# Acknowledgements

The organizers of this symposium express our deepest thanks to

Prof. Alec Marantz and MIT Department of Linguistics and Philosophy for offering to use this lecture room for this symposium and for helping us to make various arrangements for this symposium.

We also thank VMS medtech for their courtesy in providing lunch and refreshments.

*Spatial filtering in Biomagnetism*
*Satellite Symposium of Biomag 2004*
*Boston, August, 2004*

# Comparison between adaptive and non-adaptive spatial filter performances

***Kensuke Sekihara[1], Maneesh Sahani[2], Srikantan S. Nagarajan[3]***

[1]Department of Engineering, Tokyo Metropolitan Institute of Technology
[2]Keck Center for Integrative Neuroscience, University of California, San Francisco
[3]Department of Radiology, University of California, San Francisco

This talk compares the adaptive spatial filters such as minimum-variance spatial filter with the minimum-norm-based tomographic reconstruction methods, by formulating them as non-adaptive spatial filters.

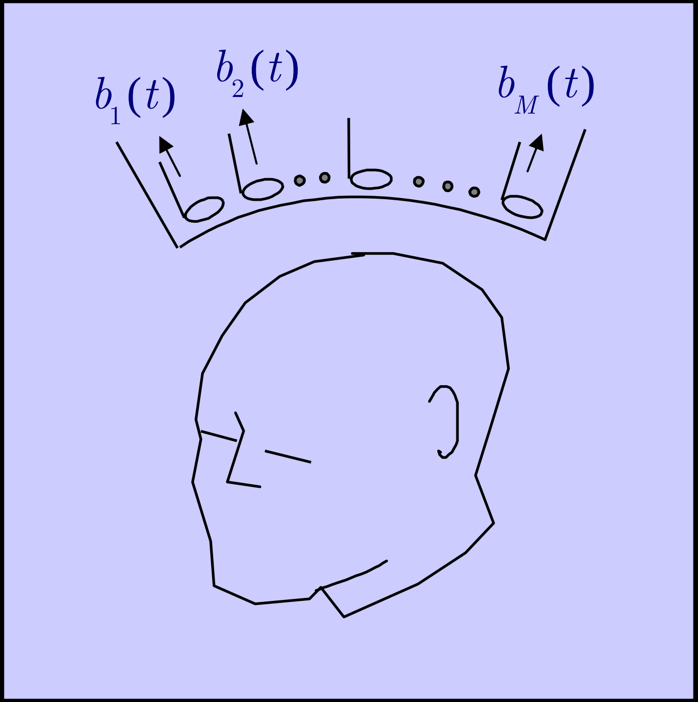
Performance measures:

- Bias in the reconstructed source location in the absence or presence of noise.

- Spatial resolution.

- Influence of source correlation.

# Definitions



- data vector: $\boldsymbol{b}(\mathrm{t}) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$

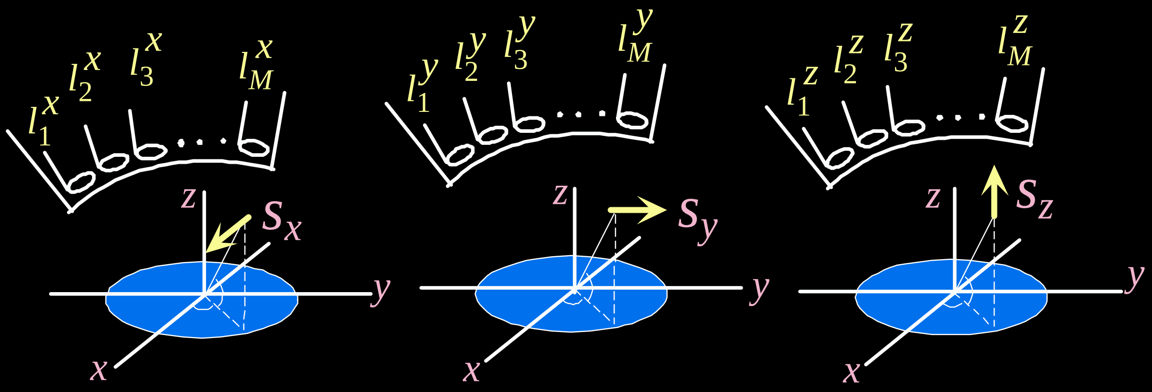
- data covariance matrix: $\boldsymbol{R}_b = \langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \rangle$

- source magnitude: $s(\boldsymbol{r},t)$

- source orientation: $\boldsymbol{\eta}(\boldsymbol{r},t) = [\eta_x(\boldsymbol{r},t), \eta_y(\boldsymbol{r},t), \eta_z(\boldsymbol{r},t)]^T$

# Sensor lead field

$$\boldsymbol{L}(\boldsymbol{r}) = \begin{bmatrix} l_1^x(\boldsymbol{r}) & l_1^y(\boldsymbol{r}) & l_1^z(\boldsymbol{r}) \\ l_2^x(\boldsymbol{r}) & l_2^y(\boldsymbol{r}) & l_2^z(\boldsymbol{r}) \\ \vdots & \vdots & \vdots \\ l_M^x(\boldsymbol{r}) & l_M^y(\boldsymbol{r}) & l_M^z(\boldsymbol{r}) \end{bmatrix}, \quad \boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{L}(\boldsymbol{r}) \begin{bmatrix} \eta_x(\boldsymbol{r}) \\ \eta_y(\boldsymbol{r}) \\ \eta_z(\boldsymbol{r}) \end{bmatrix}$$



$|s_x|=|s_y|=|s_z|=1$

# Spatial filter for bioelectromagnetic source reconstruction

The spatial filter incorporate the 3D vector nature of sources

## Scalar spatial filter

$$\hat{s}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{b}(t) \approx \boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta}_{opt})\boldsymbol{b}(t)$$

$$\Uparrow$$

$$\boldsymbol{\eta}_{opt} = \max_{\boldsymbol{\eta}} \left\langle \hat{s}(\boldsymbol{r},t)^2 \right\rangle = \max_{\boldsymbol{\eta}} \boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{R}_b\boldsymbol{w}(\boldsymbol{r},\boldsymbol{\eta})$$

## Vector spatial filter

$$[\hat{s}_x(\boldsymbol{r}),\hat{s}_y(\boldsymbol{r}),\hat{s}_z(\boldsymbol{r})] = \boldsymbol{\eta}^T\hat{s}(\boldsymbol{r},t) = [\boldsymbol{w}_x(\boldsymbol{r}),\boldsymbol{w}_y(\boldsymbol{r}),\boldsymbol{w}_z(\boldsymbol{r})]^T\boldsymbol{b}(t)$$

Non-adaptive spatial filter

$\boldsymbol{w}(\boldsymbol{r})$ is data independent

Adaptive spatial filter

$\boldsymbol{w}(\boldsymbol{r})$ is data dependent

# Gram Matrix

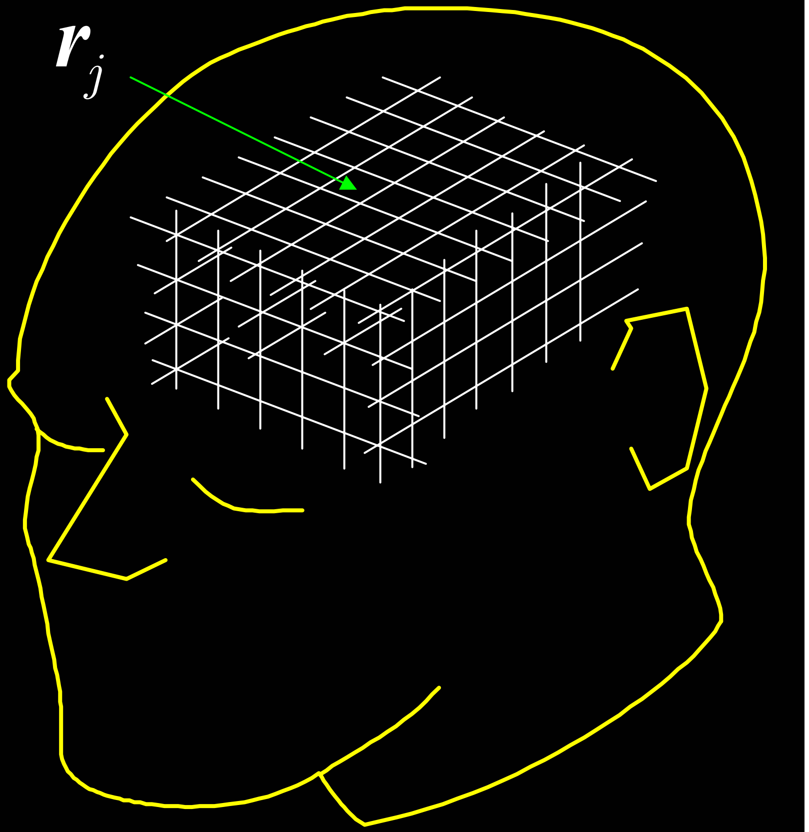
Define Gram matrix $\boldsymbol{G}$ : $\boldsymbol{G} = \int \boldsymbol{L}(\boldsymbol{r})\boldsymbol{L}^T(\boldsymbol{r})d\boldsymbol{r}$

Gram matrix $\boldsymbol{G}$ is usually calculated by introducing pixel grid $\boldsymbol{r}_j$

Define composite lead field matrix for all pixel locations such that

$$\boldsymbol{L}_N = \left[ \boldsymbol{L}(\boldsymbol{r}_1), \quad \cdots, \quad \boldsymbol{L}(\boldsymbol{r}_N) \right]$$

Then $\boldsymbol{G} = \boldsymbol{L}_N \boldsymbol{L}_N^T$



# Non-adaptive spatial filters

Minimum-norm (Hamalainen)

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r})$$

Weight normalized minimum norm (Dale et al.)

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r}) / \sqrt{\boldsymbol{l}^{T}(\boldsymbol{r})\boldsymbol{G}^{-2}\boldsymbol{l}(\boldsymbol{r})}$$

sLORETA (Pasucual-Marque)

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r}) / \sqrt{\boldsymbol{l}^{T}(\boldsymbol{r})\boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

$$\boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{L}(\boldsymbol{r})\eta_{opt}$$

# Adaptive spatial filters

Minimum variance

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{R}_b^{-1}\,\boldsymbol{l}(\boldsymbol{r}) / [\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}_b^{-1}\boldsymbol{l}(\boldsymbol{r})] \Leftarrow \min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}_b\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T\boldsymbol{l}(\boldsymbol{r}) = 1$$

Minimum variance with normalized lead field

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{R}_b^{-1}\,\tilde{\boldsymbol{l}}(\boldsymbol{r}) / [\tilde{\boldsymbol{l}}^T(\boldsymbol{r})\boldsymbol{R}_b^{-1}\tilde{\boldsymbol{l}}(\boldsymbol{r})] \Leftarrow \min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}_b\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T\boldsymbol{l}(\boldsymbol{r}) = |\boldsymbol{l}(\boldsymbol{r})|$$

Weight-normalized minimum variance (Borgiotti-Kaplan)

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{R}_b^{-1}\,\boldsymbol{l}(\boldsymbol{r}) / \sqrt{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}_b^{-2}\boldsymbol{l}(\boldsymbol{r})} \Leftarrow \min_{\boldsymbol{w}} \boldsymbol{w}^T\boldsymbol{R}_b\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T\boldsymbol{w} = 1$$

---

$$\tilde{\boldsymbol{l}}(\boldsymbol{r}) = \boldsymbol{l}(\boldsymbol{r}) / \|\boldsymbol{l}(\boldsymbol{r})\|$$

## Bias of estimated source locations

We first take a look at the bias of reconstructed source locations for these adaptive and non-adaptive spatial filters.

# Resolution kernel analysis

$$\left.\begin{aligned} \boldsymbol{b} &= \int \boldsymbol{l}(\boldsymbol{r}')s(\boldsymbol{r}')d\boldsymbol{r}' \\ \hat{s}(\boldsymbol{r}) &= \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b} \end{aligned}\right\} \rightarrow \hat{s}(\boldsymbol{r}) = \int \underbrace{\boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{l}(\boldsymbol{r}')}_{\mathbb{R}(\boldsymbol{r},\boldsymbol{r}')} s(\boldsymbol{r}')d\boldsymbol{r}'$$

$\Uparrow$

Resolution kernel

A single source at $\boldsymbol{r}_1$

$$s(\boldsymbol{r}') = \delta(\boldsymbol{r}' - \boldsymbol{r}_1) \quad \Rightarrow \quad \hat{\boldsymbol{s}}(\boldsymbol{r}) = \int \mathbb{R}(\boldsymbol{r},\boldsymbol{r}')\delta(\boldsymbol{r}' - \boldsymbol{r}_1)d\boldsymbol{r}' = \mathbb{R}(\boldsymbol{r},\boldsymbol{r}_1)$$

No location bias $\Leftrightarrow$ Resolution kernel peaks at $\boldsymbol{r}_1$

$\Updownarrow$

$$\boxed{\mathbb{R}(\boldsymbol{r}_1,\boldsymbol{r}_1) > \mathbb{R}(\boldsymbol{r},\boldsymbol{r}_1)}$$

# Non-adaptive spatial filters

The source at $\boldsymbol{r}_1$ has lead field vector $\boldsymbol{f}$ : $\boldsymbol{f} = \boldsymbol{L}(\boldsymbol{r}_1)\boldsymbol{\eta}$

The condition: $\mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1) > \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1)$

Minimum-norm

$$\boldsymbol{f}\boldsymbol{G}^{-1}\boldsymbol{f} > \boldsymbol{l}\boldsymbol{G}^{-1}\boldsymbol{f} \quad (\boldsymbol{l} = \boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}_{opt})$$

Weight normalized
minimum norm

$$\frac{\boldsymbol{f}\boldsymbol{G}^{-1}\boldsymbol{f}}{\sqrt{\boldsymbol{f}^T\boldsymbol{G}^{-2}\boldsymbol{f}}} > \frac{\boldsymbol{l}\boldsymbol{G}^{-1}\boldsymbol{f}}{\sqrt{\boldsymbol{l}^T\boldsymbol{G}^{-2}\boldsymbol{l}}}$$

sLORETA

$$(\boldsymbol{f}\boldsymbol{G}^{-1}\boldsymbol{f})(\boldsymbol{l}^T\boldsymbol{G}^{-1}\boldsymbol{l}) > (\boldsymbol{l}\boldsymbol{G}^{-1}\boldsymbol{f})^2$$

Schwartz inequality valid
because G is a positive definite matrix

# Adaptive spatial filters

The condition: $\mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1) > \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1)$

Minimum-variance

$$\frac{\|\boldsymbol{f}\|}{\|\boldsymbol{l}\|} \frac{\cos(\boldsymbol{l}, \boldsymbol{f})}{1 + \alpha[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]} < 1$$

Minimum variance
with normalized lead field

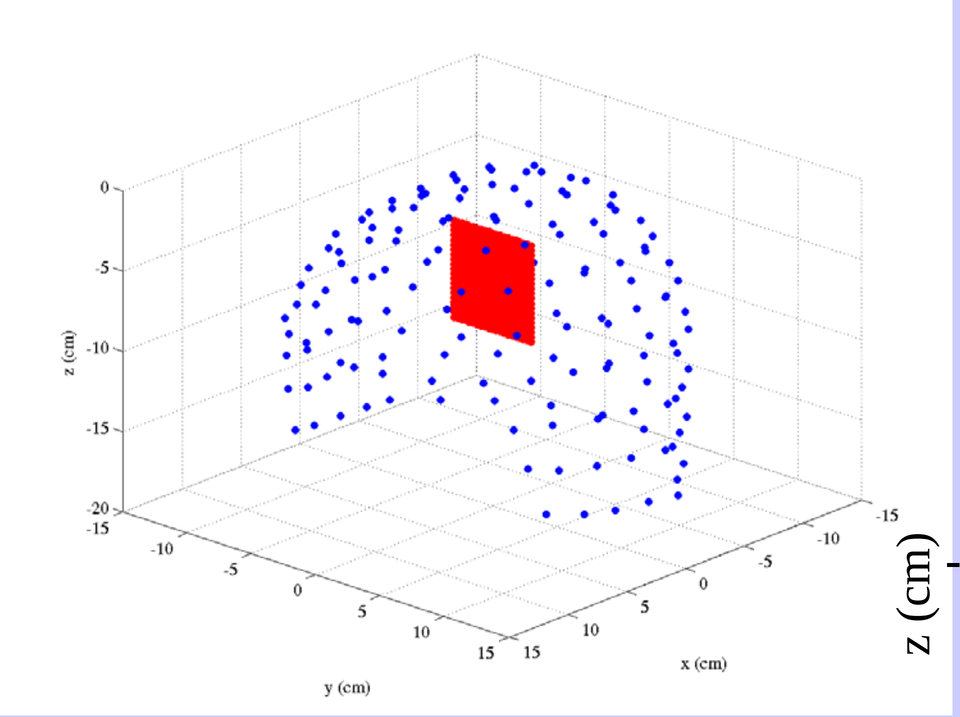
$$\frac{\cos(\boldsymbol{l}, \boldsymbol{f})}{1 + \alpha[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]} < 1$$

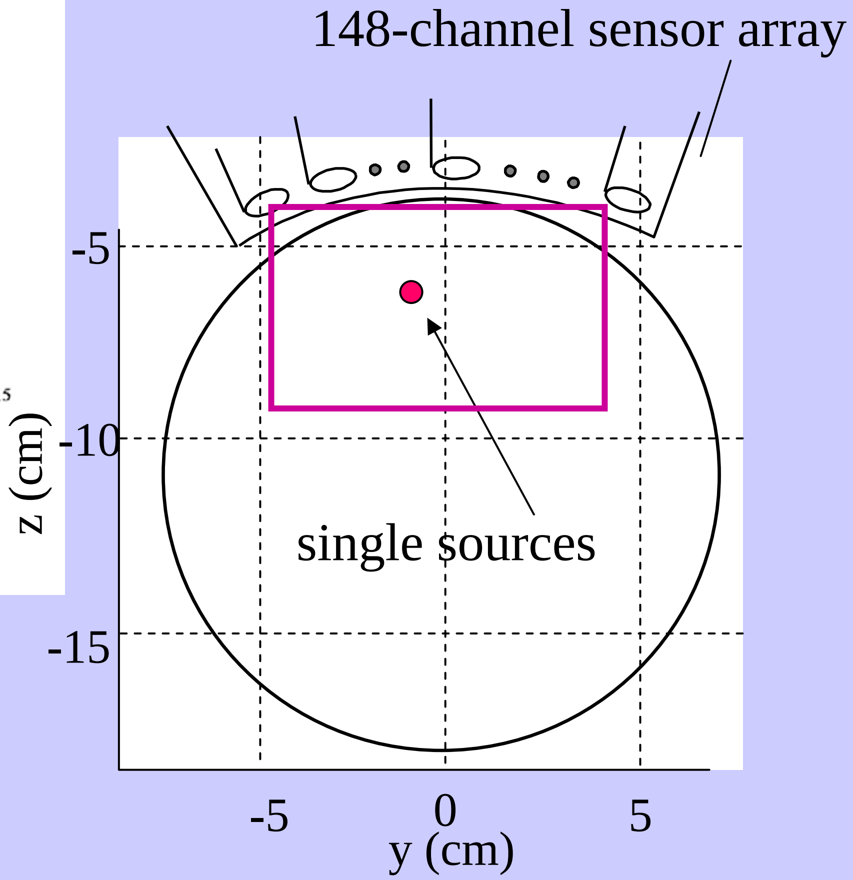
Weight-normalized
minimum variance

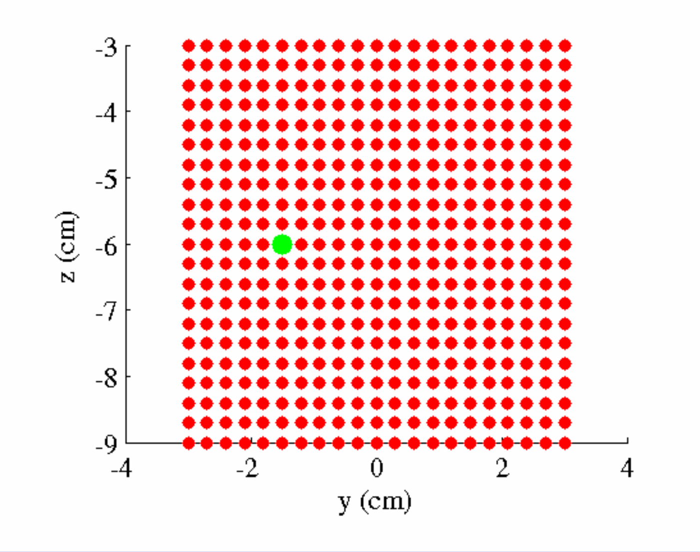
$$\frac{\cos(\boldsymbol{l}, \boldsymbol{f})}{\sqrt{1 + \alpha(\alpha + 2)[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]}} < 1$$

---

$$\alpha = (\sigma_1^2 / \sigma_0^2)\ \|\boldsymbol{f}\|^2 \,(> M), \qquad 0 \le \cos(\boldsymbol{l}, \boldsymbol{f}) = \frac{|\boldsymbol{l}^T \boldsymbol{f}|}{\|\boldsymbol{f}\| \|\boldsymbol{l}\|} \le 1$$


0
-5
-10
-15
-20
z (cm)
-15
-10
-5
0
5
10
15
y (cm)
15
10
5
0
-5
-10
-15
x (cm)


148-channel sensor array
single sources
-5
-10
-15
z (cm)
-5
0
5
y (cm)

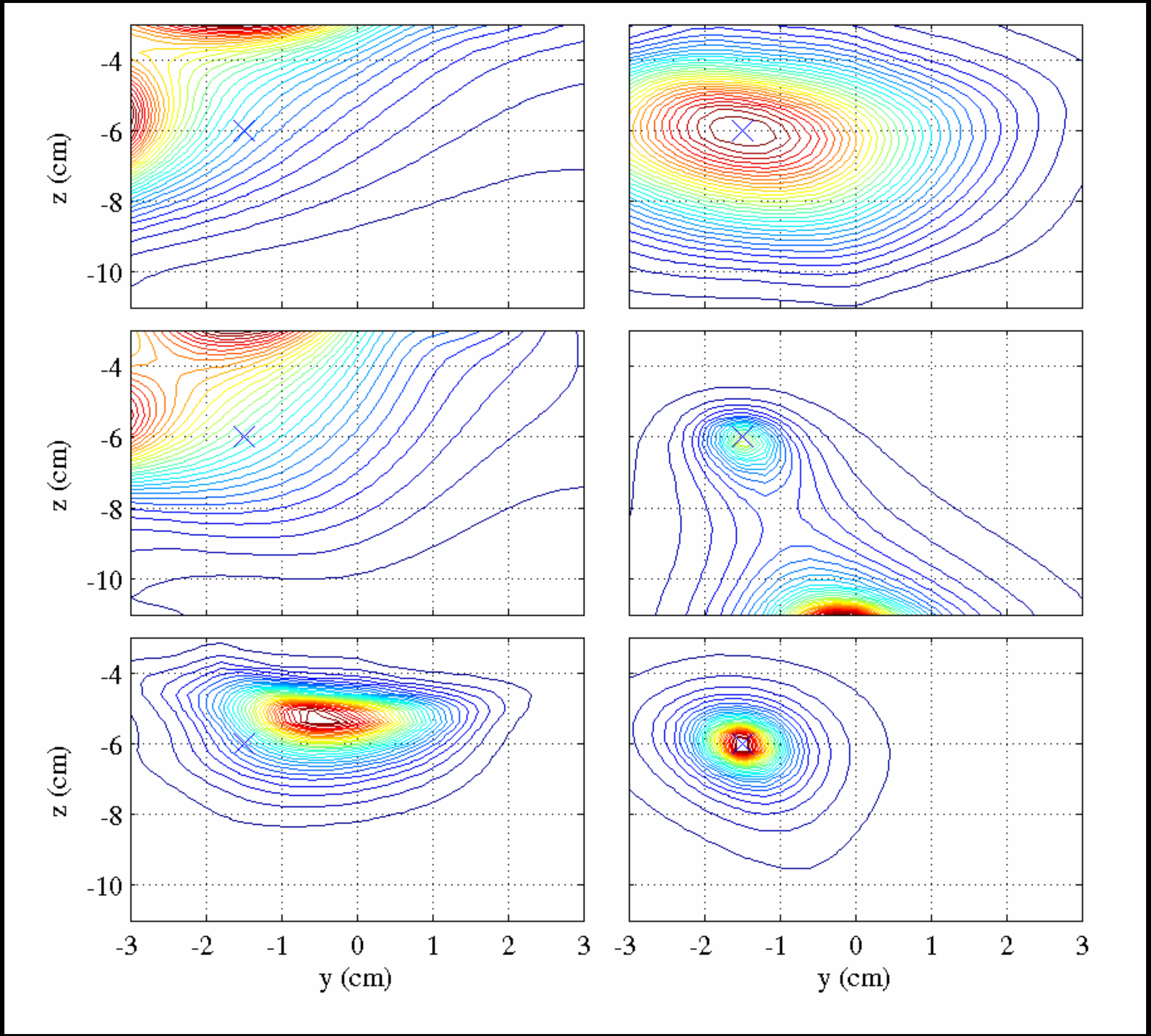

-3
-4
-5
-6
-7
-8
-9
z (cm)
-4
-2
0
2
4
y (cm)

# Reconstruction results of this single source

Minimum norm

Minimum norm (normalized leadfield)

Weight normalized minimum norm



sLORERA

Minimum variance

Minimum variance (normalized lead field)

The cross mark indicates the source location

# Effect of noise on location bias

Condition for no location bias:

$$\sigma_1^2 \mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1) + \sigma_0^2 \left\| \boldsymbol{w}(\boldsymbol{r}_1) \right\|^2 > \sigma_1^2 \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1) + \sigma_0^2 \left\| \boldsymbol{w}(\boldsymbol{r}) \right\|^2$$

sLORETA:

$$\frac{[1 + \Omega(\boldsymbol{r}) / \alpha]}{[1 + \Omega(\boldsymbol{r}_1) / \alpha]} \cos^2(\boldsymbol{l}, \boldsymbol{f} \mid G^{-1}) < 1, \quad \text{where } \Omega(\boldsymbol{r}) = \frac{\left\| \boldsymbol{f} \right\|^2 \left\| \boldsymbol{w}(\boldsymbol{r}) \right\|^2}{\mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1)^2}$$

Minimum variance with normalized lead field

$$\frac{1}{1 + \alpha[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]} < 1$$

---

$\sigma_0^2$ : noise power, $\sigma_1^2$ : signal power, input SNR: $\alpha = (\sigma_1^2 / \sigma_0^2) \left\| \boldsymbol{f} \right\|^2$

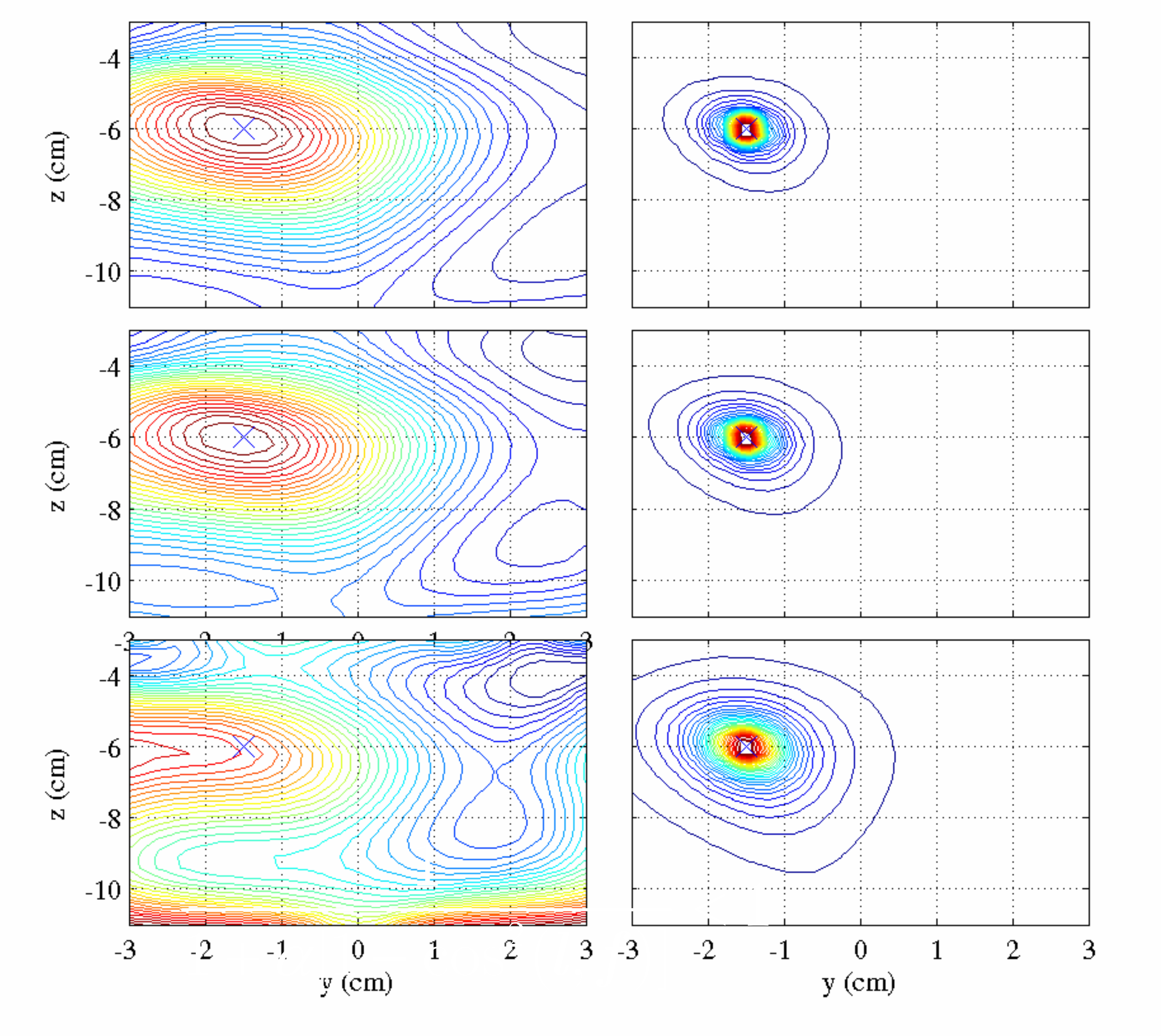
sLORETA

Minimum-variance (normalized lead field)

$\alpha = 8M(1184)$

$\alpha = 4M(592)$

$\alpha = M(148)$



# Spatial resolution comparison

When there is no location bias, the mail-lobe width of the kernel can be a measure of spatial resolution.

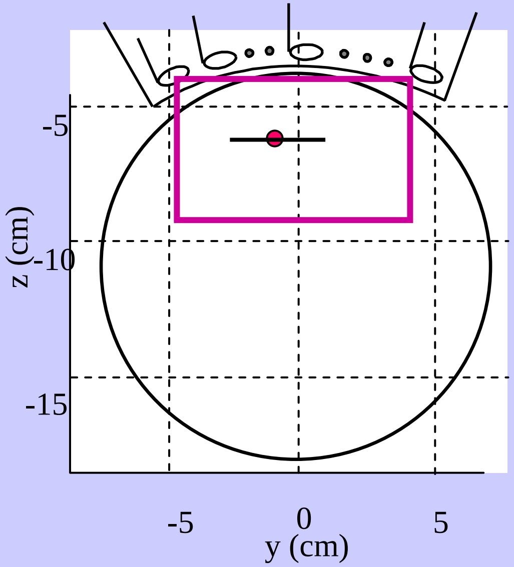
Point spread function: $\phi(\boldsymbol{r}) = \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1) \,/\, \mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1)$

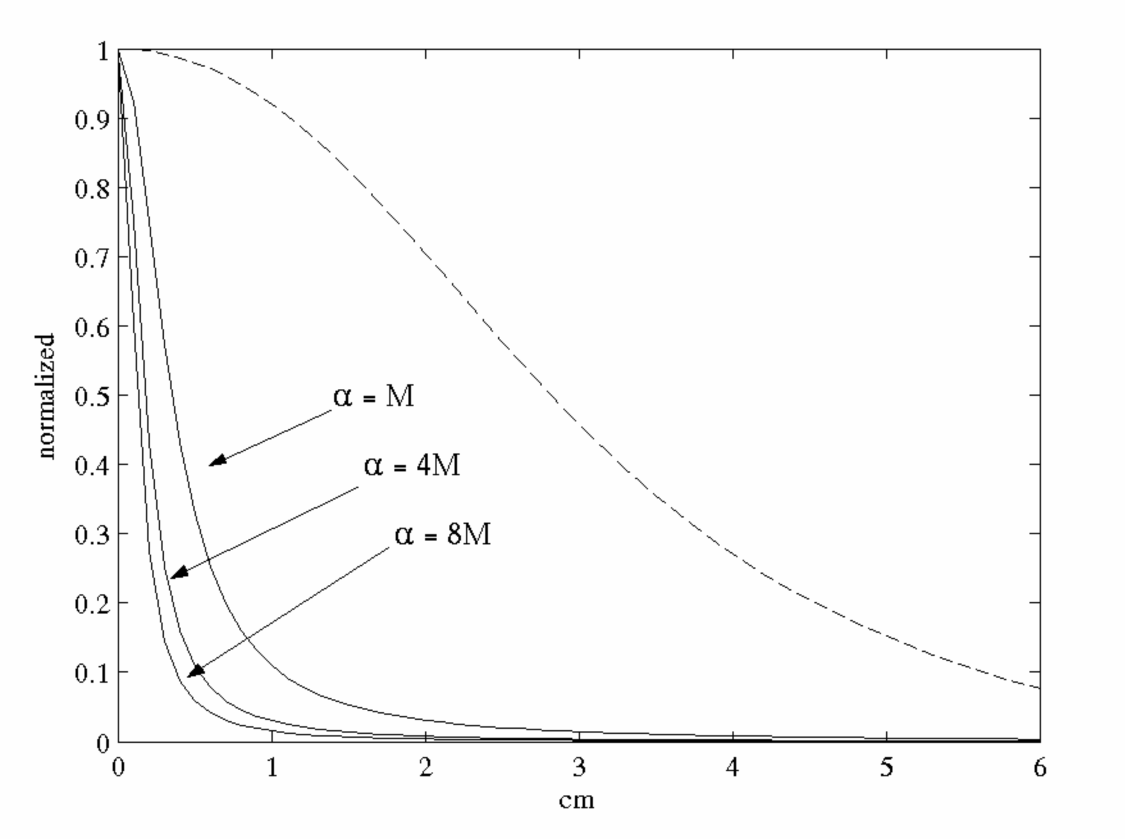
sLORETA: $$\phi(\boldsymbol{r}) = \cos(\boldsymbol{l}, \boldsymbol{f} \mid \boldsymbol{G}^{-1}) \left( = \frac{\left| \boldsymbol{l}^T \boldsymbol{G}^{-1} \boldsymbol{f} \right|}{\sqrt{(\boldsymbol{f}^T \boldsymbol{G}^{-1} \boldsymbol{f})(\boldsymbol{l}^T \boldsymbol{G}^{-1} \boldsymbol{l})}} \right)$$

Minimum variance: $$\phi(\boldsymbol{r}) = \frac{\cos(\boldsymbol{l}, \boldsymbol{f})}{1 + \underset{\Uparrow}{\alpha[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]}}$$

Because α is a large value, this part causes a rapid decay of the point spread function

# Point spread function





- - - - sLORETA

—— Minimum-variance

# Reconstruction experiments when two sources exist

Minimum-variance
(normalized lead field)

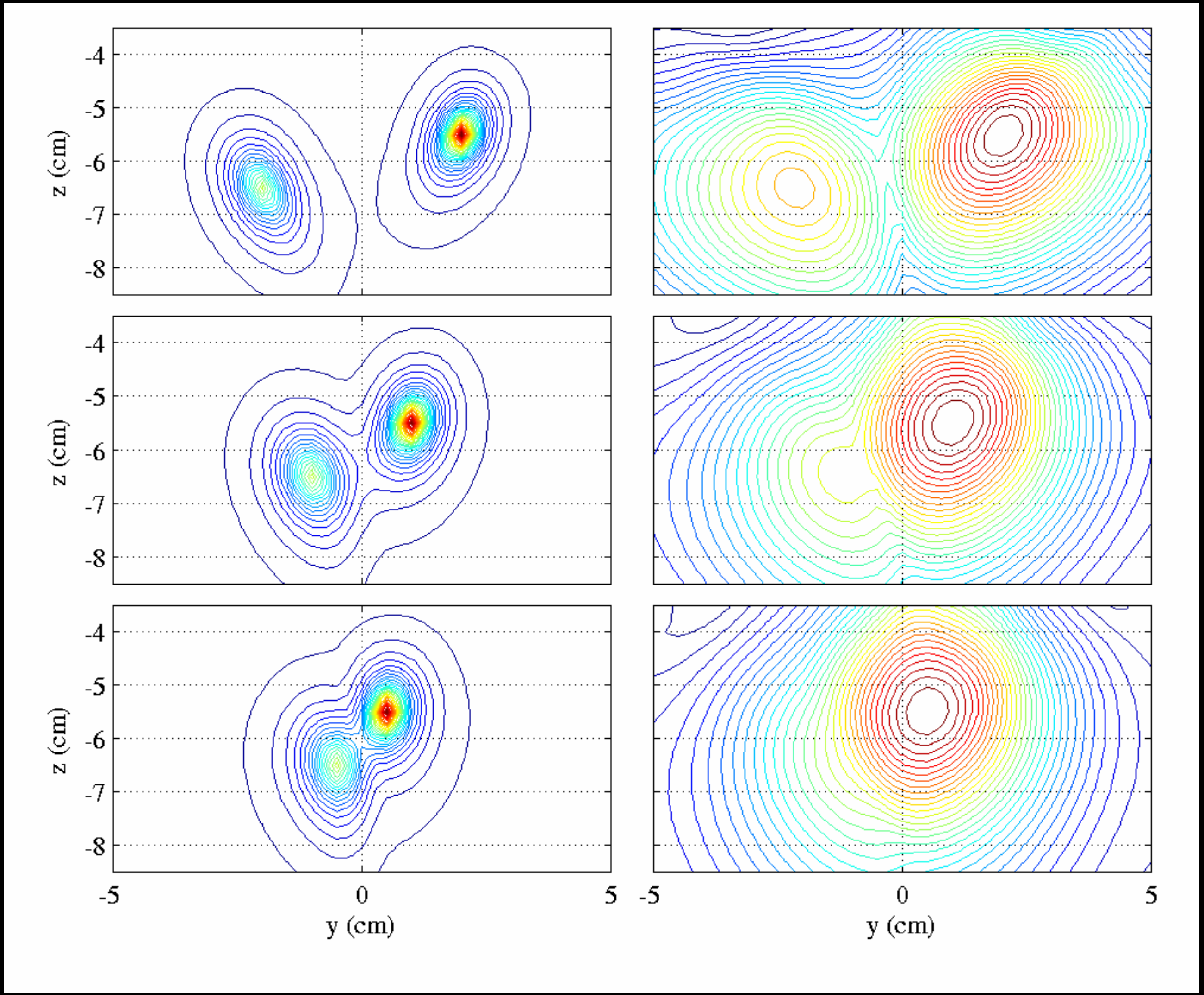
sLORETA

Source distance

4.2cm

2.1cm

1.4cm



input SNR: $\alpha = M$

# Source correlation influence for adaptive spatial filters

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \delta_{pq}$$ (Sources are uncorrelated)

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}}$$ (Sources are partially correlated)

When $Q$ sources are correlated with the $p$th source,

$$\hat{s}(\boldsymbol{r}_p, t) = s(\boldsymbol{r}_p, t) + \sum_{q=1}^{Q} \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}} s(\boldsymbol{r}_q, t)$$

spatial-filter output

leakages from other correlated sources

---

$\boldsymbol{R}_S$ : source covariance matrix, $[\boldsymbol{R}_S^{-1}]_{pq}$:the $(p, q)$ element of $\boldsymbol{R}_S^{-1}$

# Signal cancellation

When two correlated sources exist

$$\hat{s}(\boldsymbol{r}_1,t)=s(\boldsymbol{r}_1,t)-(\frac{\alpha_1\mu}{\alpha_2})s(\boldsymbol{r}_2,t)$$

$$\hat{s}(\boldsymbol{r}_2,t)=-(\frac{\alpha_2\mu}{\alpha_1})s(\boldsymbol{r}_1,t)+s(\boldsymbol{r}_2,t)$$

$$\Downarrow$$

$$\left\langle \hat{s}(\boldsymbol{r}_1,t)^2\right\rangle=(1-\mu^2)\left\langle s(\boldsymbol{r}_1,t)^2\right\rangle$$

$$\left\langle \hat{s}(\boldsymbol{r}_2,t)^2\right\rangle=(1-\mu^2)\left\langle s(\boldsymbol{r}_2,t)^2\right\rangle$$

$$\Uparrow$$

Source power decreases by a factor of $(1-\underset{\uparrow}{\mu}^2)$

source correlation coefficient

# Reconstruction experiments when two correlated sources exist

Minimum-variance
(normalized lead field)

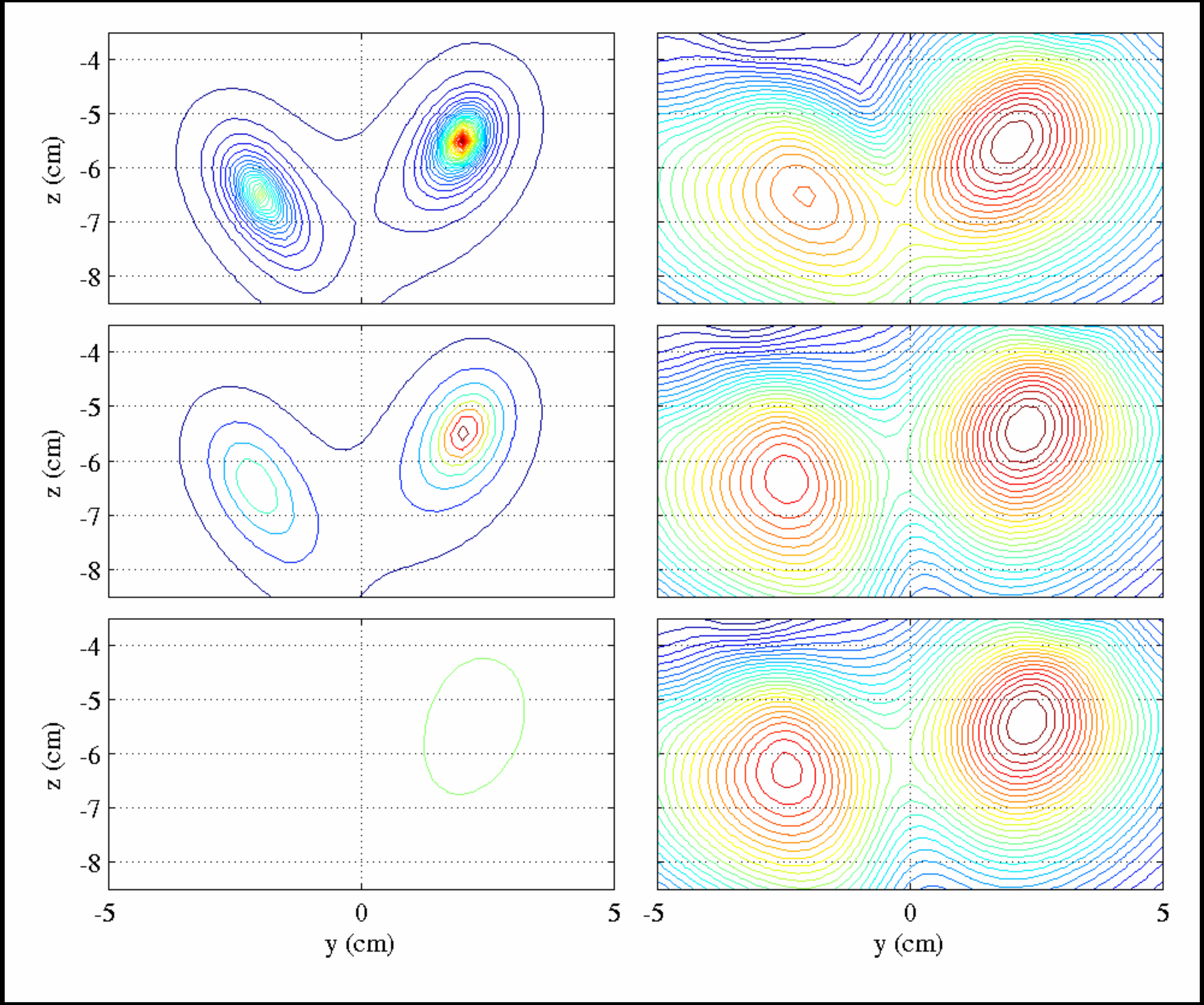
sLORETA

Correltion
coefficient

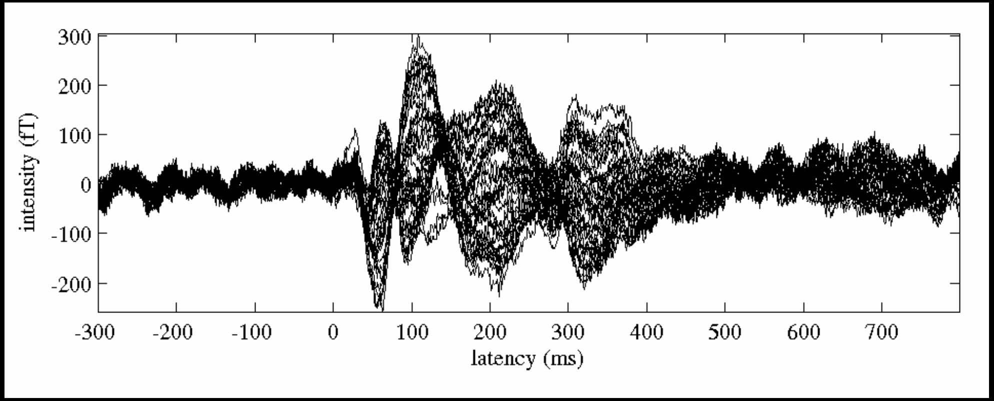
0

0.85

0.98

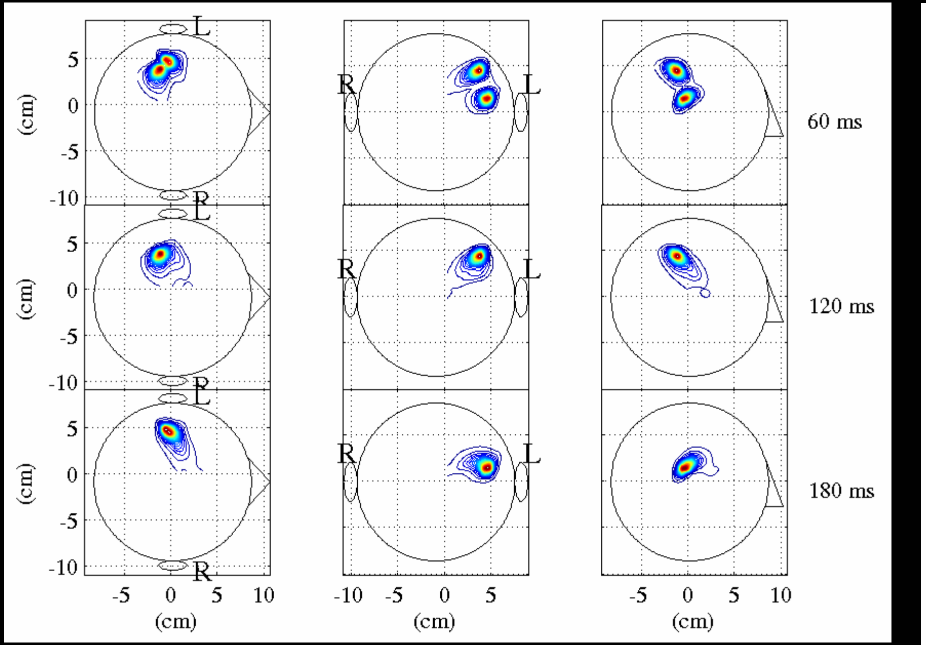


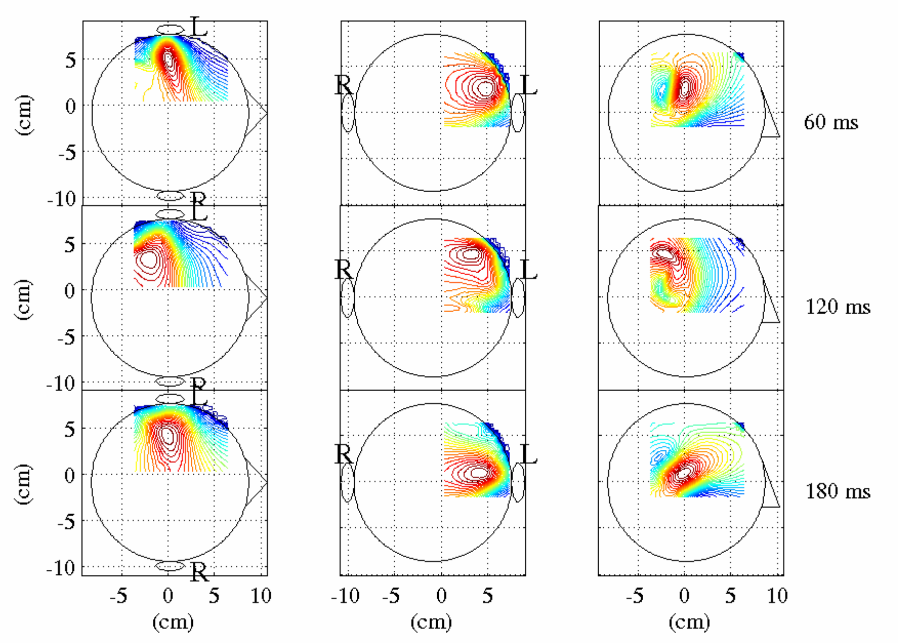
# Auditory somatosensory response



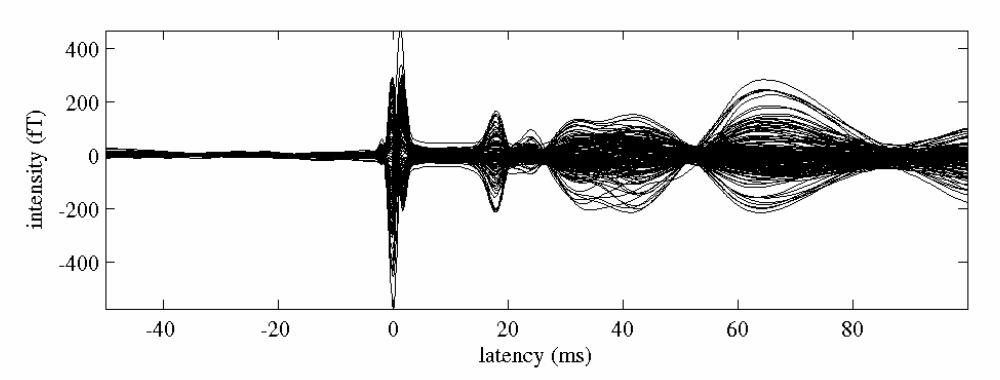
Minimum-variance
(normalized lead field)

sLORETA

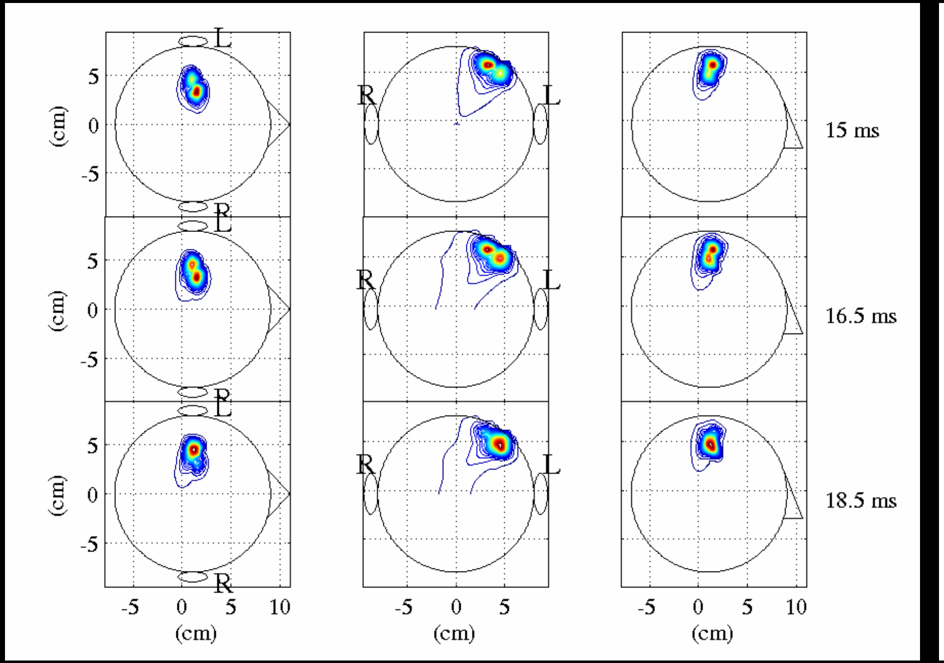




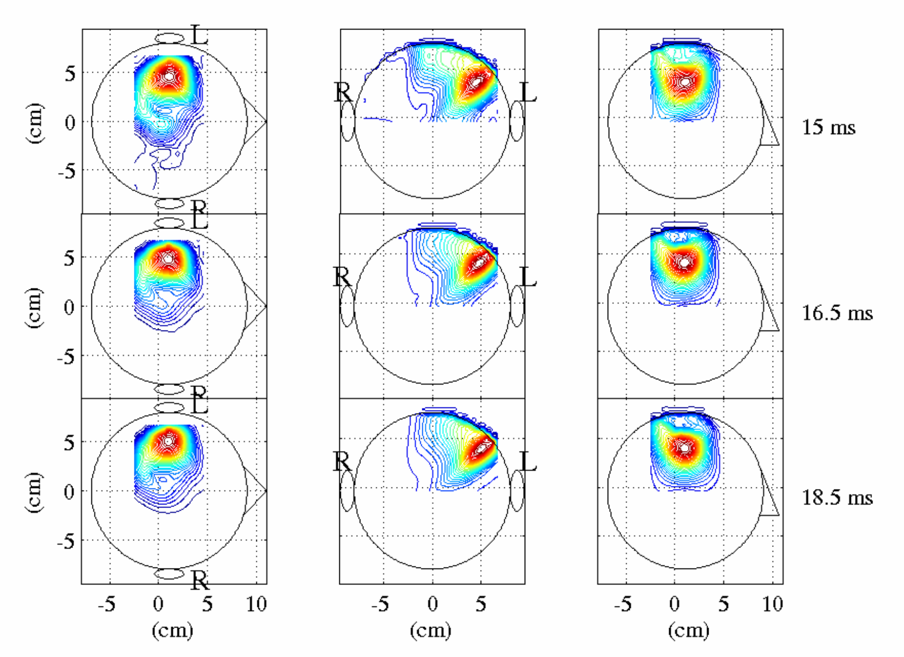
# Somatosensory response (right median nerve stimulation)



Minimum-variance
(normalized lead field)



sLORETA



# Summary

- The resolution kernel analysis validates the previous findings that sLORETA has no location bias. Minimum-variance filter has no location bias, if it is used with normalized lead field.

- Noise may cause the location bias for sLORETA. Minimum-variance spatial filter with normalized lead field has no location bias even in the presence of noise.

- Minimum-variance filter generally has significantly higher spatial resolution than sLORETA.

- Performance of sLORETA is not affected by source correlation.

- High spatial resolution of the minimum-variance filter is demonstrated by the somatosensory and somatosensory-auditory applications.

# Summary

- The resolution kernel analysis validates the previous findings that sLORETA has no location bias. Minimum-variance filter has no location bias, if it is used with normalized lead field.

- Noise may cause the location bias for sLORETA. Minimum-variance spatial filter with normalized lead field has no location bias even in the presence of noise.

- Minimum-variance filter generally has significantly higher spatial resolution than sLORETA.

- Performance of sLORETA is not affected by source correlation.

- High spatial resolution of the minimum-variance filter is demonstrated by the somatosensory and somatosensory-auditory applications.

# Summary

- The resolution kernel analysis validates the previous findings that sLORETA has no location bias. Minimum-variance filter has no location bias, if it is used with normalized lead field.

- Noise may cause the location bias for sLORETA. Minimum-variance spatial filter with normalized lead field has no location bias even in the presence of noise.

- Minimum-variance filter generally has significantly higher spatial resolution than sLORETA.

- Performance of sLORETA is not affected by source correlation.

- High spatial resolution of the minimum-variance filter is demonstrated by the somatosensory and somatosensory-auditory applications.

# Summary

- The resolution kernel analysis validates the previous findings that sLORETA has no location bias.  Minimum-variance filter has no location bias, if it is used with  normalized lead field.

- Noise may cause the location bias for sLORETA.  Minimum-variance spatial filter with  normalized lead field has no location bias even in the presence of noise.

- Minimum-variance filter generally has significantly higher spatial resolution than sLORETA.

- Performance of sLORETA is not affected by source correlation.

- High spatial resolution of the minimum-variance filter is demonstrated by the somatosensory and somatosensory-auditory applications.

# Summary

•The resolution kernel analysis validates the previous findings that sLORETA has no location bias. Minimum-variance filter has no location bias, if it is used with normalized lead field.

•Noise may cause the location bias for sLORETA. Minimum-variance spatial filter with normalized lead field has no location bias even in the presence of noise.

•Minimum-variance filter generally has significantly higher spatial resolution than sLORETA.

•Performance of sLORETA is not affected by source correlation.

•High spatial resolution of the minimum-variance filter is demonstrated by the somatosensory and somatosensory-auditory applications.

# Collaborators

Kanazawa Institute of Technology
Human Science Laboratory
Dr. Isao Hashimoto

University of Maryland
Linguistics and Cognitive Neuroscience Laboratory
Dr. David Poeppel

Massachusetts Institute of Technology,
Department of Linguistics and Philosophy
Dr. Alec Marantz

Visit

http://www.tmit.ac.jp/~sekihara/

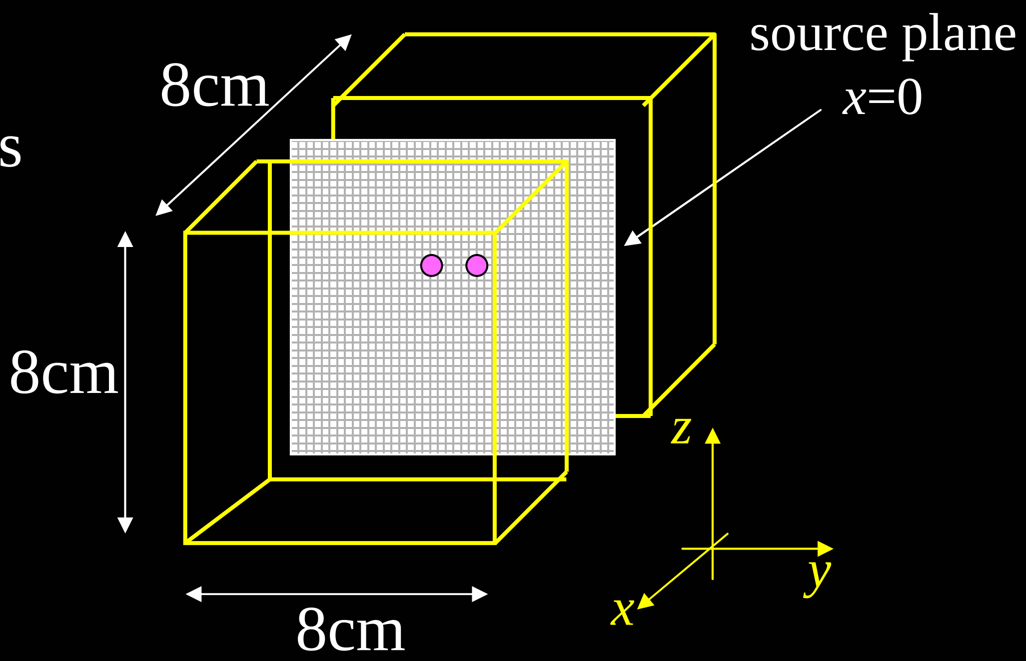
The PDF version of this power-point presentation as well as PDFs of my recent publications are available.

Generate many random dipoles in a volume:

$$-4 < x < -1, \quad 1 < x < 4$$

$$-4 < y < 4$$

$$-10 < z < -2$$



Number of noise random dipoles: 200

$P_N$ : The power of noise dipoles is set at

$0.005 \times$ second source power

or $0.025 \times$ second source power

Time courses of the noise sources are incoherent to each other.

# Violation of the low-rank signal assumption

Minimum-variance
(normalized lead field)

sLORETA

**No noise source**

**Noise source power:
0.005**

**Noise source power:
0.025**