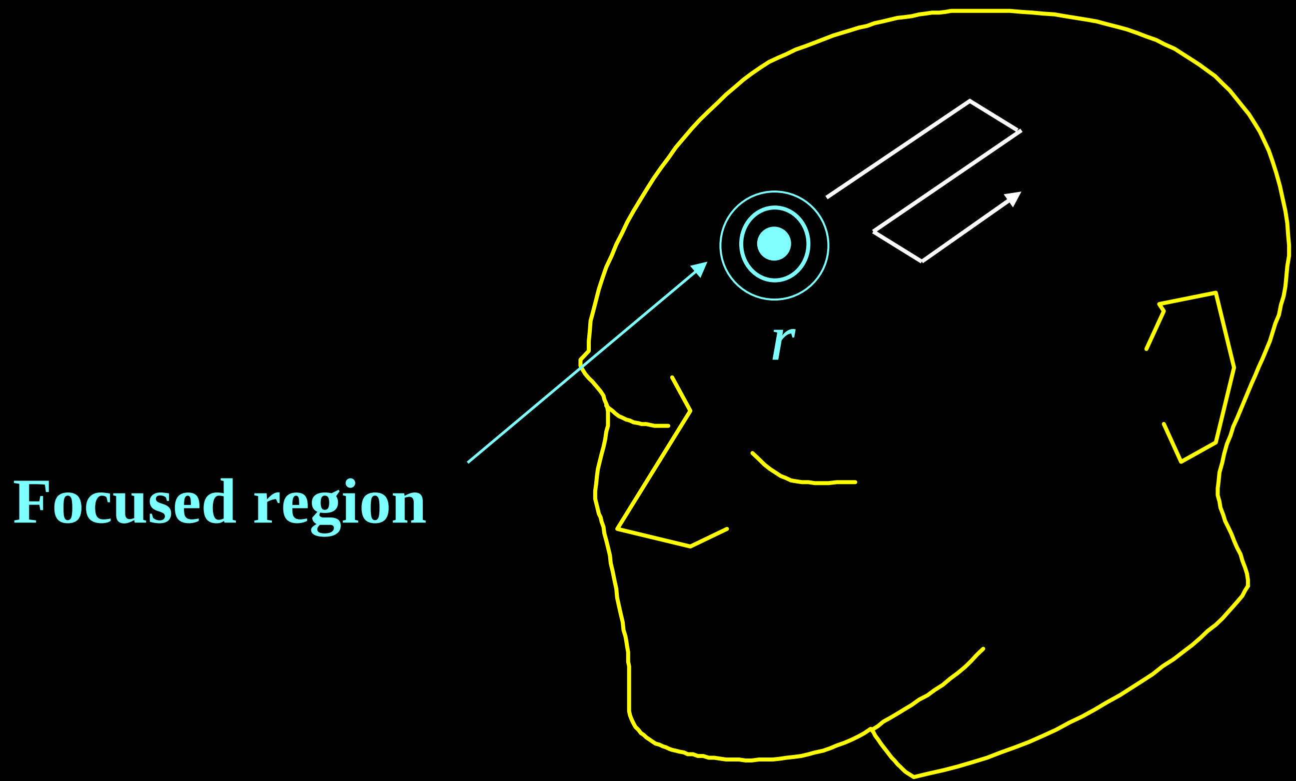
$$\min_{\boldsymbol{w}} \left[\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r}_p) \right] \Rightarrow \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q)=0,\ q \neq p$$

Therefore, this minimization gives the weight satisfying

$$\begin{aligned} \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) &= 1 \quad \text{for } p = q \\ &= 0 \quad \text{for } p \neq q \end{aligned}$$

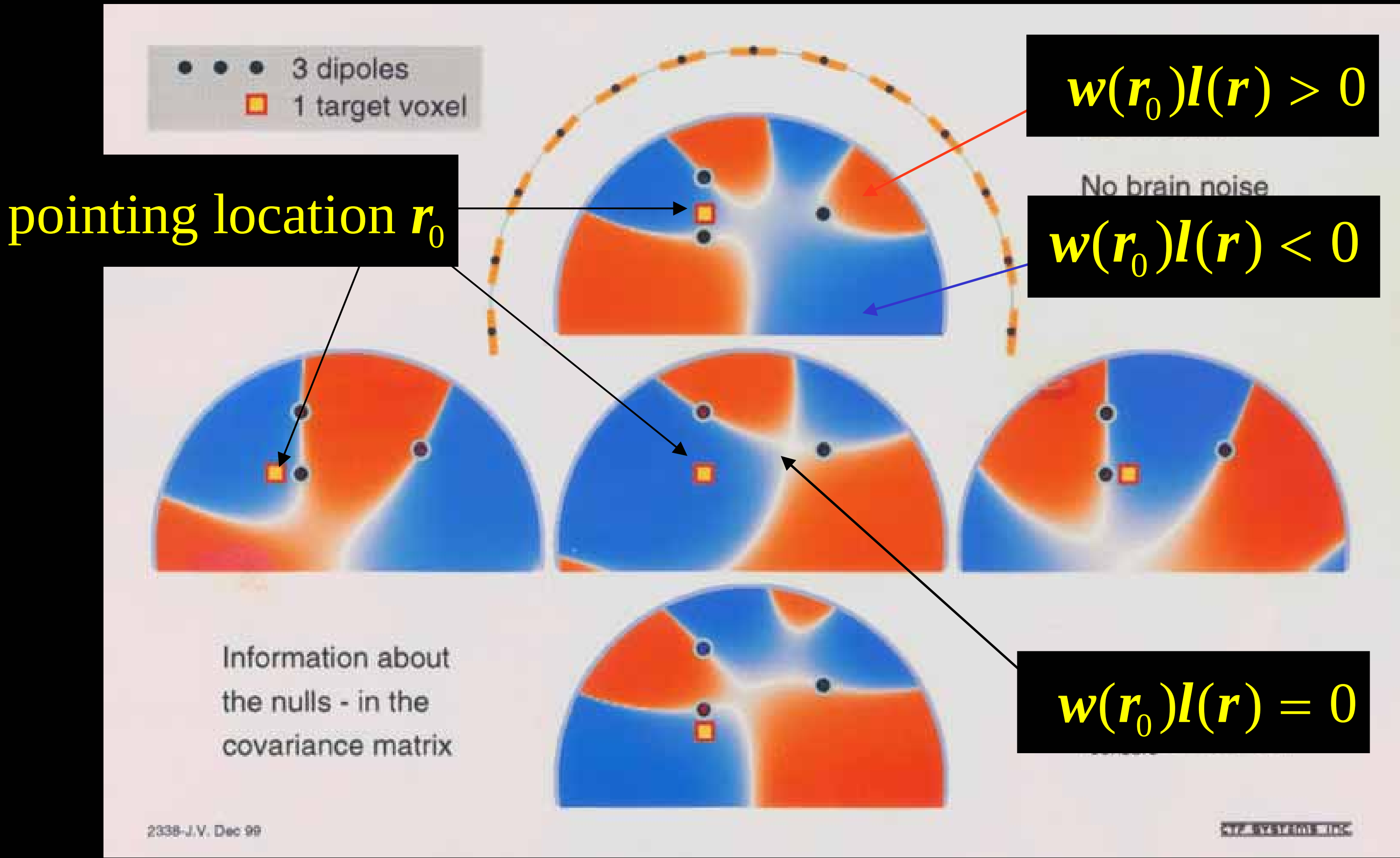
Spatial filter technique

•Form spatial filter weight $w(r)$ that focuses the sensitivity of the sensor array at a small area at ***r***.



Adaptive beamformer sensitivity pattern: plot of $\boldsymbol{w}(\boldsymbol{r}_0)\boldsymbol{l}(\boldsymbol{r})$

The density of the colors is propotional to $\boldsymbol{w}(\boldsymbol{r}_0)\boldsymbol{l}(\boldsymbol{r})$.



The weight sets null-sensitivity at regions where sources exist.

Low-rank signal assumption

Consider a easiest case where we know locations and orientations of all Q sources

weight $\boldsymbol{w}(\boldsymbol{r}_1)$ (containing M unknowns) can be obtained by solving a set of Q linear equations:

$$
\begin{aligned}
\boldsymbol{w}^T(\boldsymbol{r}_1)\boldsymbol{l}(\boldsymbol{r}_1) &= w_1(\boldsymbol{r}_1)l_1(\boldsymbol{r}_1) + \ldots + w_M(\boldsymbol{r}_1)l_M(\boldsymbol{r}_1) = 1 \\
\boldsymbol{w}^T(\boldsymbol{r}_1)\boldsymbol{l}(\boldsymbol{r}_2) &= w_1(\boldsymbol{r}_1)l_1(\boldsymbol{r}_2) + \ldots + w_M(\boldsymbol{r}_1)l_M(\boldsymbol{r}_2) = 0 \\
&\vdots \\
\boldsymbol{w}^T(\boldsymbol{r}_1)\boldsymbol{l}(\boldsymbol{r}_Q) &= w_1(\boldsymbol{r}_1)l_1(\boldsymbol{r}_Q) + \ldots + w_M(\boldsymbol{r}_1)l_M(\boldsymbol{r}_Q) = 0
\end{aligned}
$$

when $Q > M$, there is no solution for $\boldsymbol{w}^T(\boldsymbol{r}_1)$

Low-rank signal

Number of sensors M > Number of sources Q

$$\boldsymbol{R} = \boldsymbol{U}\left[\begin{array}{ccc|cc} \lambda_1 & 0 & \cdots & \cdot & 0 \\ 0 & \ddots & & 0 & \cdot \\ \vdots & & \lambda_Q & & \vdots \\ \hline \cdot & 0 & & \ddots & 0 \\ 0 & \cdot & \cdots & 0 & \lambda_M \end{array}\right]\boldsymbol{U}^T = \boldsymbol{U}\begin{bmatrix} \boldsymbol{\Lambda}_S & 0 \\ 0 & \boldsymbol{\Lambda}_N \end{bmatrix}\boldsymbol{U}^T$$

$$\boldsymbol{U} = [\underbrace{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_Q}_{\boldsymbol{E}_S} \mid \underbrace{\boldsymbol{e}_{Q+1}, \ldots, \boldsymbol{e}_M}_{\boldsymbol{E}_N}] = [\boldsymbol{E}_S \mid \boldsymbol{E}_N]$$

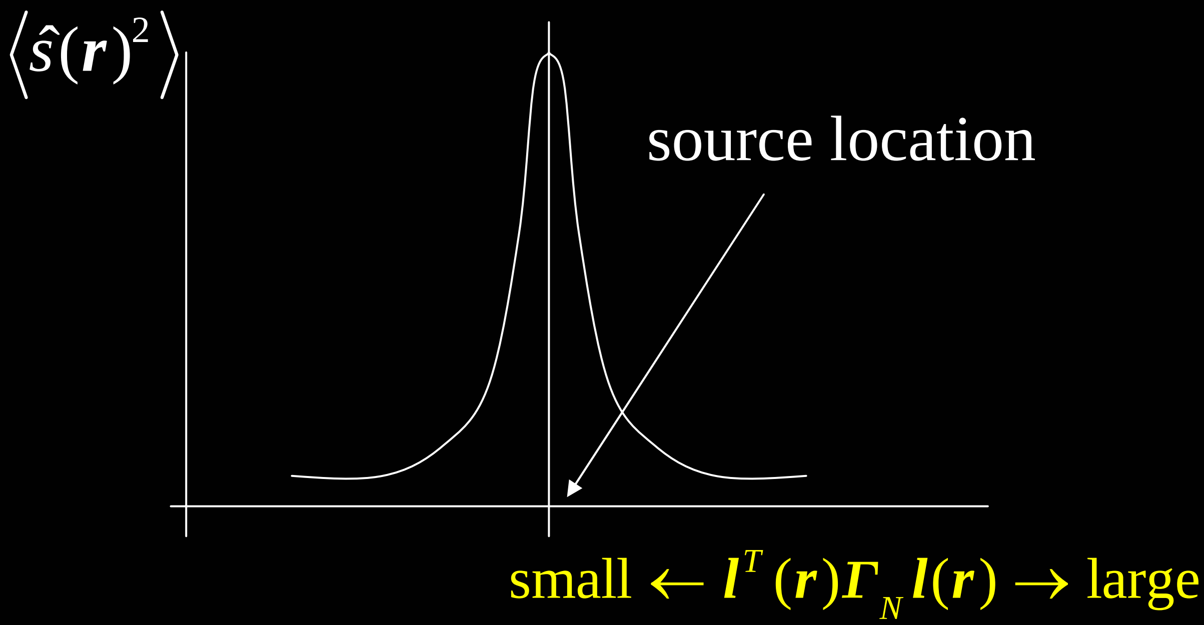
$$\boxed{\boldsymbol{\Gamma}_S^{-1} = \boldsymbol{E}_S \boldsymbol{\Lambda}_S^{-1} \boldsymbol{E}_S^T \ \text{ and } \ \boldsymbol{\Gamma}_N^{-1} = \boldsymbol{E}_N \boldsymbol{\Lambda}_N^{-1} \boldsymbol{E}_N^T \ \Rightarrow R^{-1} = \boldsymbol{\Gamma}_S^{-1} + \boldsymbol{\Gamma}_N^{-1}}$$

Orthogonality principle

$$\boxed{\boldsymbol{E}_N^T \boldsymbol{l}(\boldsymbol{r}_q) = \boldsymbol{\Gamma}_N^{-1} \boldsymbol{l}(\boldsymbol{r}_q) = 0 \text{ at any source location } \boldsymbol{r}_q}$$

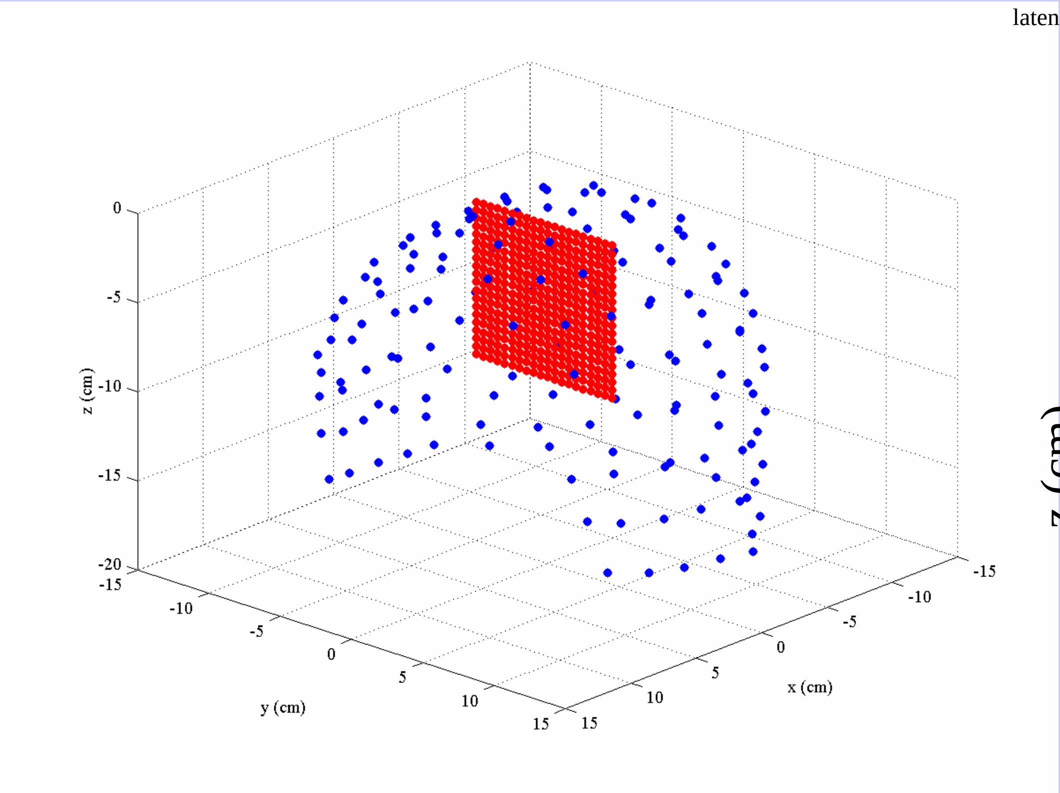
Minimum-variance spatial filter output:

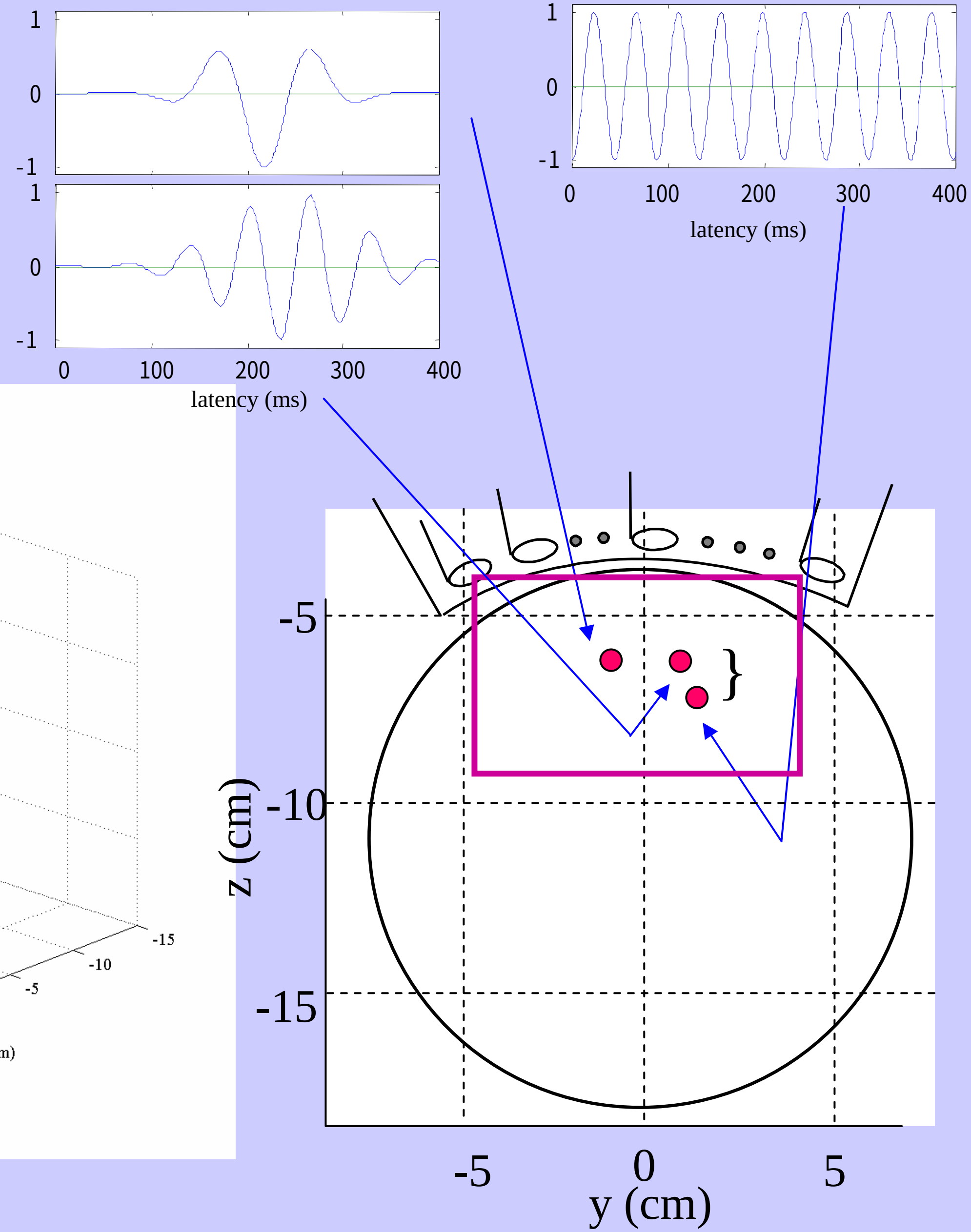
$$\left\langle \hat{s}(\boldsymbol{r})^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})} = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S^{-1}\boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_N^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

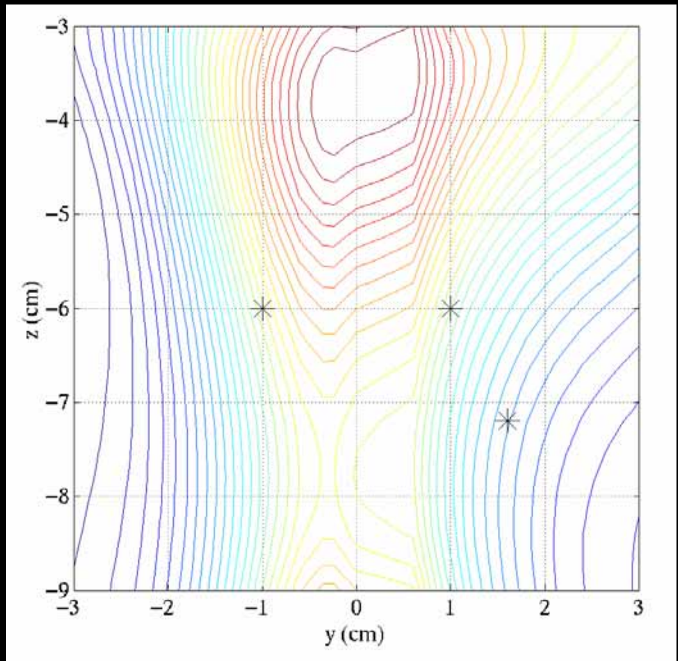


assumed source waveform

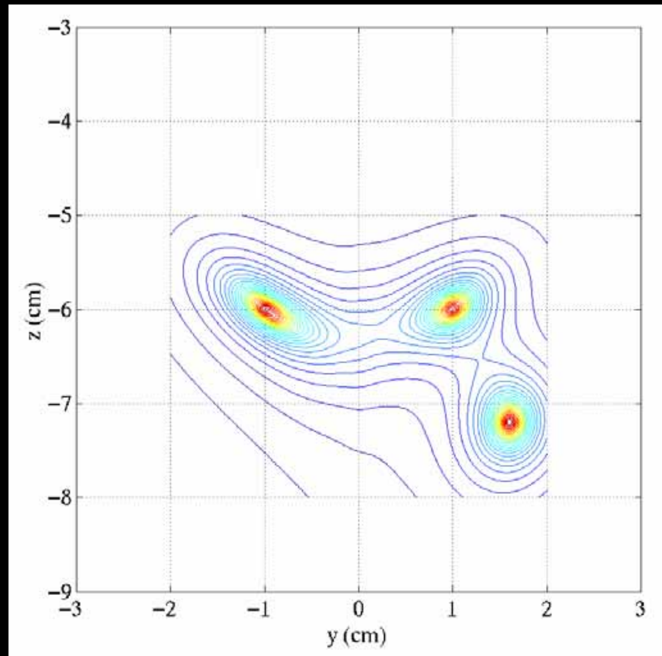
148-channel sensor array







Minimum-norm
reconstruction



Minimum-variance
spatial filter reconstruction

Adaptive-beamformer techniques were originally developed in the fields of radar, sonar, and seismic exploration.

Two major problems arise when applying minimum-variance beamformer to neuromagnetic source reconstruction.

- Output SNR degradation.
- Vector source detection.

Output SNR degradation.

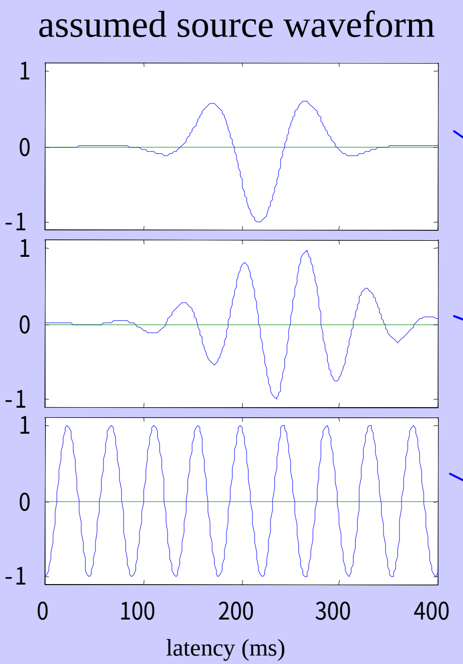
Minimum-variance beamformer is very sensitive to errors in forward modeling or errors in sample covariance matrix.

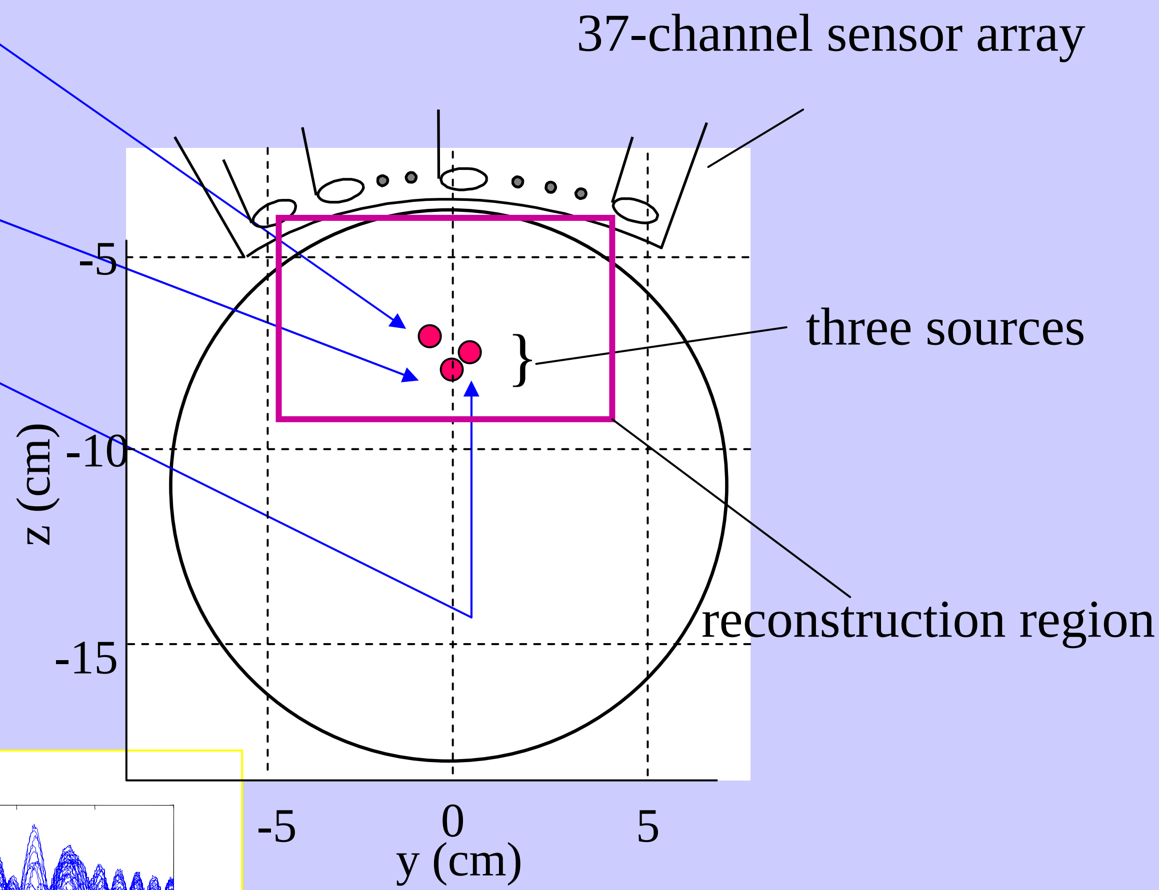
$\Downarrow$

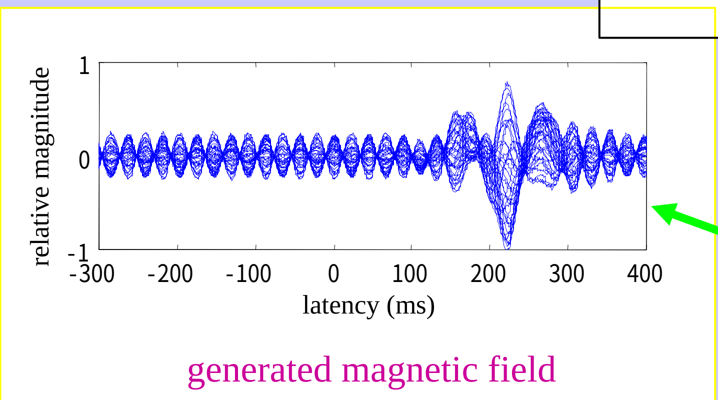
Because such errors are almost inevitable in neuromagnetic measurements, minimum-variance beamformer generally provides noisy spatio-temporal reconstruction results.

$\Downarrow$

Introducing eigenspace projection

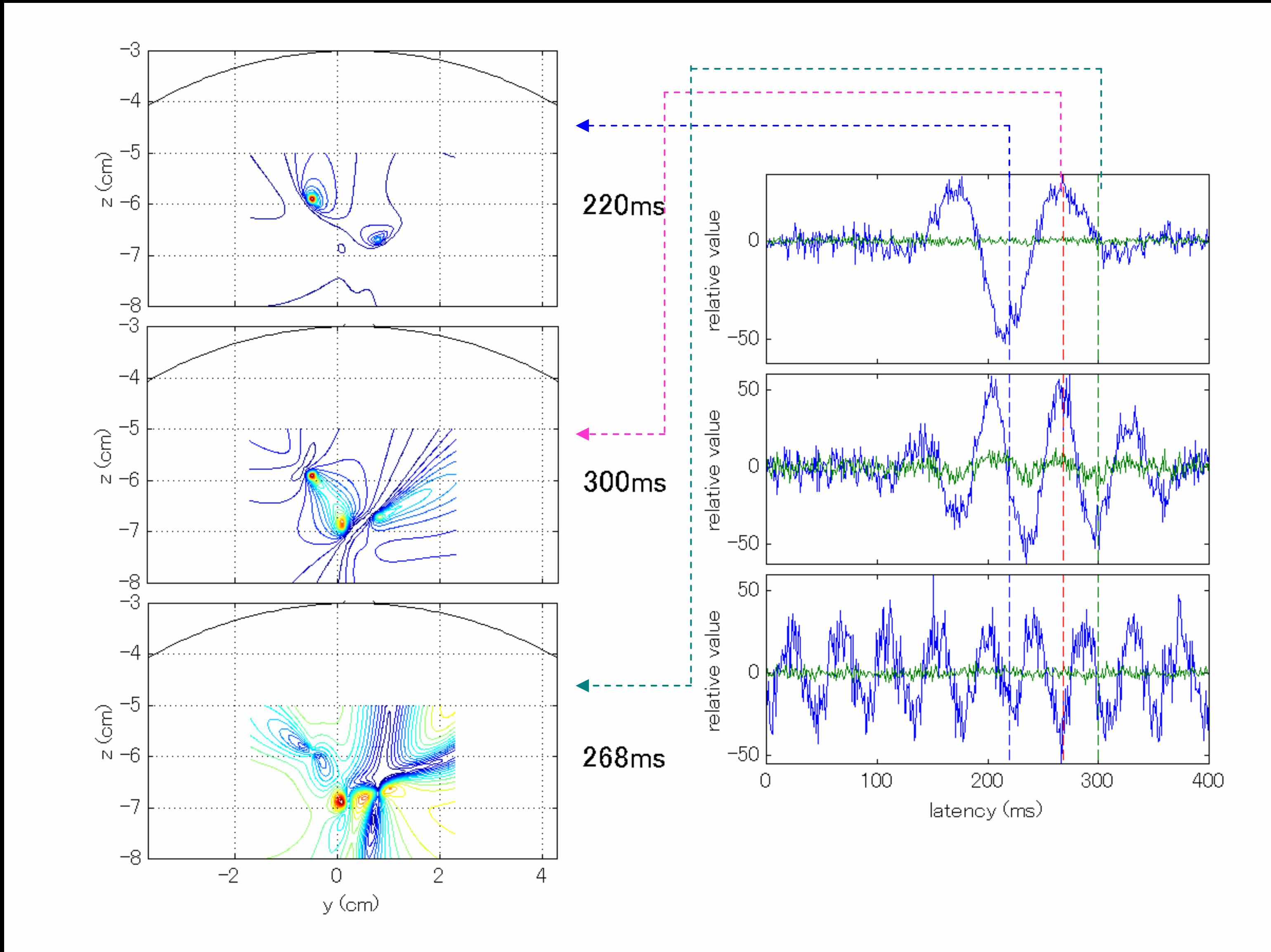






Add very small amount of noise
to obtain the simulated MEG recordings

Spatio-tempotal reconstruction



Recall some definitions:

$$R = U\left[\begin{array}{ccc|cc} \lambda_1 & 0 & \cdots & \cdot & 0 \\ 0 & \ddots & & 0 & \cdot \\ \vdots & & \lambda_P & & \vdots \\ \hline \cdot & 0 & & \ddots & 0 \\ 0 & \cdot & \cdots & 0 & \lambda_M \end{array}\right]U^T = U\begin{bmatrix} \boldsymbol{\Lambda}_S & 0 \\ 0 & \boldsymbol{\Lambda}_N \end{bmatrix}U^T, \text{ and } U = [\underbrace{e_1, \ldots, e_P}_{E_S} \mid \underbrace{e_{P+1}, \ldots, e_M}_{E_N}]$$

Also, $\boldsymbol{\Gamma}_S^{-1} = \boldsymbol{E}_S \boldsymbol{\Lambda}_S^{-1} \boldsymbol{E}_S^T$, $\boldsymbol{\Gamma}_N^{-1} = \boldsymbol{E}_N \boldsymbol{\Lambda}_N^{-1} \boldsymbol{E}_N^T$

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overall error in estimating $l(r)$

$$\text{Output SNR} \propto \frac{[l^T(r)\boldsymbol{\Gamma}_S^{-1} l(r)]^2}{[l^T(r)\boldsymbol{\Gamma}_S^{-2} l(r) + \varepsilon^T \boldsymbol{\Gamma}_N^{-2} \varepsilon]}$$

Even when ε is small, $\varepsilon^T \boldsymbol{\Gamma}_N^{-2} \varepsilon$ may not be small,
because $\varepsilon^T \boldsymbol{\Gamma}_N^{-2} \varepsilon \approx \|\varepsilon\|^2 / \lambda_{p+j}^2 \leftarrow$ noise level eigenvalue

Eigenspace projection

The error term $\varepsilon^T \Gamma_N^{-2} \varepsilon$ arises from the noise subspace component of $\boldsymbol{w}(\boldsymbol{r})$.

Extension to eigenspace projection beamformer

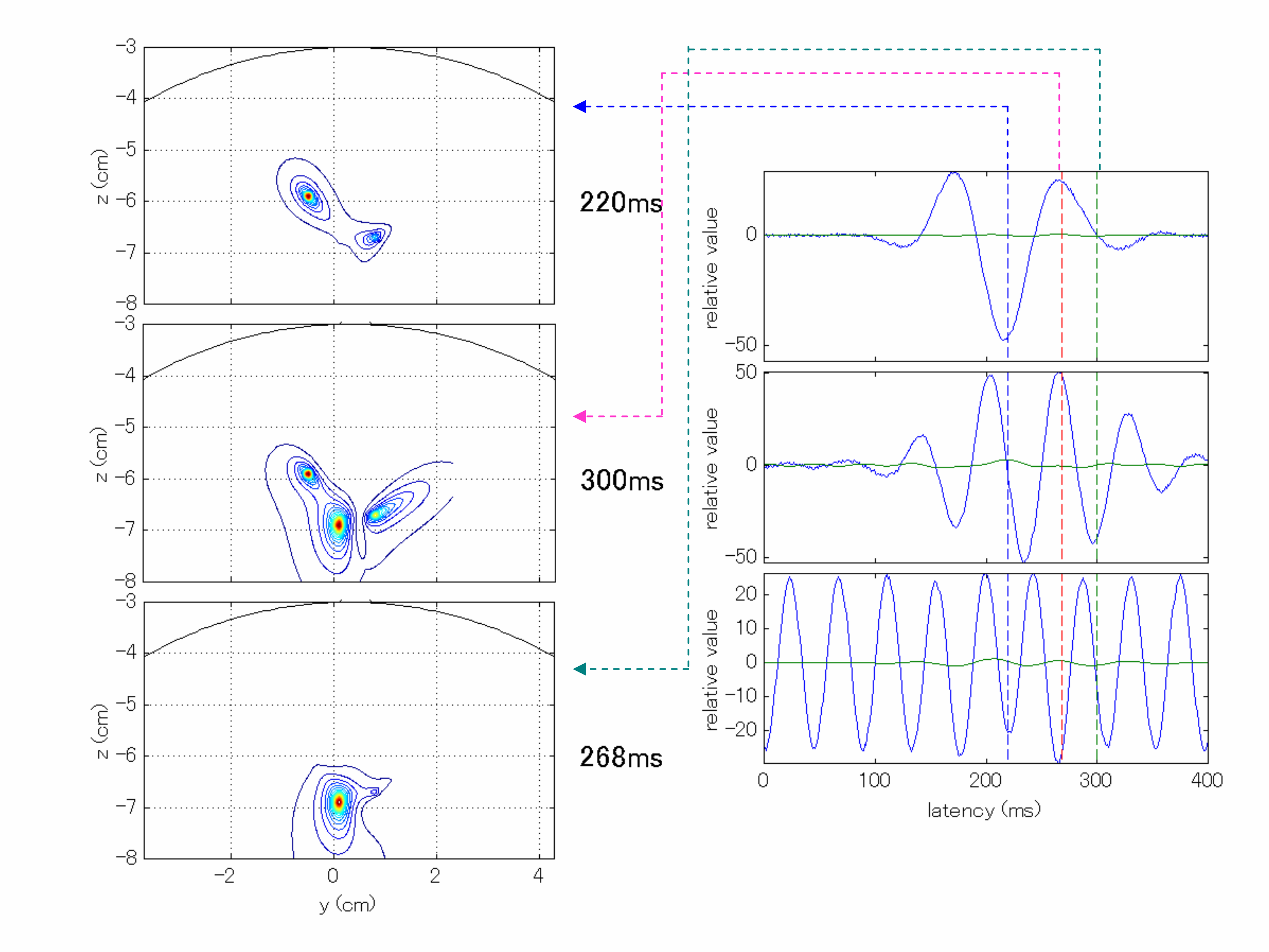
$$\bar{w}_\mu = E_S E_S^T w_\mu, \quad \text{where } \mu = x, y \text{ or } z$$

Output SNR $\propto \dfrac{[l^T(r)\Gamma_S^{-1} l(r)]^2}{[l^T(r)\Gamma_S^{-2} l(r) + \varepsilon^T \Gamma_N^{-2} \varepsilon\ ]}$ (non-eigenspace projected)

$$\Downarrow$$

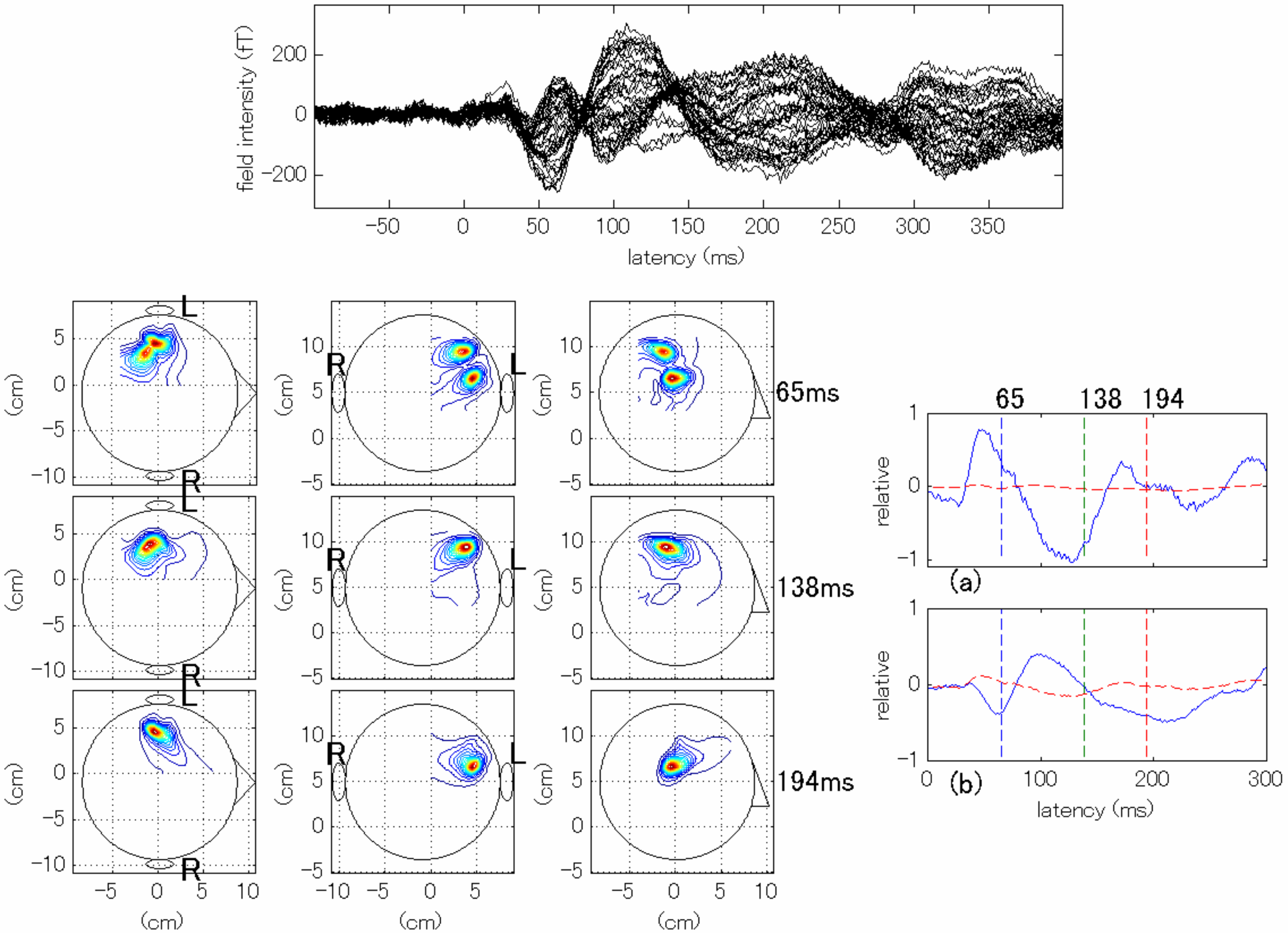
Output SNR $\propto \dfrac{[l^T(r)\Gamma_S^{-1} l(r)]^2}{[l^T(r)\Gamma_S^{-2} l(r)]}$ (eigenspace projected)

Spatio-tempotal reconstruction with eigen-space projection

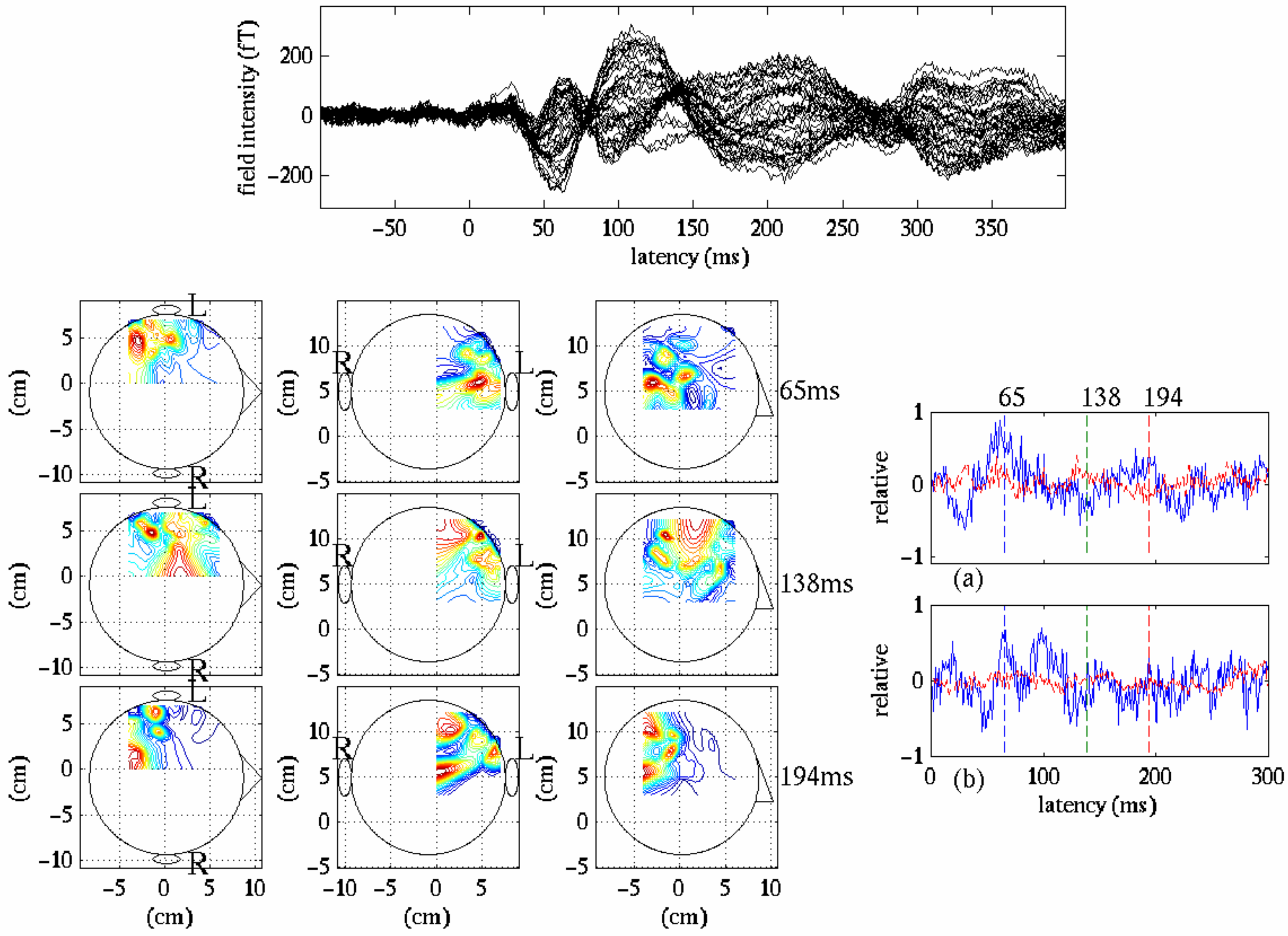


Application to 37-channel auditory-somatosensory recording

eigenspace-projection results



Application to 37-channel auditory-somatosensory recording
Non-eigenspace projected results



Summary

•This talk reviews the application of adaptive beamformer to reconstruction of brain activities.

•Eigenspace projection is shown to overcome the SNR degradation caused by errors in forward modeling; such errors are unavoidable in MEG measurements.

The implicit assumptions for the adaptive beamformer are that sources are uncorrelated and that the signal is low rank.

The influences caused when these assumptions are invalidated have been discussed elsewhere. The PDF versions of the presentations regarding those matters are available in

http://www.tmit.ac.jp/~sekihara/

The PDF version of this power-point presentation as well as PDFs of the recent publications are also available.

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