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Adaptive beamformer source reconstruction:

Our recent developments toward spatio-temporal reconstruction of brain activities

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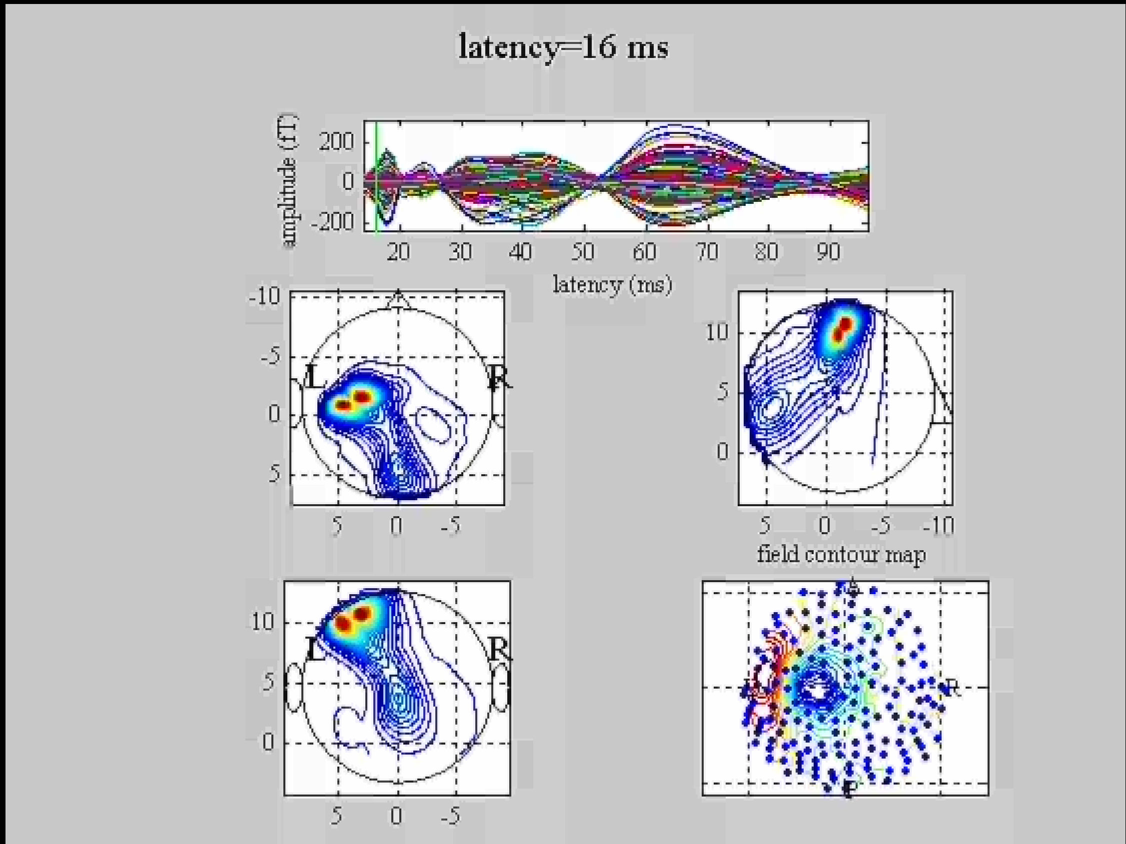
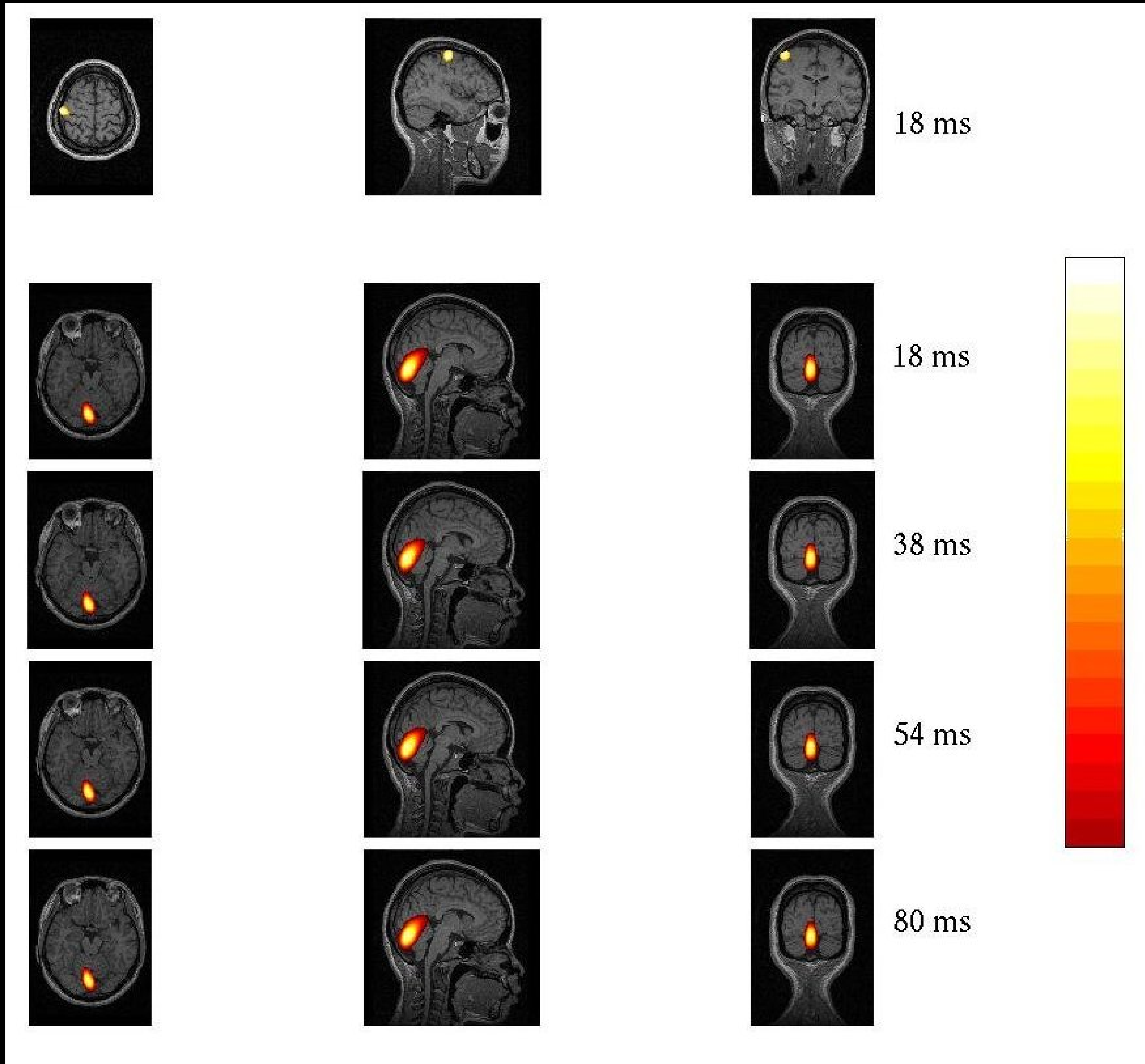
Right median nerve stimulation

measured by a 160-channel whole-head sensor array

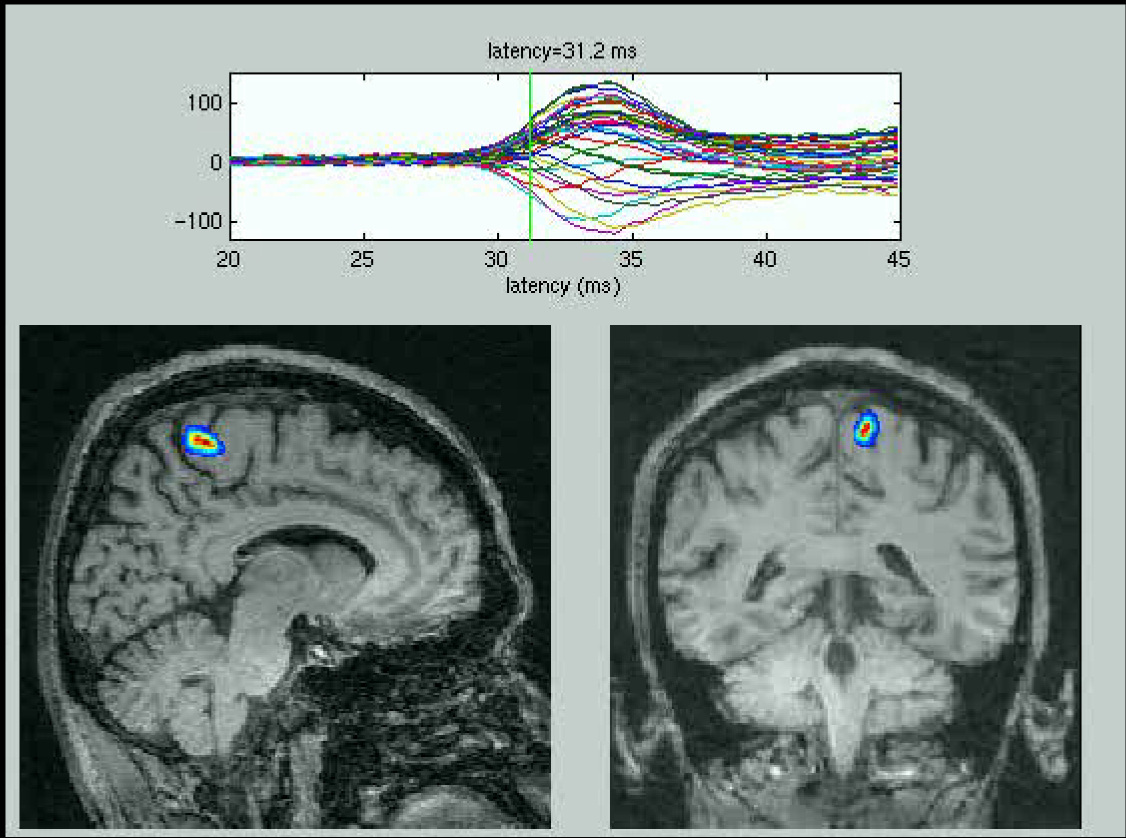


Hashimoto et al., "Muscle afferent inputs from the hand activate human cerebellum sequentially through parallel and climbing fiber systems", Clin. Neurophysiol. Nov;114, pp.2107-17, 2003 .

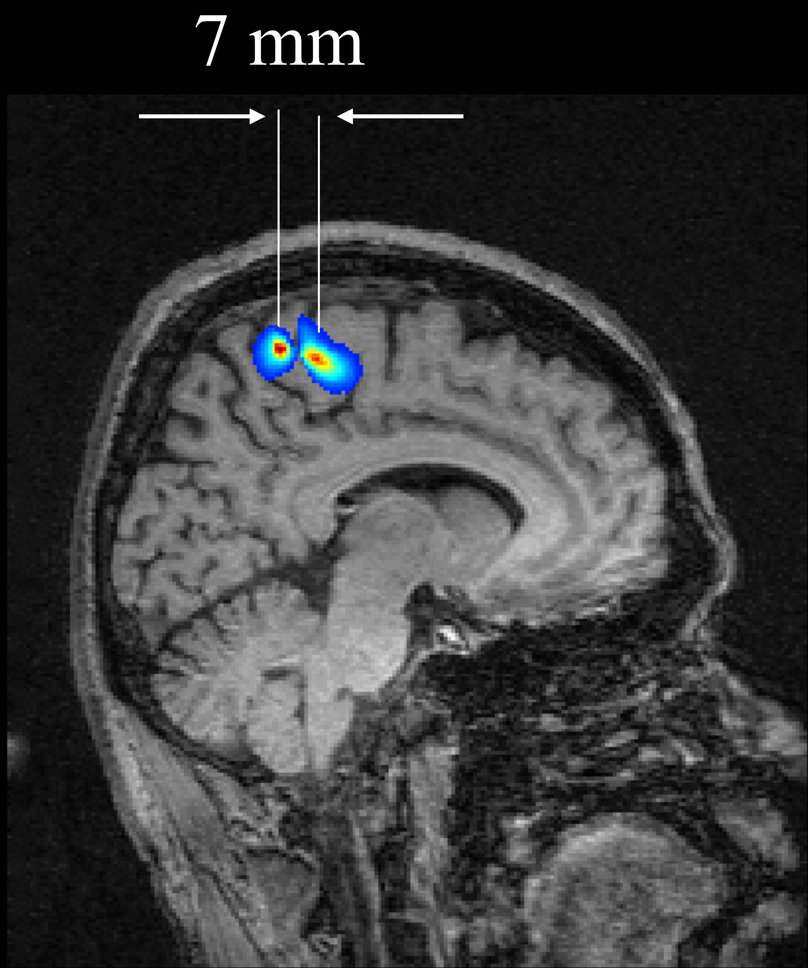

18 ms
18 ms
38 ms
54 ms
80 ms

Right posterior tibial nerve stimulation

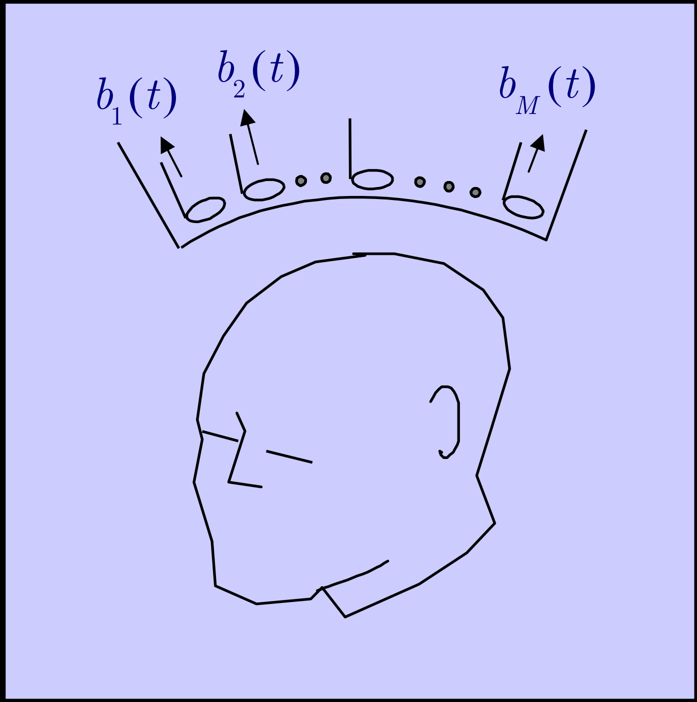
measured by a 37-channel sensor array



Hashimoto et al., “Serial activation of distinct cytoarchitectonic areas of the human {SI} cortex after posterior tibial nerve stimulation,” NeuroReport 12, pp1857-1862, 2001


7 mm

Definitions

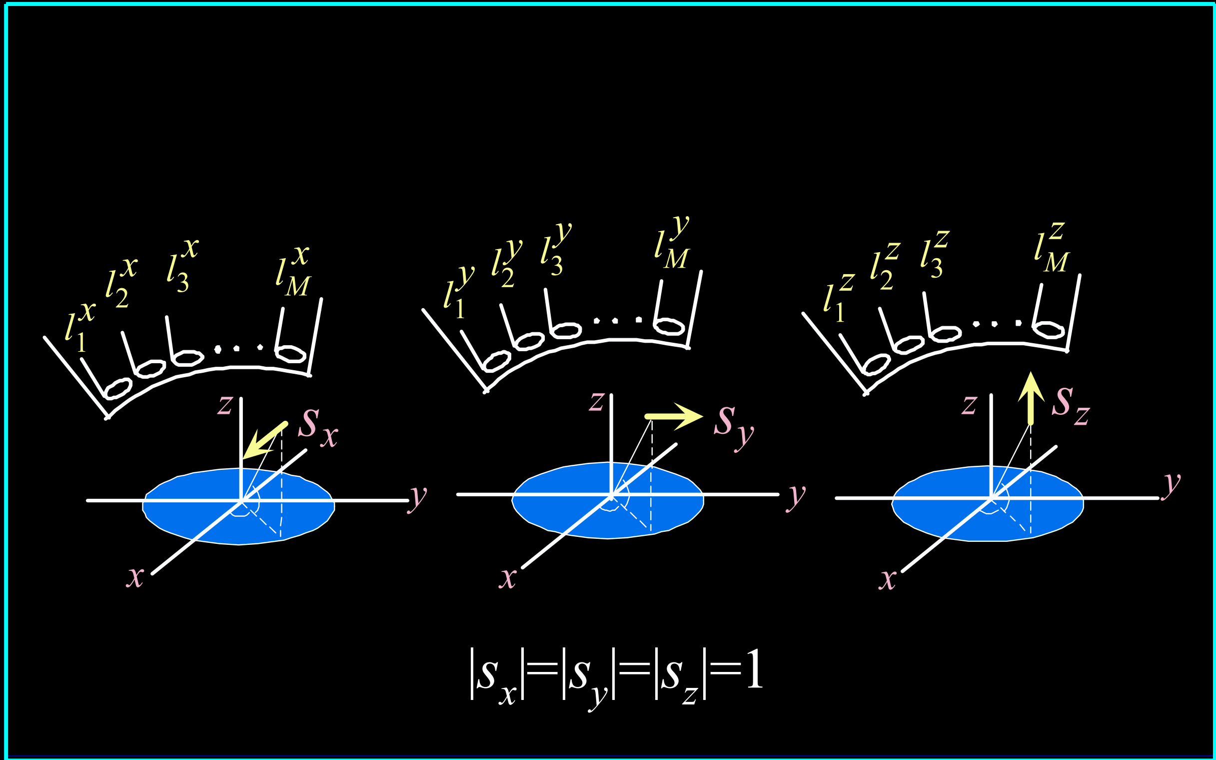


- data vector: $\boldsymbol{b}(\mathrm{t}) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$

- data covariance matrix: $\boldsymbol{R} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

- source magnitude: $s(\boldsymbol{r},t)$

- source orientation: $\boldsymbol{\eta}(\boldsymbol{r},t) = [\eta_x(\boldsymbol{r},t), \eta_y(\boldsymbol{r},t), \eta_z(\boldsymbol{r},t)]^T$



Lead field vector for the source orientation $\boldsymbol{\eta}(\boldsymbol{r})$

$$\boldsymbol{L}(\boldsymbol{r}) = \begin{bmatrix} l_1^x(\boldsymbol{r}) & l_1^y(\boldsymbol{r}) & l_1^z(\boldsymbol{r}) \\ l_2^x(\boldsymbol{r}) & l_2^y(\boldsymbol{r}) & l_2^z(\boldsymbol{r}) \\ \vdots & \vdots & \vdots \\ l_M^x(\boldsymbol{r}) & l_M^y(\boldsymbol{r}) & l_M^z(\boldsymbol{r}) \end{bmatrix}, \quad \boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{L}(\boldsymbol{r}) \begin{bmatrix} \eta_x(\boldsymbol{r}) \\ \eta_y(\boldsymbol{r}) \\ \eta_z(\boldsymbol{r}) \end{bmatrix}$$

Adaptive spatial filter

Spatial filter

$$\hat{s}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = [w_1(\boldsymbol{r}),\ldots,w_M(\boldsymbol{r})]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r)b_m(t)$$

Minimum-variance beamformer

$$\min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T\boldsymbol{l}(\boldsymbol{r}) = 1 \quad \Rightarrow \quad \boldsymbol{w}^T(\boldsymbol{r}) = \frac{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

$$\left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \frac{1}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

Following problems arise when applying minimum-variance beamformer to MEG/EEG source reconstruction.

(1) Output SNR degradation.

(2) Vector source detection.

(3) Statistical significance evaluation.

Output SNR degradation.

Minimum-variance beamformer is very sensitive to errors in forward modeling or errors in sample covariance matrix.

⇓

Because such errors are inevitable in neuromagnetic measurements, minimum-variance beamformer generally provides noisy spatio-temporal reconstruction results.

⇓

Introducing eigenspace projection

Some definitions:

$$\boldsymbol{R} = \boldsymbol{U}\left[\begin{array}{ccc|cc} \lambda_1 & 0 & \cdots & \cdot & 0 \\ 0 & \ddots & & 0 & \cdot \\ \vdots & & \lambda_P & & \vdots \\ \hline \cdot & 0 & & \ddots & 0 \\ 0 & \cdot & \cdots & 0 & \lambda_M \end{array}\right]\boldsymbol{U}^T = \boldsymbol{U}\begin{bmatrix} \boldsymbol{\Lambda}_S & 0 \\ 0 & \boldsymbol{\Lambda}_N \end{bmatrix}\boldsymbol{U}^T, \text{ and } \boldsymbol{U} = [\underbrace{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_P}_{\boldsymbol{E}_S} \mid \underbrace{\boldsymbol{e}_{P+1}, \ldots, \boldsymbol{e}_M}_{\boldsymbol{E}_N}]$$

Also, $\boldsymbol{\Gamma}_S = \boldsymbol{E}_S \boldsymbol{\Lambda}_S^{-1} \boldsymbol{E}_S^T$, $\boldsymbol{\Gamma}_N = \boldsymbol{E}_N \boldsymbol{\Lambda}_N^{-1} \boldsymbol{E}_N^T$

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error in estimating $\boldsymbol{l}(\boldsymbol{r})$

$$\text{Output SNR} \propto \frac{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S \boldsymbol{l}(\boldsymbol{r})]^2}{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S \boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{\varepsilon}^T \boldsymbol{\Gamma}_N \boldsymbol{\varepsilon}]}$$

Even when $\boldsymbol{\varepsilon}$ is small, $\boldsymbol{\varepsilon}^T \boldsymbol{\Gamma}_N \boldsymbol{\varepsilon}$ may not be small,
because $\boldsymbol{\varepsilon}^T \boldsymbol{\Gamma}_N \boldsymbol{\varepsilon} \approx \|\boldsymbol{\varepsilon}\|^2 / \lambda_{p+j}^2 \leftarrow$ noise level eigenvalue

Eigenspace projection

The error term $\varepsilon^T \Gamma_N \varepsilon$ arises from the noise subspace component of $\boldsymbol{w}(\boldsymbol{r})$.

Extension to eigenspace projection beamformer

$$\bar{\boldsymbol{w}}_\mu = \boldsymbol{E}_S \boldsymbol{E}_S^T \boldsymbol{w}_\mu, \quad \text{where } \mu = x, y \text{ or } z$$

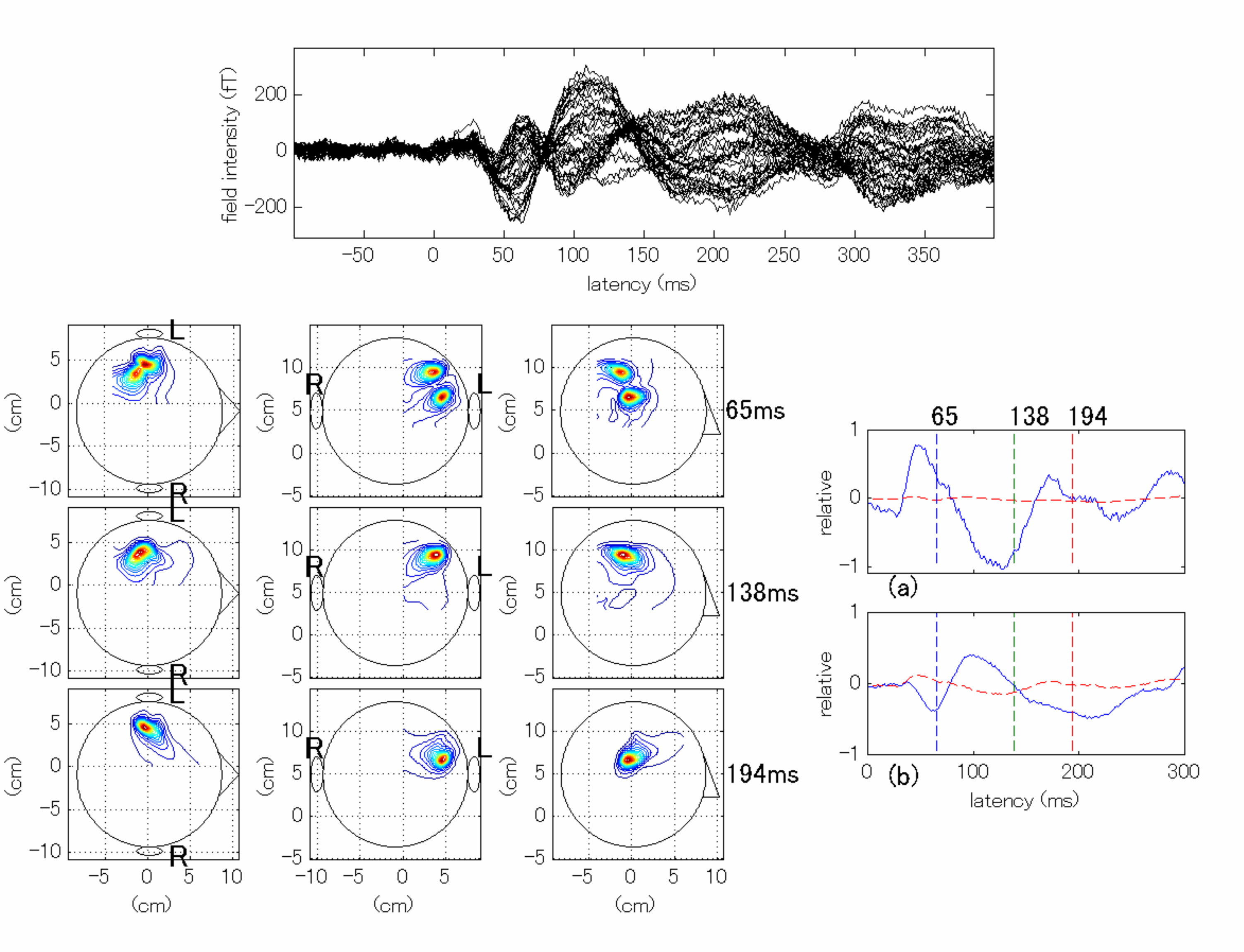
$$\text{Output SNR} \propto \frac{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S \boldsymbol{l}(\boldsymbol{r})]^2}{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S \boldsymbol{l}(\boldsymbol{r}) + \boldsymbol{\varepsilon}^T \boldsymbol{\Gamma}_N \boldsymbol{\varepsilon}]} \quad \text{(non-eigenspace projected)}$$

$$\Downarrow$$

$$\text{Output SNR} \propto \frac{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S \boldsymbol{l}(\boldsymbol{r})]^2}{[\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{\Gamma}_S \boldsymbol{l}(\boldsymbol{r})]} \quad \text{(eigenspace projected)}$$

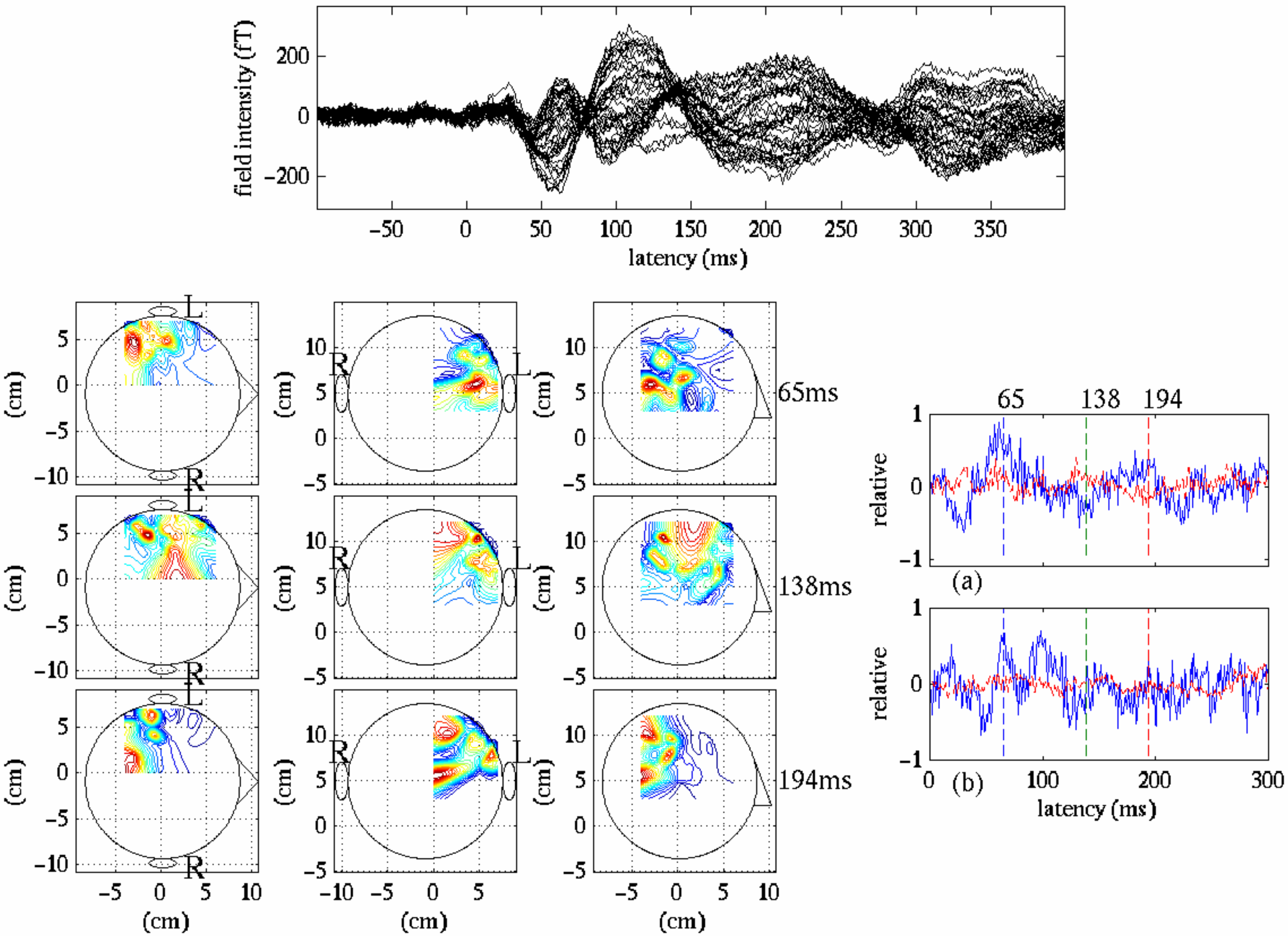
Application to 37-channel auditory-somatosensory recording

eigenspace-projection results



Application to 37-channel auditory-somatosensory recording

Non-eigenspace projected results



Vector source detection

The electromagnetic sources are three dimensional vectors.

$\Downarrow$

The minimum-variance beamformer formulation should be extended to incorporate the vector nature of sources.

Two-types of extensions has been proposed: scalar and vector formulations.

Scalar MV beamformer formulation

$$\boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta}) = \frac{\boldsymbol{l}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}^{-1}}{\boldsymbol{l}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r},\boldsymbol{\eta})} = \frac{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}}{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}$$

uses a single weight vector, but it depends not only on $\boldsymbol{r}$ but also on $\boldsymbol{\eta}$.

S. E. Robinson et al., Recent Advances in Biomagnetism, Tohoku University Press, 1999

Vector MV beamformer formulation

$$[\boldsymbol{w}_x(\boldsymbol{r}), \boldsymbol{w}_y(\boldsymbol{r}), \boldsymbol{w}_z(\boldsymbol{r})]^T = [\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})]^{-1}\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}$$

uses three weight vectors which detect x, y, and z source components.

M. E. Spencer et al., 26th Annual Asilomer Conference on Signals, Systems, and Computers, 1992
B. D. van Veen et al., IEEE Trans. Biomed. Eng., 1997

Output power

Scalar formulation:

$$\max_{\boldsymbol{\eta}} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \max_{\boldsymbol{\eta}} \frac{1}{\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r}) \boldsymbol{\eta}}$$

$$= \left[\min_{\boldsymbol{\eta}} (\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r}) \boldsymbol{\eta}) \right]^{-1} = \frac{1}{\gamma_{min}}$$

Vector formulation:

$$\max_{\boldsymbol{\eta}} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \max_{\boldsymbol{\eta}} \left\| [\hat{s}_x(\boldsymbol{r}), \hat{s}_y(\boldsymbol{r}), \hat{s}_z(\boldsymbol{r})] \boldsymbol{\eta} \right\|^2$$

$$= \max_{\boldsymbol{\eta}} [\boldsymbol{\eta}^T [\boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})]^{-1} \boldsymbol{\eta}] = \frac{1}{\gamma_{min}}$$

γ_{min} : minimum eigenvalue of $[\boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})]$

Two types of formulations give the same output power.

Asymptotic output SNR

Scalar beamformer

$$Z_S^{opt}(\boldsymbol{r}) = \max_{\boldsymbol{\eta}} \frac{\left\langle \hat{s}(\boldsymbol{r},\boldsymbol{\eta},t)^2 \right\rangle}{\sigma_0^2 \left\|\boldsymbol{w}(\boldsymbol{r},\boldsymbol{\eta})\right\|^2} = \frac{1}{\sigma_0^2}\left[\min_{\boldsymbol{\eta}} \frac{\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-2}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}{\boldsymbol{\eta}^T \boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}\right]^{-1} = \frac{1}{\sigma_0^2 \alpha_{min}}$$

Vector beamformer

$$Z_V^{opt}(\boldsymbol{r}) = \max_{\boldsymbol{\eta}} \frac{\left\|[\hat{s}_x(\boldsymbol{r}),\hat{s}_y(\boldsymbol{r}),\hat{s}_z(\boldsymbol{r})]\boldsymbol{\eta}\right\|^2}{\sigma_0^2 \left\|\boldsymbol{W}(\boldsymbol{r})\boldsymbol{\eta}\right\|^2} = \frac{1}{\sigma_0^2 \alpha_{min}}$$

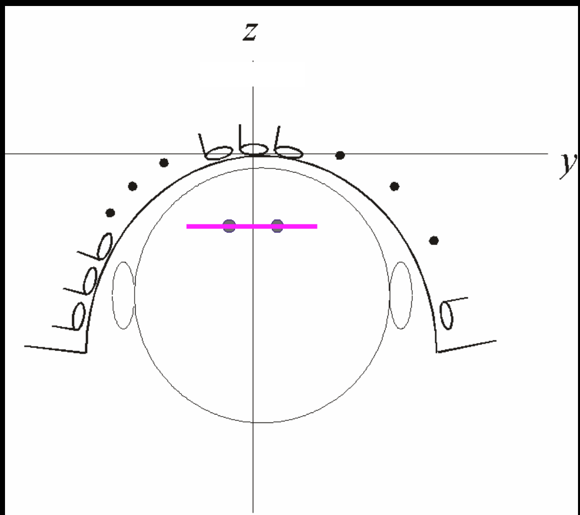
α_{min} : minimum eigenvalue of $\left[\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\right]^{-1}\left[\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-2}\boldsymbol{L}(\boldsymbol{r})\right]$

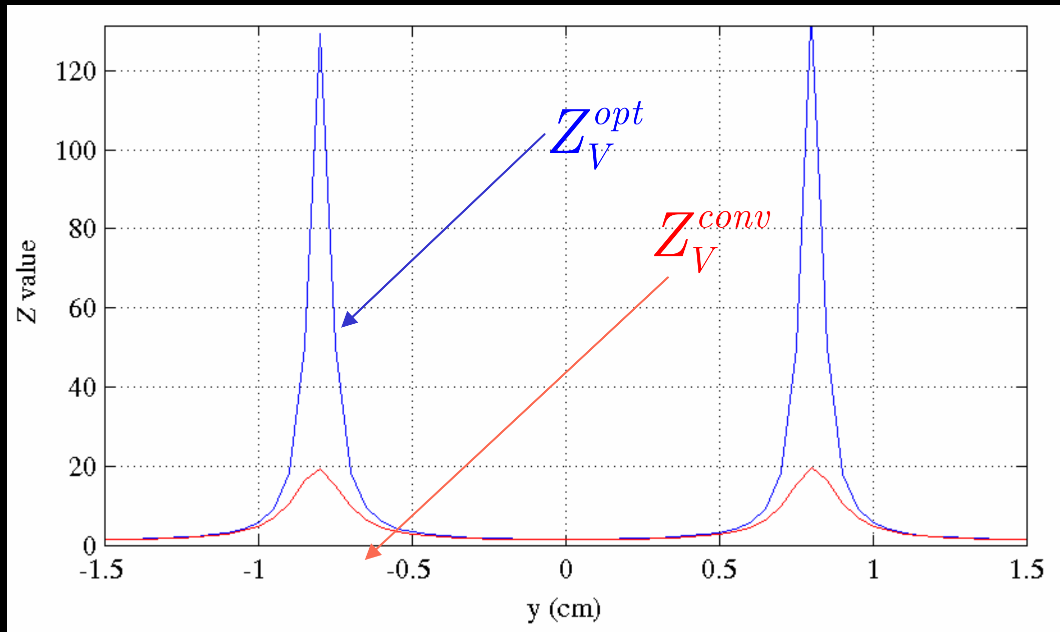
Two types of formulations give the same asymptotic output SNR.

Asymptotic output SNR for vector beamformer

$$Z_V^{conv}(\boldsymbol{r}) = \frac{\|\hat{s}_x(\boldsymbol{r})\|^2 + \|\hat{s}_y(\boldsymbol{r})\|^2 + \|\hat{s}_z(\boldsymbol{r})\|^2}{\sigma_0^2(\|\boldsymbol{w}_x(\boldsymbol{r})\|^2 + \|\boldsymbol{w}_y(\boldsymbol{r})\|^2 + \|\boldsymbol{w}_z(\boldsymbol{r})\|^2)}$$ (conventional)

$$Z_V^{opt}(\boldsymbol{r}) = \max_{\boldsymbol{\eta}} \frac{\|[\hat{s}_x(\boldsymbol{r}), \hat{s}_y(\boldsymbol{r}), \hat{s}_z(\boldsymbol{r})]\boldsymbol{\eta}\|^2}{\sigma_0^2\|\boldsymbol{W}(\boldsymbol{r})\boldsymbol{\eta}\|^2} = \frac{1}{\sigma_0^2\alpha_{min}}$$ (orientation optimized)





Orientation optimized results always gives the best SNR

Orientation optimized weight

MV beamformer

$$w^T(r) = \frac{\eta_{opt}^T L^T(r) R^{-1}}{\eta_{opt}^T L^T(r) R^{-1} L(r) \eta_{opt}}$$

where $\left[L^T(r) R^{-1} L(r)\right] \eta_{opt} = \gamma_{min} \eta_{opt}$

Weight normalized MV (Borgiotti-Kaplan) beamformer

$$w^T(r) = \frac{\eta_{opt}^T L^T(r) R^{-1}}{\sqrt{\eta_{opt}^T L^T(r) R^{-2} L(r) \eta_{opt}}}$$

where $\left[L^T(r) R^{-1} L(r)\right]^{-1} \left[L^T(r) R^{-2} L(r)\right] \eta_{opt} = \alpha_{min} \eta_{opt}$

Evaluation of the statistical significance: Parametric method

Data model (signal plus additive noise): $\boldsymbol{b}(t) = \boldsymbol{b}_S(t) + \boldsymbol{n}(t)$

Beamformer reconstruction:

$$\hat{s}(\boldsymbol{r}, t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}_S(t) + \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{n}(t)$$

Thus, if $\boldsymbol{n}(t) \sim N(0, \sigma_0^2)$, then $\hat{s}(\boldsymbol{r}, t) \sim N(\boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}_S(t), \sigma_0^2 \left\| \boldsymbol{w}^T(\boldsymbol{r}) \right\|^2)$

Problem:

$$\hat{s}(\boldsymbol{r}, t) = \boldsymbol{w}^T(\boldsymbol{r}, \boldsymbol{\eta})\boldsymbol{b}(t) = \boldsymbol{w}^T(\boldsymbol{r}, \boldsymbol{\eta})\boldsymbol{b}_S(t) + \Delta\boldsymbol{w}^T(\boldsymbol{r}, \boldsymbol{\eta})\boldsymbol{b}_S(t) + \boldsymbol{w}^T(\boldsymbol{r}, \boldsymbol{\eta})\boldsymbol{n}(t) + \Delta\boldsymbol{w}^T(\boldsymbol{r}, \boldsymbol{\eta})\boldsymbol{n}(t)$$

This is because the weight is obtained using a sample covariance.

Evaluation of the statistical significance:

Our proposed method

The null hypothesis: no signal source

Test this hypothesis at each voxel using a non-parametric statistics

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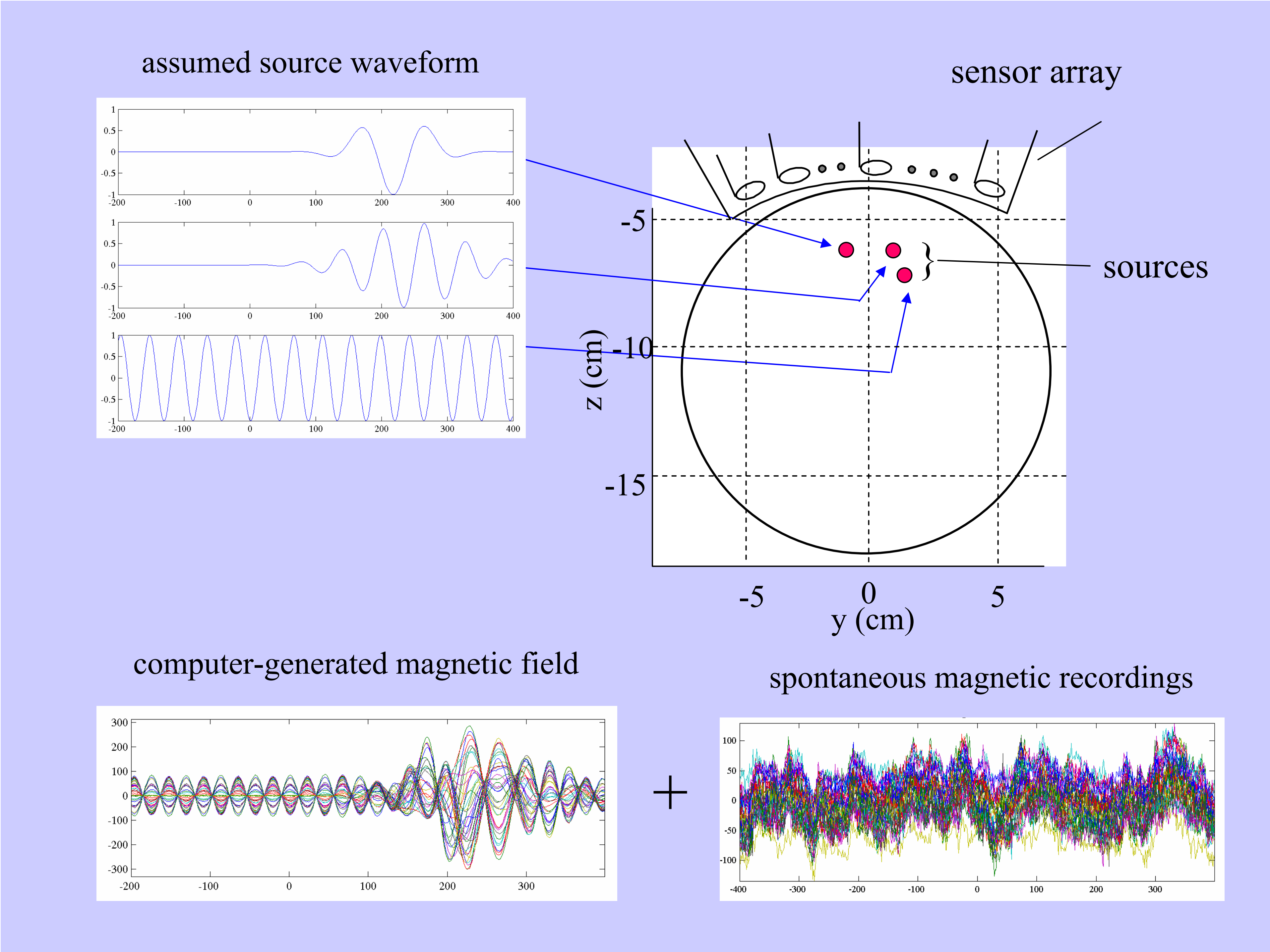
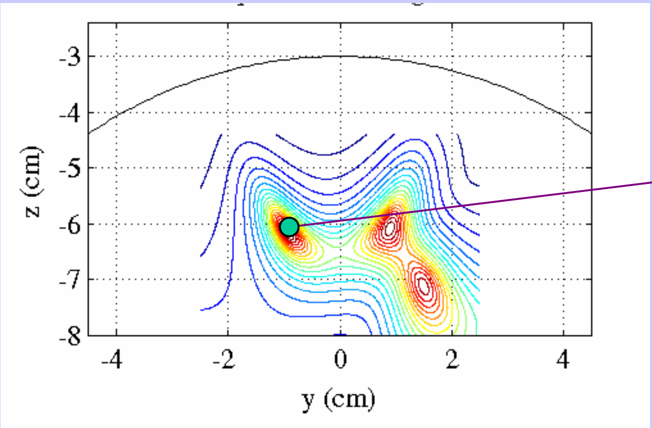
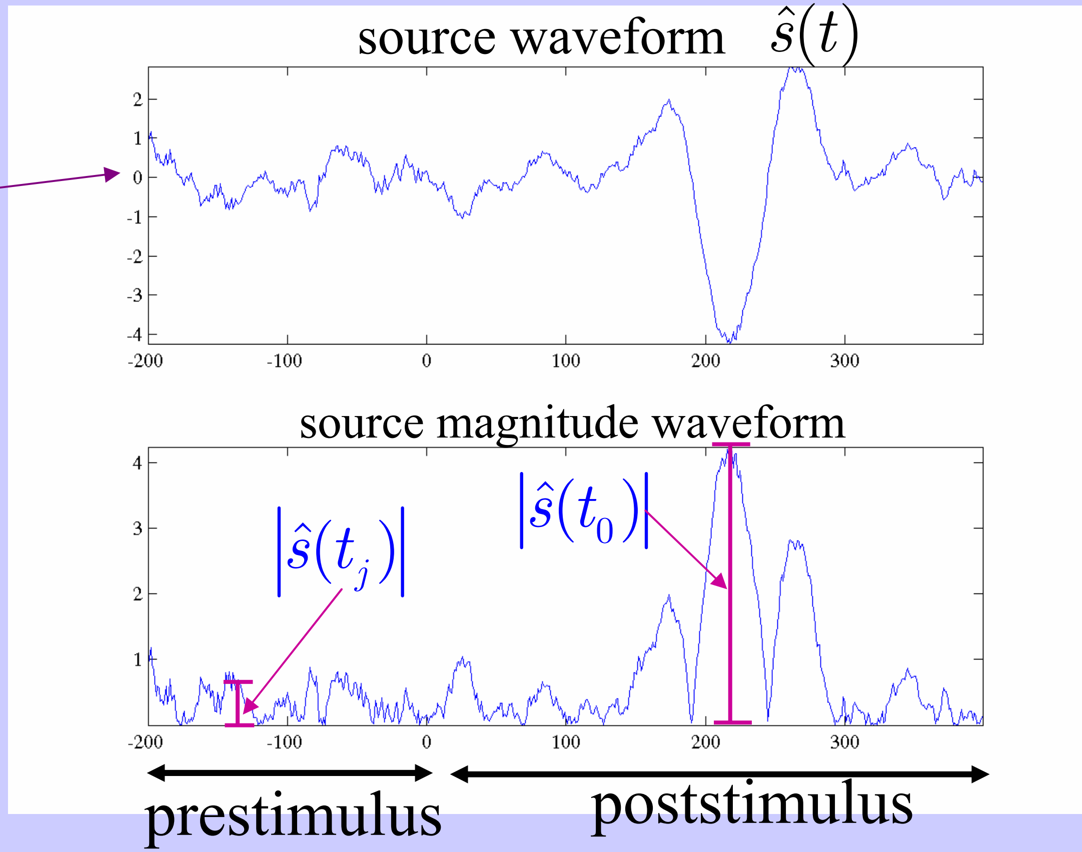
Signal sources: sources existing in the test (task) period but not in the control period.


assumed source waveform
sensor array
sources
-5
-10
-15
z (cm)
-5
0
5
y (cm)
computer-generated magnetic field
+
spontaneous magnetic recordings

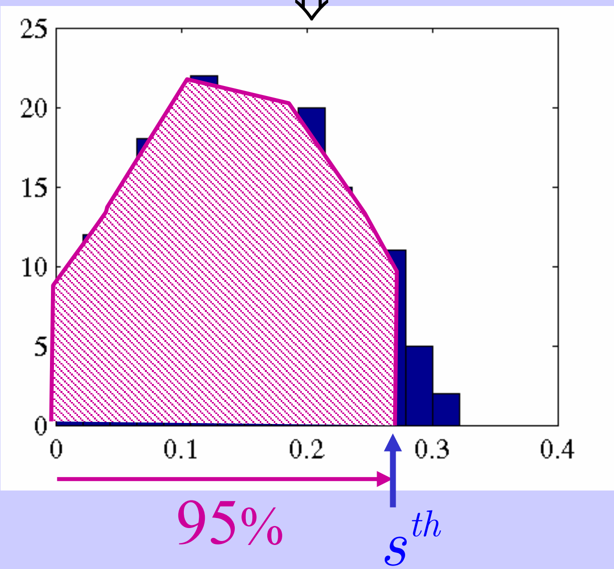




histogram of $\left|\hat{s}(t_j)\right|$

$t_j \in$ prestimulus period

$\Downarrow$



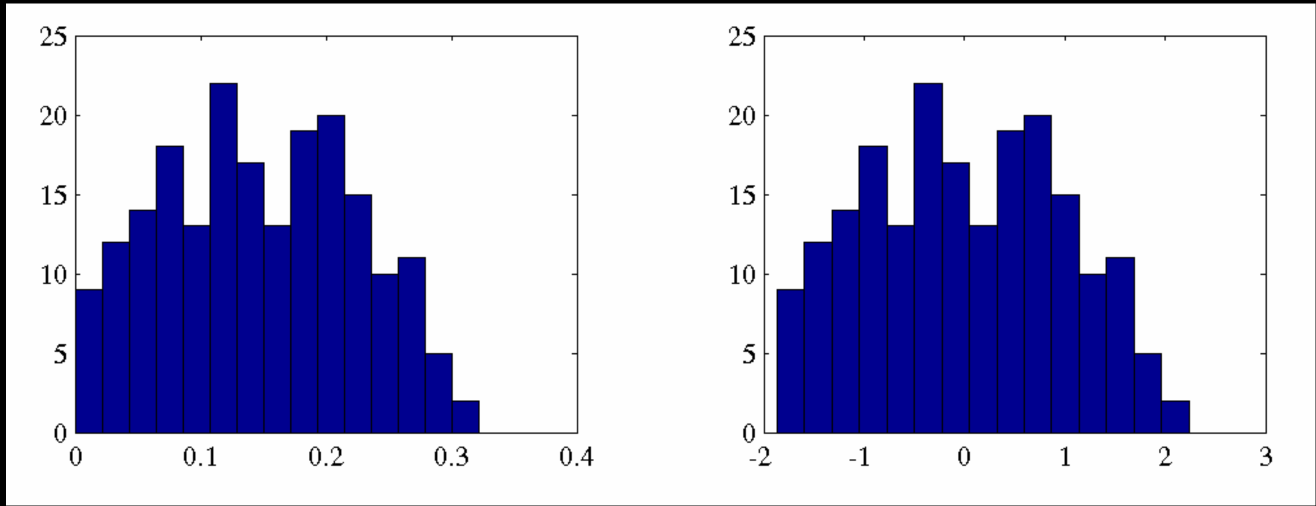
$\Leftarrow$ use this distribution as an empirical distribution to test H_0 at each (x,y,z).

If $\left|\hat{s}(t_0)\right| > s^{th}, \left|\hat{s}(t_0)\right|$ is significant.

Multiple comparison procedure

Standardization of the distribution

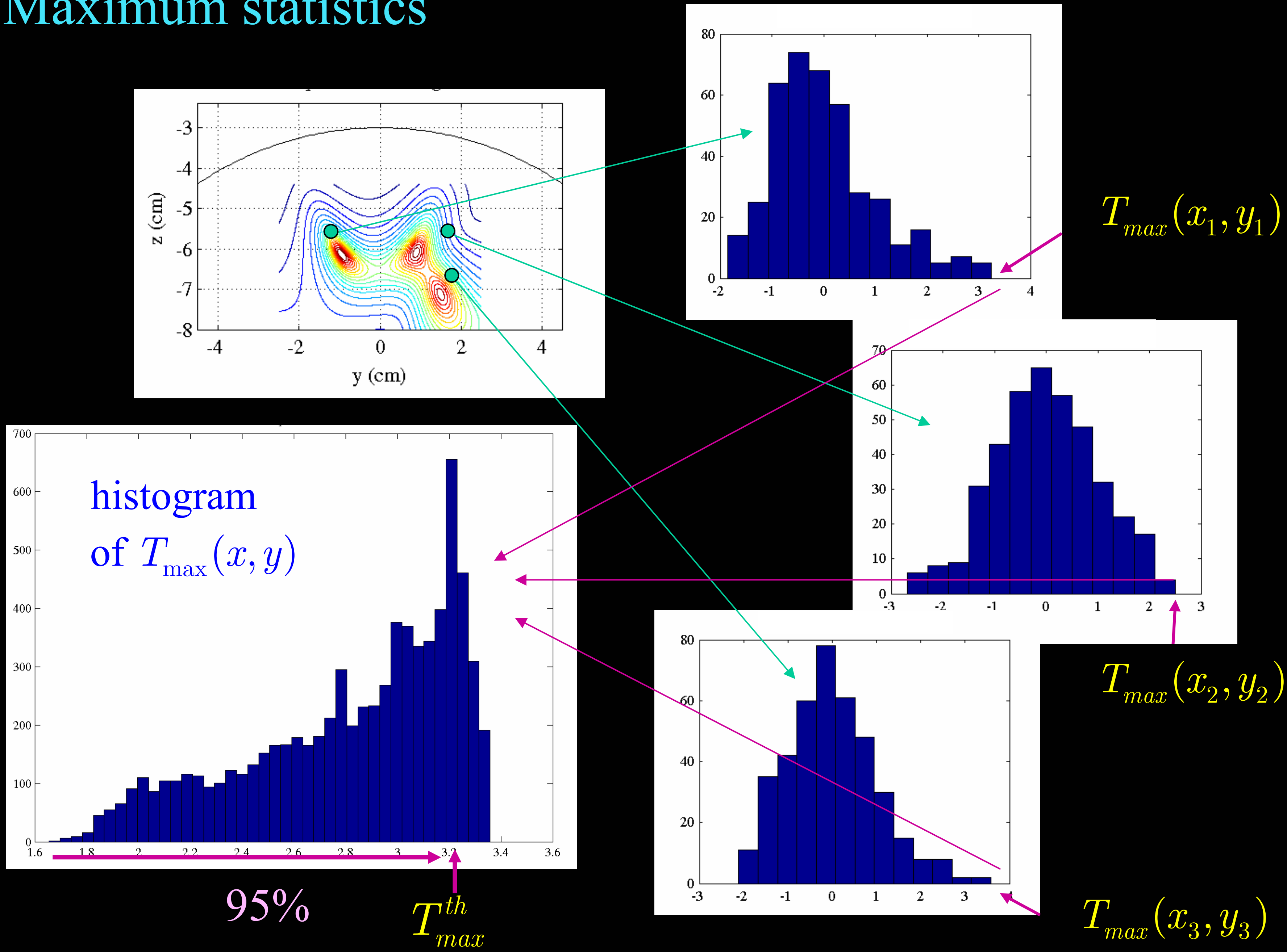
histogram of $|s(t_j)|$ $\Rightarrow$ histogram of $T(t_j)$



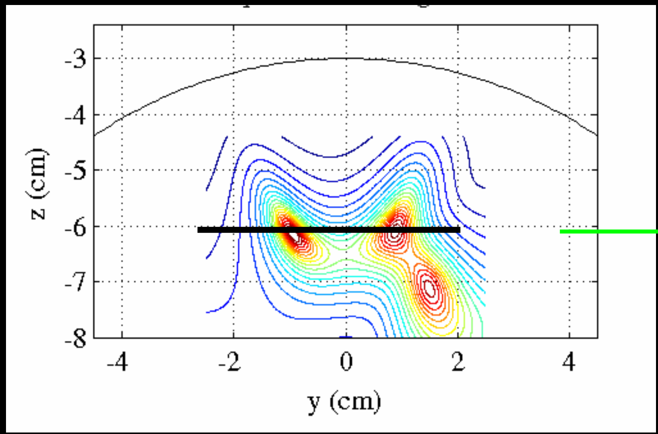
$$T(t_j) = \frac{|s(t_j)| - \langle |s(t_j)| \rangle}{\sigma} \quad \text{where} \quad \sigma^2 = \langle s(t_j)^2 \rangle - \langle |s(t_j)| \rangle^2$$

$\langle \cdot \rangle$ indicates the average over the prestimulus period.

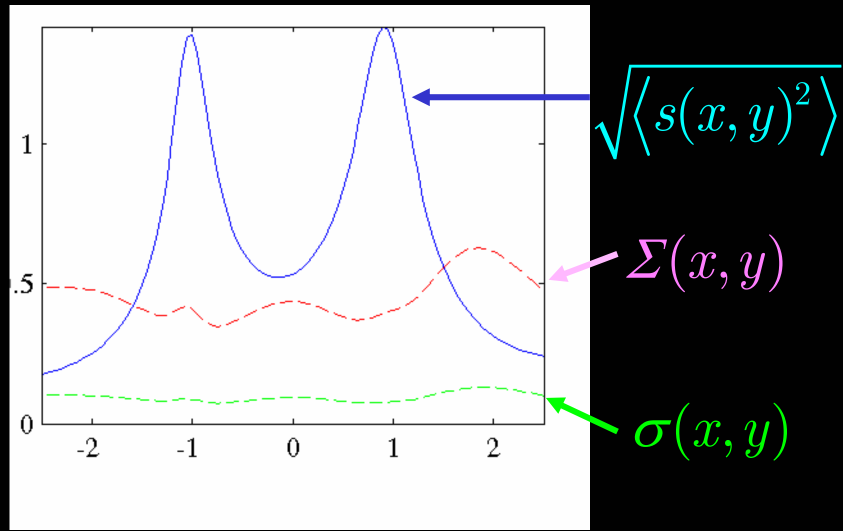
Maximum statistics

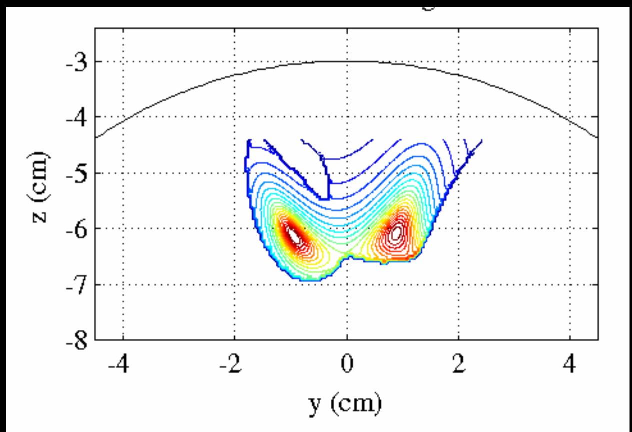


Statistical thresholding



cross sectional view

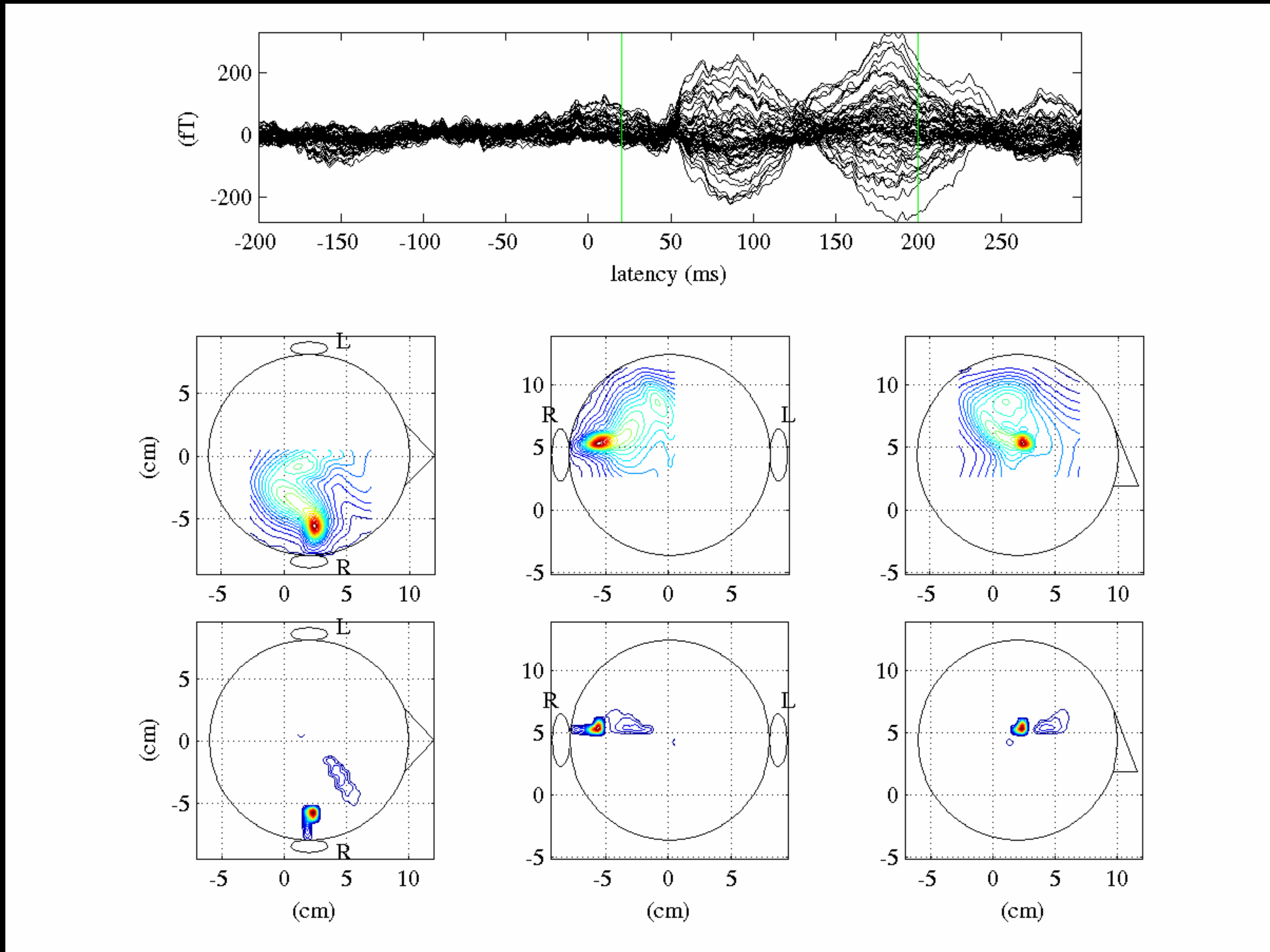




Threshold:

$$\Sigma(x,y) = T_{max}^{th}\sigma(x,y) + \left\langle \left| s(x,y) \right| \right\rangle$$

Application to auditory-button-press measurements



Summary

My talk has addressed the problems caused from:

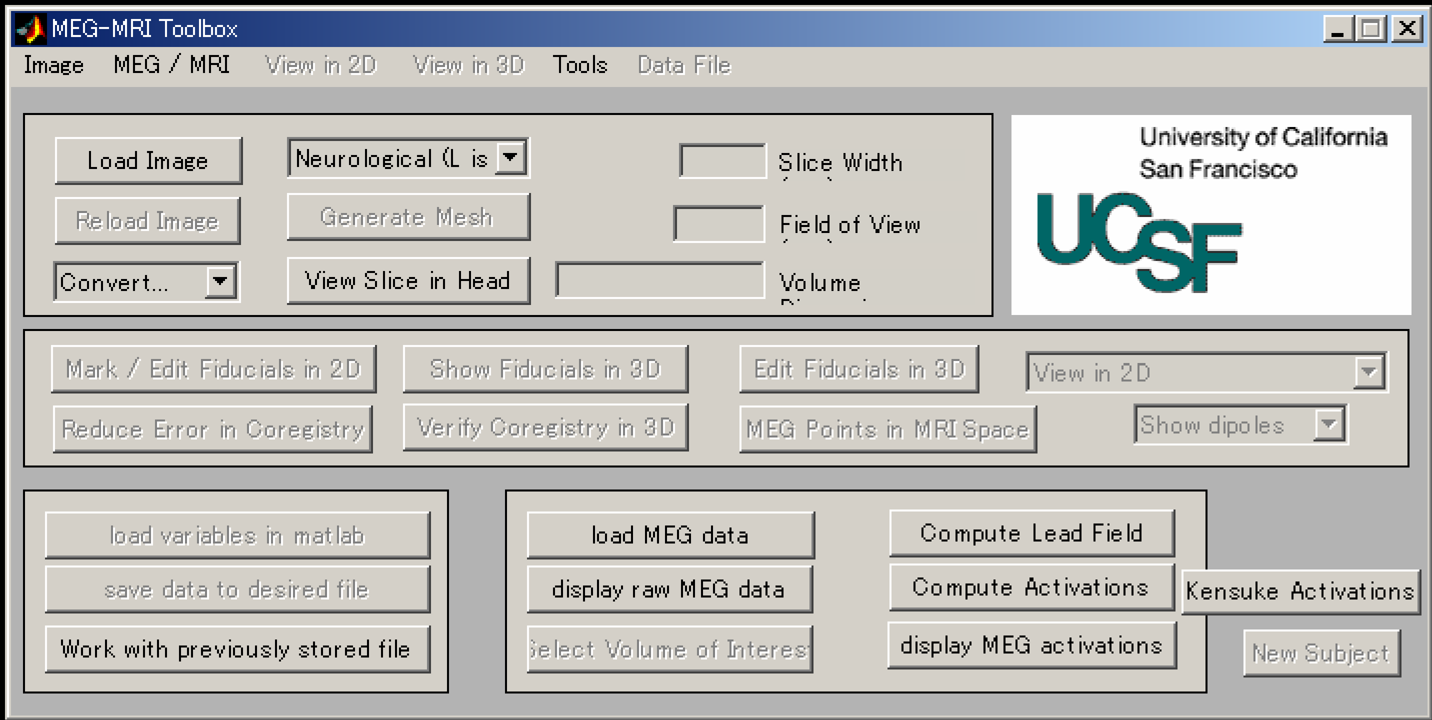
- output SNR degradation
- vector source detection
- statistical significance evaluation

and described our solutions for them:

- eigenspace projection
- orientation optimized weight
- a novel nonparametric statistical method.

MEG-MRI Toolbox

(requires SPM 2)



will be soon available from UCSF.

Visit

http://www.tmit.ac.jp/~sekihara/

The PDF version of this power-point presentation as well as PDFs of the recent publications are available.

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