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Adaptive spatial filter technique: its application to MEG/EEG functional source imaging

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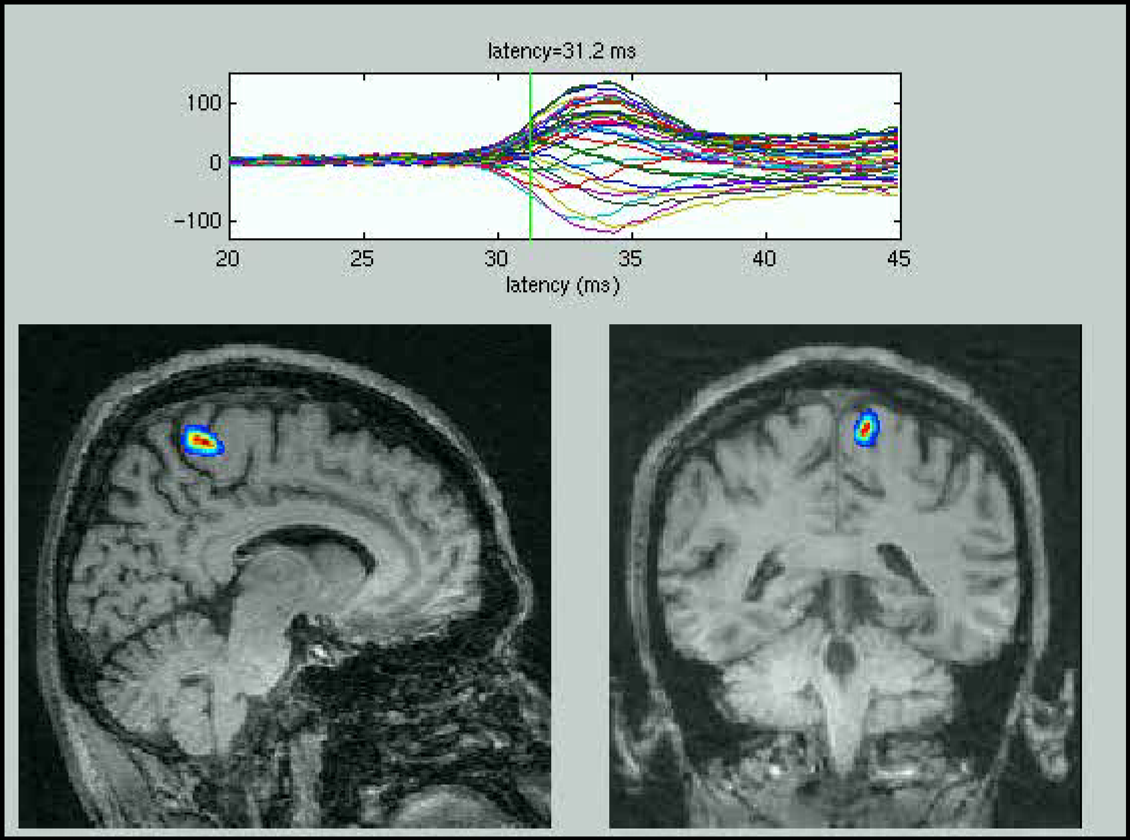
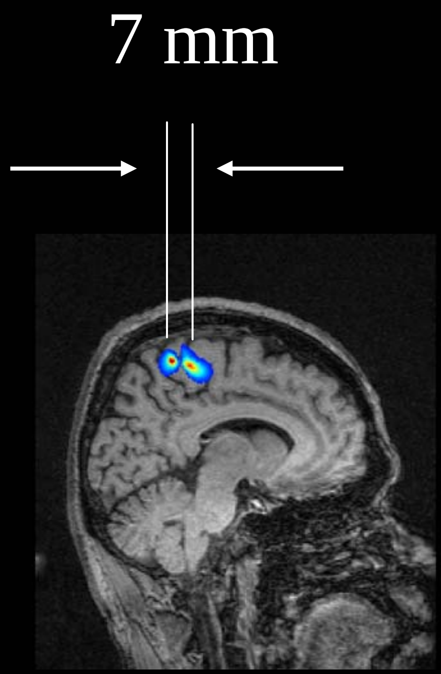
Right posterior tibial nerve stimulation

measured by a 37-channel sensor array

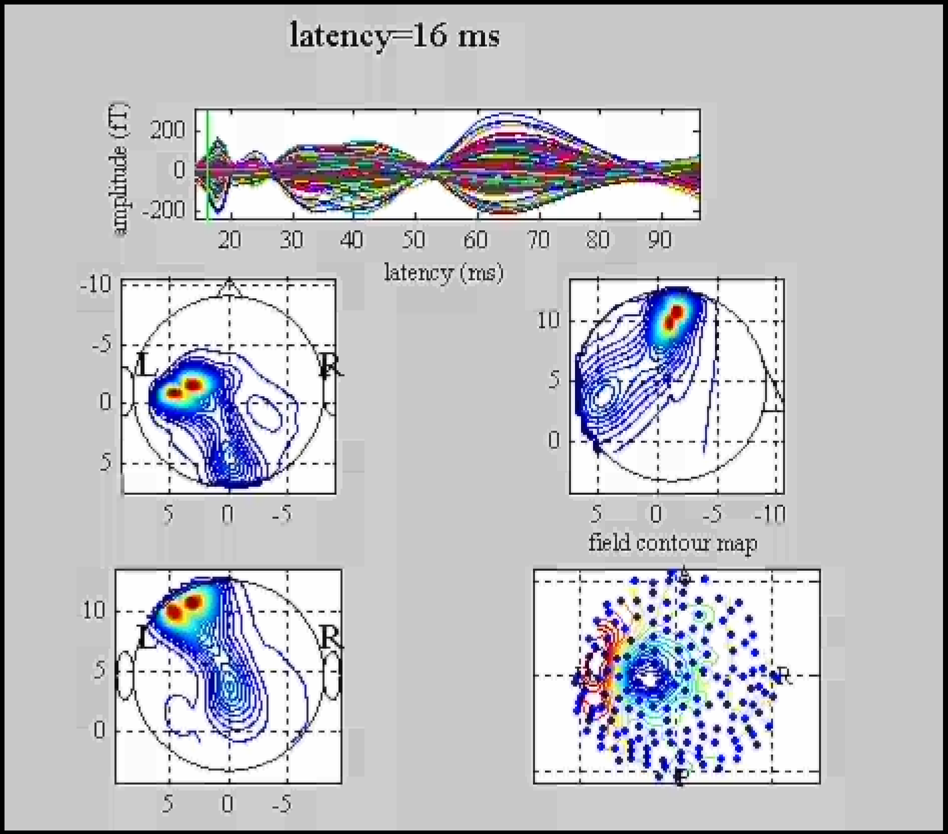




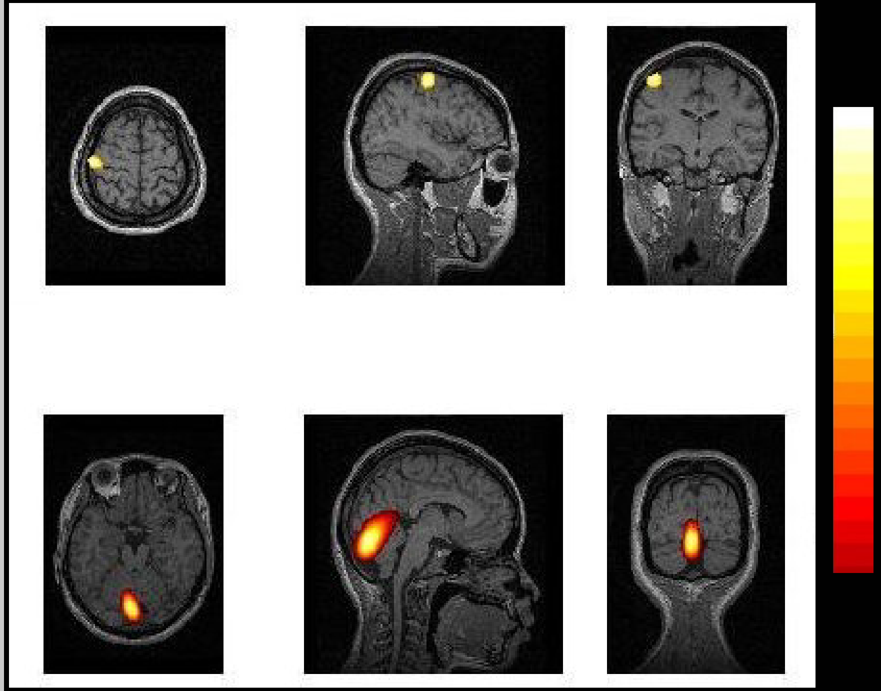
Hashimoto et al., "Serial activation of distinct cytoarchitectonic areas of the human SI cortex after posterior tibial nerve stimulation," NeuroReport 12, pp1857-1862, 2001

Right median nerve stimulation

measured by a 160-channel whole-head sensor array

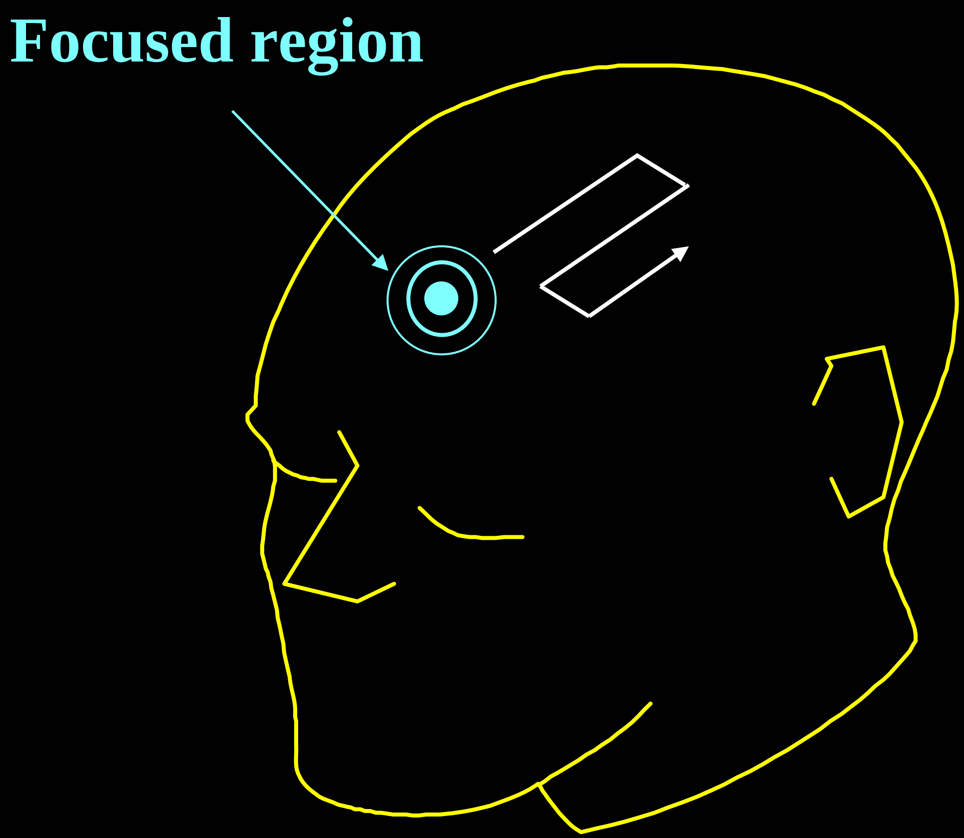


18 ms

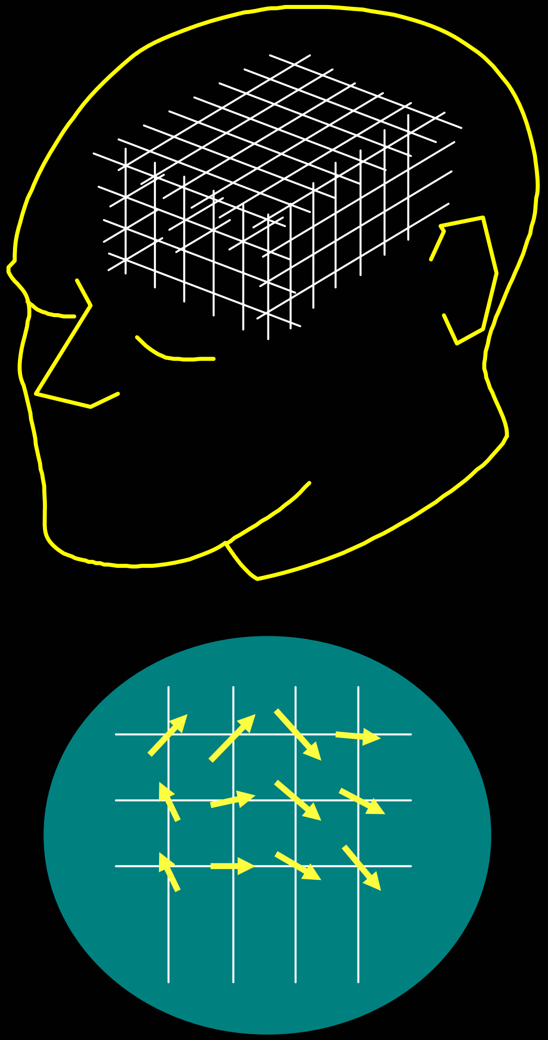


Hashimoto et al., **Clin. Neurophysiol. Nov;114, pp.2107-17, 2003**

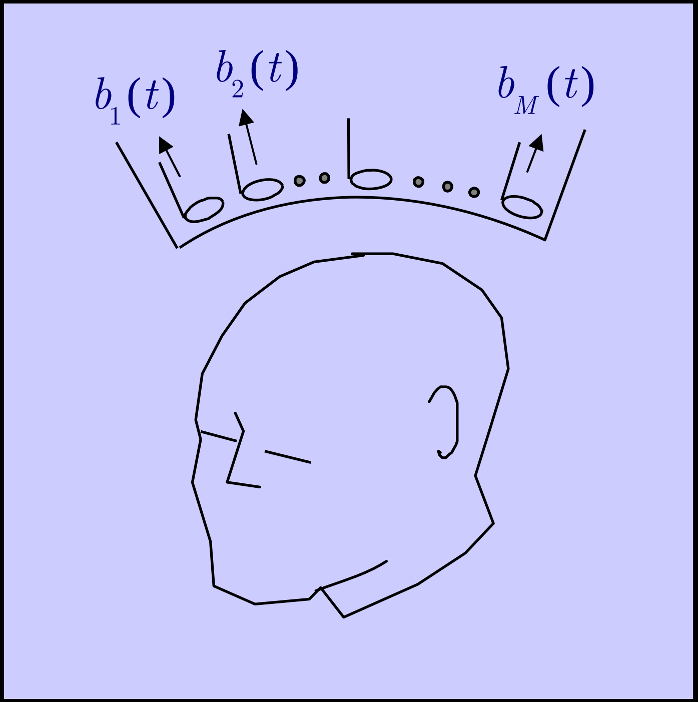
Spatial filter



Tomographic reconstruction



Definitions



- data vector: $\boldsymbol{b}(\mathrm{t}) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$

- data covariance matrix: $\boldsymbol{R} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

- source magnitude: $s(\boldsymbol{r},t)$

- source orientation: $\boldsymbol{\eta}(\boldsymbol{r},t) = [\eta_x(\boldsymbol{r},t), \eta_y(\boldsymbol{r},t), \eta_z(\boldsymbol{r},t)]^T$

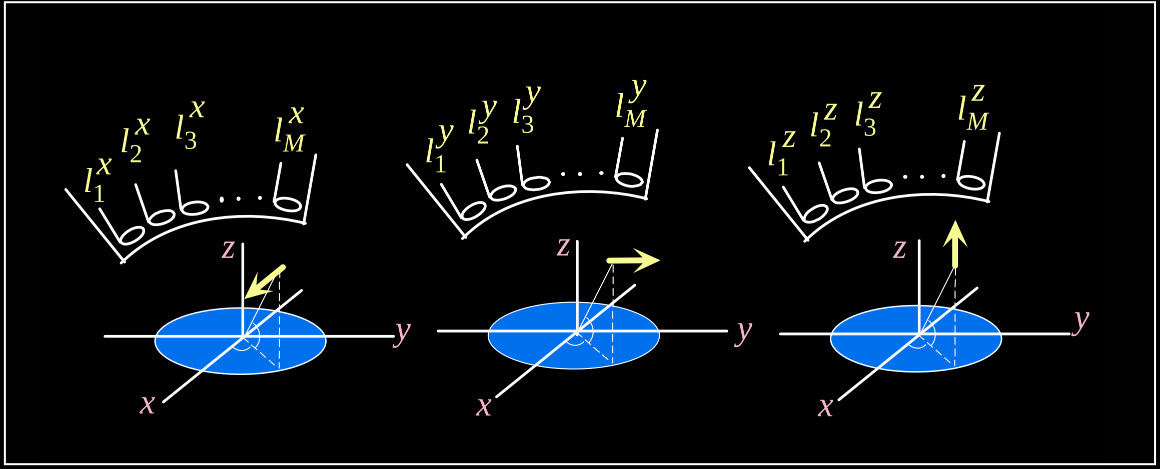
Sensor lead field

Lead field for source orientation: $\boldsymbol{\eta}(\boldsymbol{r})$

$$\boldsymbol{L}(\boldsymbol{r}) = \begin{bmatrix} l_1^x(\boldsymbol{r}) & l_1^y(\boldsymbol{r}) & l_1^z(\boldsymbol{r}) \\ l_2^x(\boldsymbol{r}) & l_2^y(\boldsymbol{r}) & l_2^z(\boldsymbol{r}) \\ \vdots & \vdots & \vdots \\ l_M^x(\boldsymbol{r}) & l_M^y(\boldsymbol{r}) & l_M^z(\boldsymbol{r}) \end{bmatrix}, \quad \boldsymbol{l}(\boldsymbol{r}) = \boldsymbol{L}(\boldsymbol{r}) \begin{bmatrix} \eta_x(\boldsymbol{r}) \\ \eta_y(\boldsymbol{r}) \\ \eta_z(\boldsymbol{r}) \end{bmatrix}$$

Measurement equation

$$\boldsymbol{b}(t) = \int \boldsymbol{l}(\boldsymbol{r}) s(\boldsymbol{r}, t) d\boldsymbol{r}$$



Spatial filter-basic formulation

weight vector

$$\hat{s}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = [w_1(\boldsymbol{r}),\ldots,w_M(\boldsymbol{r})]\begin{bmatrix} b_1(t) \\ \vdots \\ b_M(t) \end{bmatrix} = \sum_{m=1}^{M} w_m(r) b_m(t)$$

Estimate of source activity

Spatial filter for 3D-vector sources

$$\hat{s}(\boldsymbol{r},\boldsymbol{\eta},t) = \boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta})\boldsymbol{b}(t)$$

Weight depends on $\boldsymbol{\eta}$, as well as $\boldsymbol{r}$.

When no information on $\boldsymbol{\eta}$ is available, use $\boldsymbol{\eta}_{opt} = \arg\max_{\eta} \left\langle \hat{s}(\boldsymbol{r},\boldsymbol{\eta},t)^2 \right\rangle$

$$\hat{s}(\boldsymbol{r},t) = \boldsymbol{w}^T(\boldsymbol{r},\boldsymbol{\eta}_{opt})\boldsymbol{b}(t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t)$$

Non-adaptive spatial filter:

$w(r)$ is data independent

Adaptive spatial filter:

$w(r)$ is data dependent

Non-adaptive spatial filter--Spatial matched filter

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{l}(\boldsymbol{r}) \,/\, \|\boldsymbol{l}(\boldsymbol{r})\|$$

and

$$\hat{s}(\boldsymbol{r}, t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t) = \boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{b}(t) / \|\boldsymbol{l}(\boldsymbol{r})\|$$

Filter outputs are the inner product between the lead field and data vector.

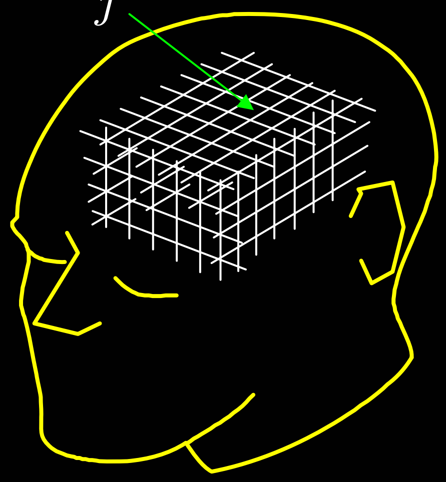
↓

The inner product represents the similarity between them

↓

The filter outputs form a peak at each source location if they are sufficiently isolated.

Non-adaptive spatial filter-- minimum-norm filter



Composite lead field matrix

$$\boldsymbol{L}_N = \left[\boldsymbol{l}(\boldsymbol{r}_1), \quad \cdots, \quad \boldsymbol{l}(\boldsymbol{r}_N) \right]$$

Minimum-norm solution

$$\begin{bmatrix} \hat{s}(\boldsymbol{r}_1) \\ \vdots \\ \boxed{\hat{s}(\boldsymbol{r}_j)} \\ \vdots \\ \hat{s}(\boldsymbol{r}_N) \end{bmatrix} = \begin{bmatrix} \boldsymbol{l}^T(\boldsymbol{r}_1) \\ \vdots \\ \boldsymbol{l}^T(\boldsymbol{r}_j) \\ \vdots \\ \boldsymbol{l}^T(\boldsymbol{r}_N) \end{bmatrix} (\underbrace{\boldsymbol{L}_N \boldsymbol{L}_N^T}_{\boldsymbol{G}})\boldsymbol{b}$$

Solution at $\boldsymbol{r}_j$:

$$\hat{s}(\boldsymbol{r}_j) = \boldsymbol{l}^T(\boldsymbol{r}_j)\boldsymbol{G}^{-1}\boldsymbol{b}$$

Spatial filter formulation

$$\hat{s}(\boldsymbol{r}) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}; \quad \boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r})$$

Non-adaptive spatial filter
Variants of minimum-norm filter

Weight normalized minimum norm filter (Dale et al.)

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r}) \,/\, \sqrt{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{G}^{-2}\boldsymbol{l}(\boldsymbol{r})]}$$

sLORETA (Pasucual-Marque)

$$\boldsymbol{w}(\boldsymbol{r}) = \boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r}) \,/\, \sqrt{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{G}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

A. M. Dale et al., Neuron,Vol.26, pp.55-67, 2000

R. D. Pascual-Marqui, Methods and Findings in Experimental and Clinical Pharmacology,Vol.24, pp.5-12, 2002

Adaptive spatial filter--minimum-variance filter

$$\arg\min_{\boldsymbol{w}} \boldsymbol{w}^T \boldsymbol{R}\boldsymbol{w} \text{ subject to } \boldsymbol{w}^T \boldsymbol{l}(\boldsymbol{r}) = 1$$

$$\Downarrow$$

$$\boldsymbol{w}(\boldsymbol{r}) = \frac{\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}{\boldsymbol{l}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

Assumption: $\left\langle s(\boldsymbol{r}_p, t) s(\boldsymbol{r}_q, t) \right\rangle = 0$ for $p \neq q$

Output power when the filter is focused on the pth source

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r}_p) = \left\langle s(\boldsymbol{r}_p, t)^2 \right\rangle + \sum_{q \neq p} \left\langle s(\boldsymbol{r}_q, t)^2 \right\rangle \left\| \boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) \right\| \searrow$$

zero

Minimizing output power gives the weight satisfying

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = 1 \text{ for } p = q$$

$$= 0 \text{ for } p \neq q$$

Minimum-variance filter for 3D-vector sources

Weight depending both on $\boldsymbol{\eta}$ and $\boldsymbol{r}$.

$$\boldsymbol{w}(\boldsymbol{r},\boldsymbol{\eta}) = \frac{\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}$$

Output power

$$\left\langle \hat{s}(\boldsymbol{r},\boldsymbol{\eta},t)^2 \right\rangle = \frac{1}{\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}}$$

Therefore

$$\boldsymbol{\eta}_{opt} = \arg\max_{\boldsymbol{\eta}} \left\langle \hat{s}(\boldsymbol{r},\boldsymbol{\eta},t)^2 \right\rangle = \arg\min_{\boldsymbol{\eta}} [\boldsymbol{\eta}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{\eta}] = \boldsymbol{u}_{min}$$

Eigenvector for the minimum eigenvalue of $\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})$

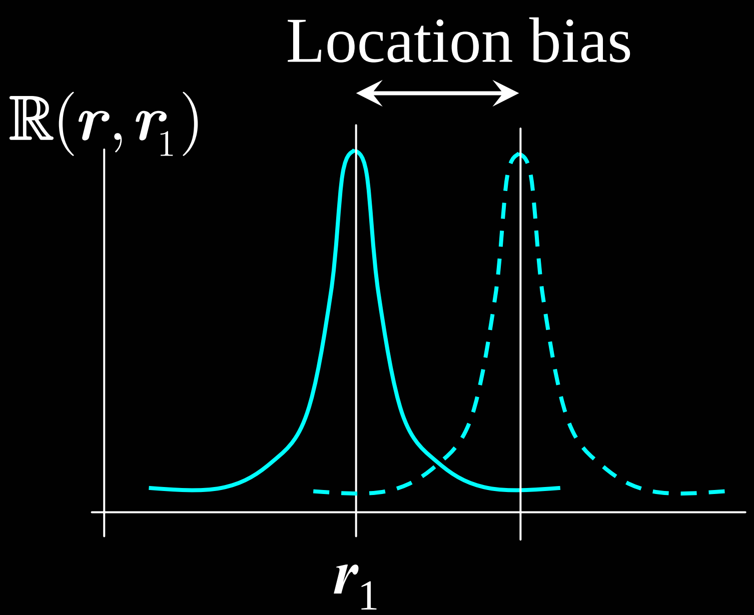
$$\boldsymbol{w}(\boldsymbol{r}) = \frac{\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{u}_{min}}{\boldsymbol{u}_{min}^T\boldsymbol{L}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})\boldsymbol{u}_{min}}$$

Comparison between spatial filter performances
-Location bias-

Resolution kernel analysis

$$\hat{s}(\boldsymbol{r}) = \int \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}')s(\boldsymbol{r}')d\boldsymbol{r}'$$

$\mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{l}(\boldsymbol{r}_1)$ (when a point source is located at $\boldsymbol{r}_1$)



No location bias

$\Updownarrow$

Resolution kernel peaks at $\boldsymbol{r}_1$

$\Updownarrow$

$$\mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1) > \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1)$$

The condition: $\mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1) > \mathbb{R}(\boldsymbol{r}, \boldsymbol{r}_1)$

Spatial matched filter $\quad \|\boldsymbol{f}\|\|\boldsymbol{l}\| > \boldsymbol{l}^T\boldsymbol{f} \quad$ always valid

Minimum-norm $\quad \boldsymbol{f}^T\boldsymbol{G}^{-1}\boldsymbol{f} > \boldsymbol{l}^T\boldsymbol{G}^{-1}\boldsymbol{f} \quad$ not necessarily valid

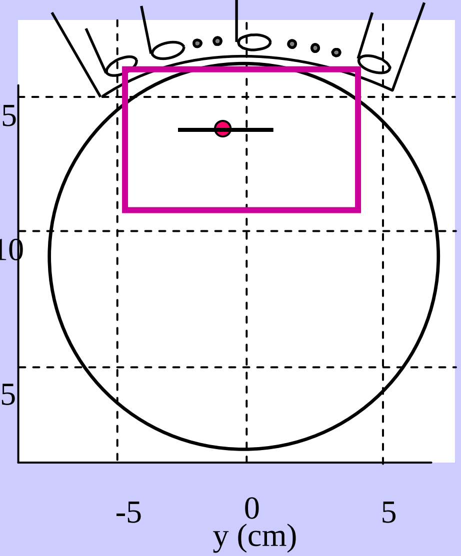
Weight normalized minimum norm $\quad \dfrac{\boldsymbol{f}^T\boldsymbol{G}^{-1}\boldsymbol{f}}{\sqrt{\boldsymbol{f}^T\boldsymbol{G}^{-2}\boldsymbol{f}}} > \dfrac{\boldsymbol{l}^T\boldsymbol{G}^{-1}\boldsymbol{f}}{\sqrt{\boldsymbol{l}^T\boldsymbol{G}^{-2}\boldsymbol{l}}} \quad$ not necessarily valid

sLORETA $\quad (\boldsymbol{f}^T\boldsymbol{G}^{-1}\boldsymbol{f})(\boldsymbol{l}^T\boldsymbol{G}^{-1}\boldsymbol{l}) > (\boldsymbol{l}^T\boldsymbol{G}^{-1}\boldsymbol{f})^2$

always valid (Schwartz inequality)

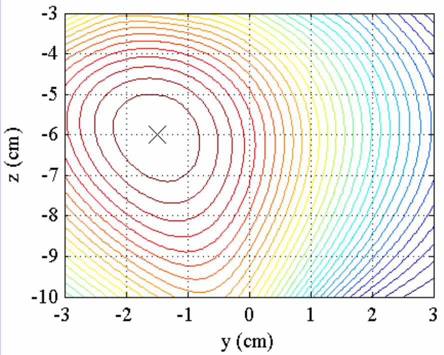
Minimum variance $\quad \dfrac{\cos(\boldsymbol{l}, \boldsymbol{f})}{1 + \alpha[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]} < 1 \quad$ always valid

$\boldsymbol{f}$: source lead field vector at $\boldsymbol{r}_1$ $\quad \alpha = (\sigma_1^2 / \sigma_0^2)\ \|\boldsymbol{f}\|^2\ (> M), \quad \cos(\boldsymbol{l}, \boldsymbol{f}) = \left|\boldsymbol{l}^T\boldsymbol{f}\right| / \|\boldsymbol{f}\|\|\boldsymbol{l}\|$

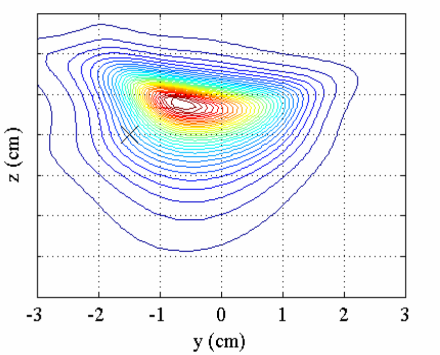


Reconstruction of Point Source

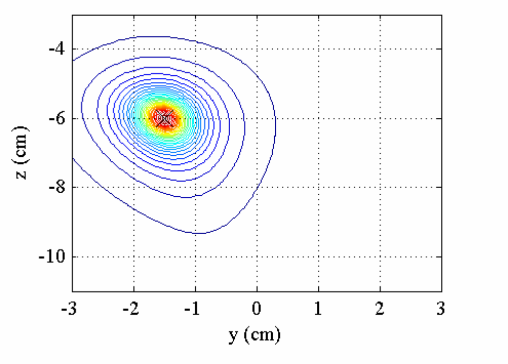
Spatial matched filter



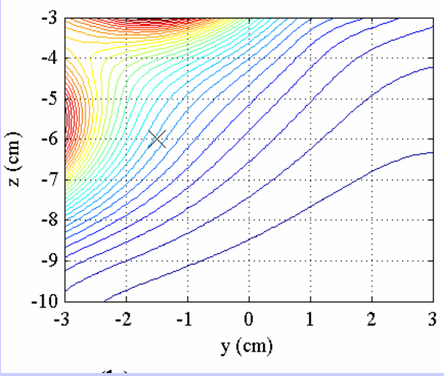
Weight-normalized minimum norm



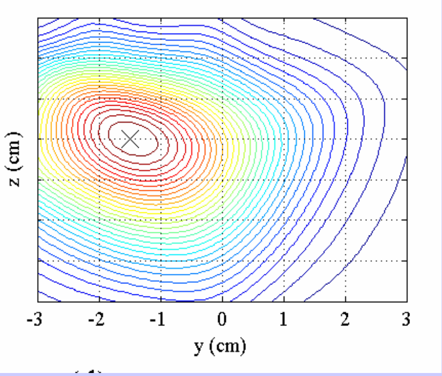
Minimum variance



Minimum norm



sLORERA



Comparison between spatial filter performances
-Spatial resolution-

Point-spread function: $\phi(\boldsymbol{r}) = \mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}) \,/\, \mathbb{R}(\boldsymbol{r}_1, \boldsymbol{r}_1)$

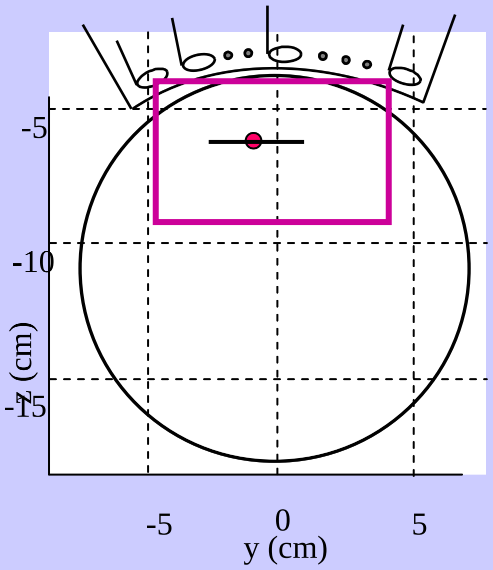
Spatial Matched Filter: $\phi(\boldsymbol{r}) = \cos(\boldsymbol{l}, \boldsymbol{f})$

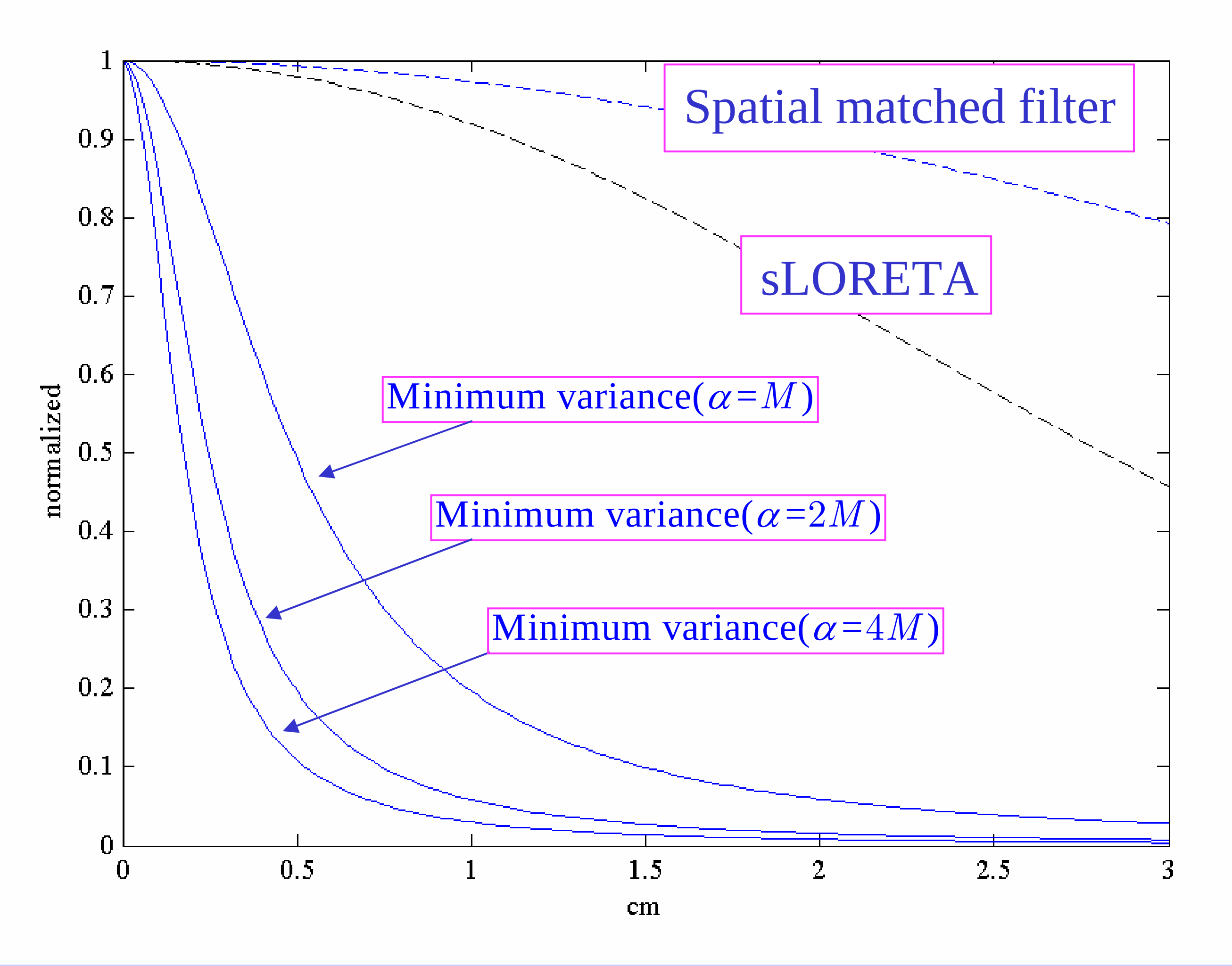
sLORETA: $\phi(\boldsymbol{r}) = \cos(\boldsymbol{l}, \boldsymbol{f} \mid \boldsymbol{G}^{-1})$

Minimum variance: $\phi(\boldsymbol{r}) = \dfrac{\cos(\boldsymbol{l}, \boldsymbol{f})}{1 + \alpha[1 - \cos^2(\boldsymbol{l}, \boldsymbol{f})]}$

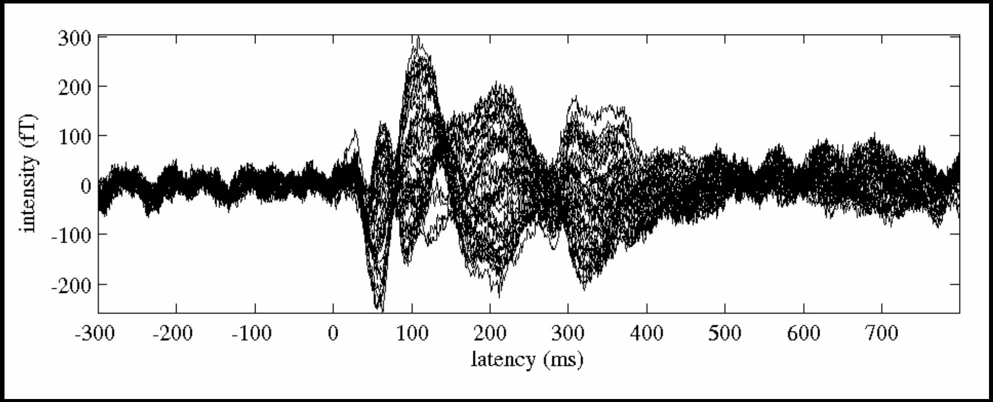
Because α >*M*, this part causes a rapid decay.

Point spread function

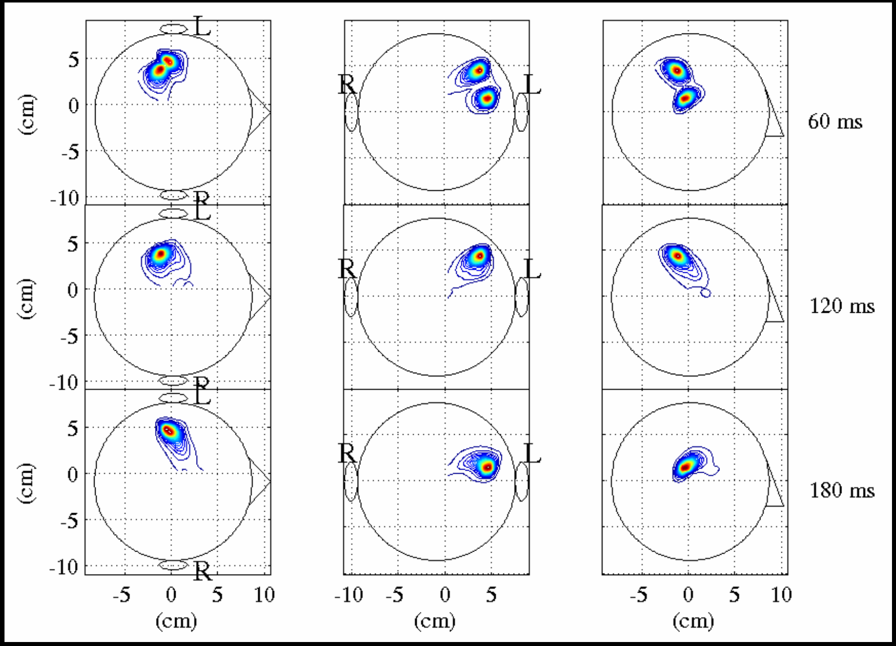




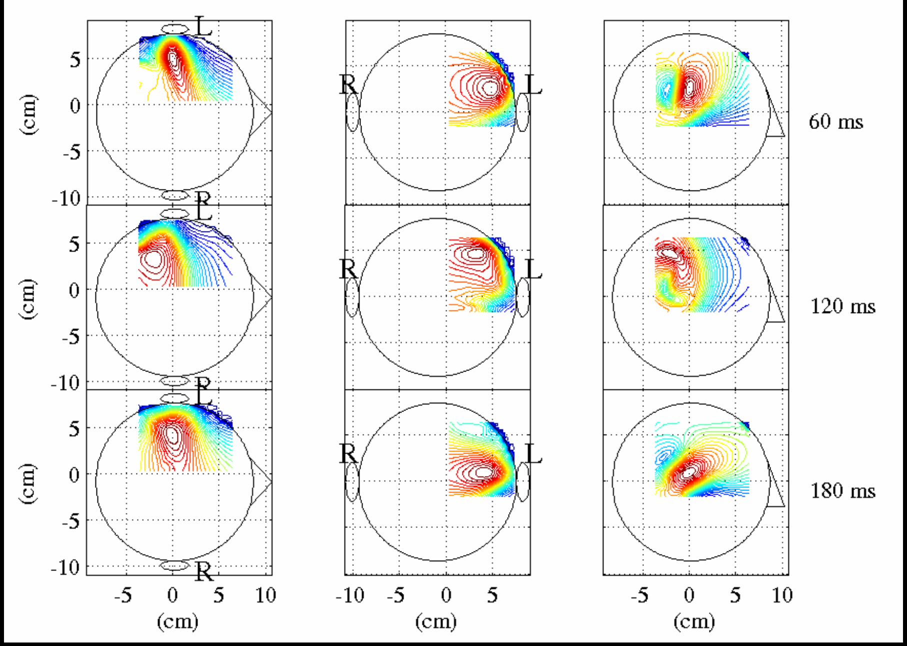
Auditory somatosensory response



Minimum-variance (normalized lead field)



sLORETA



Problems for adaptive spatial filter

Source correlation

Background brain interference

Influence of Source correlation

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \delta_{pq}$$ (Sources are uncorrelated)

$$\boldsymbol{w}^T(\boldsymbol{r}_p)\boldsymbol{l}(\boldsymbol{r}_q) = \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}}$$ (Sources are partially correlated)

Reconstructed source time course:

$$\hat{s}(\boldsymbol{r}_p, t) = s(\boldsymbol{r}_p, t) + \sum_{q=1}^{Q} \frac{[\boldsymbol{R}_S^{-1}]_{pq}}{[\boldsymbol{R}_S^{-1}]_{pp}} s(\boldsymbol{r}_q, t)$$

spatial-filter output leakages from other correlated sources

$\boldsymbol{R}_S$: source covariance matrix, $[\boldsymbol{R}_S^{-1}]_{pq}$:the (p,q) element of $\boldsymbol{R}_S^{-1}$

Signal cancellation

When two correlated sources exist

$$\hat{s}(\boldsymbol{r}_1,t) = s(\boldsymbol{r}_1,t) - (\frac{\alpha_1\mu}{\alpha_2})s(\boldsymbol{r}_2,t)$$

$$\hat{s}(\boldsymbol{r}_2,t) = -(\frac{\alpha_2\mu}{\alpha_1})s(\boldsymbol{r}_1,t) + s(\boldsymbol{r}_2,t)$$

$$\Downarrow$$

$$\left\langle \hat{s}(\boldsymbol{r}_1,t)^2 \right\rangle = (1-\mu^2)\left\langle s(\boldsymbol{r}_1,t)^2 \right\rangle$$

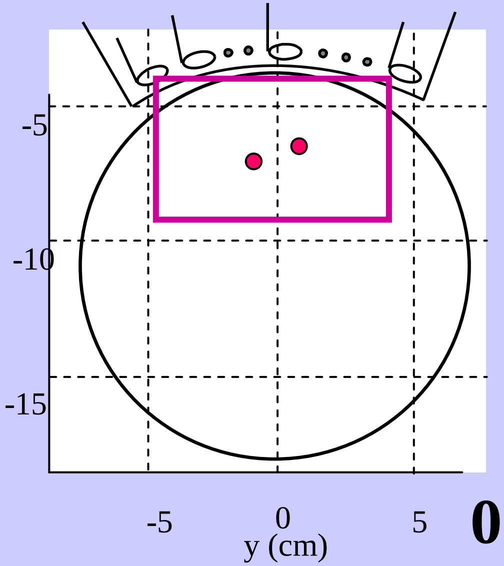
$$\left\langle \hat{s}(\boldsymbol{r}_2,t)^2 \right\rangle = (1-\mu^2)\left\langle s(\boldsymbol{r}_2,t)^2 \right\rangle$$

$$\Uparrow$$

Source power decreases by a factor of $(1-\mu^2)$

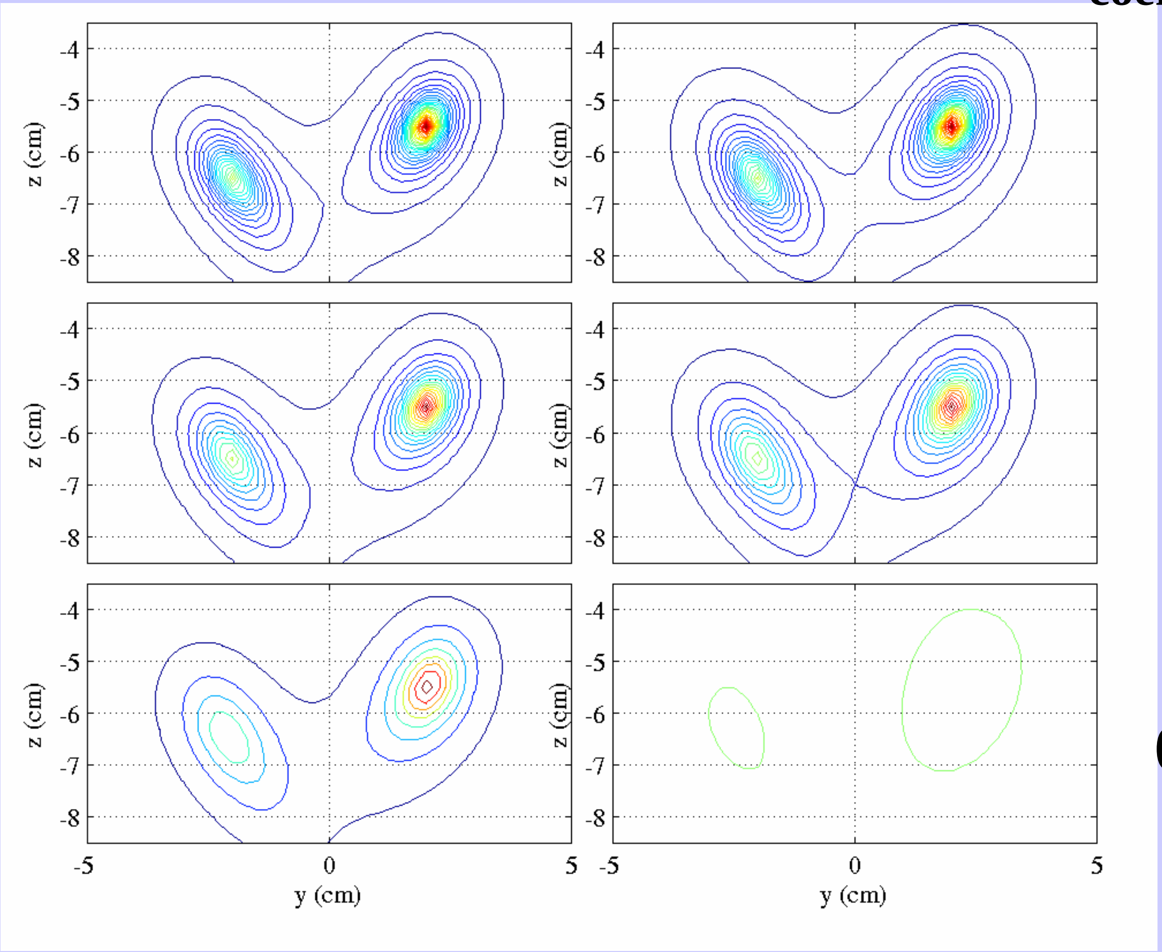
$$\boxed{\alpha_1 = \left\langle \hat{s}(\boldsymbol{r}_1,t)^2 \right\rangle,\ \alpha_2 = \left\langle \hat{s}(\boldsymbol{r}_2,t)^2 \right\rangle,\ \mu = \left\langle \hat{s}(\boldsymbol{r}_1,t)\hat{s}(\boldsymbol{r}_2,t) \right\rangle / \alpha_1\alpha_2}$$

Reconstruction experiments when two sources are correlated



Correlation coefficient

0



0.4

0.6

0.7

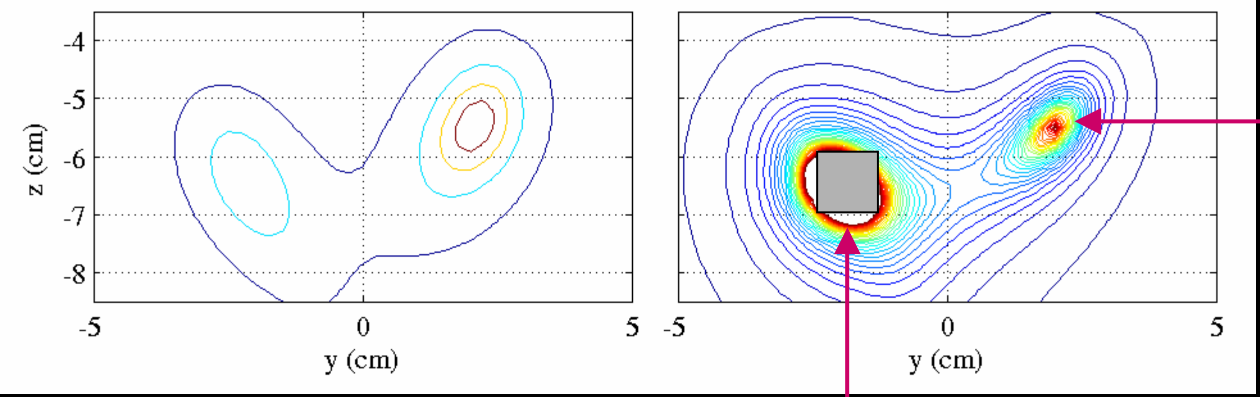
0.85

0.98

Coherent source suppression:

If locations of coherent interferences are approximately known, its influence can be suppressed.

Correlation coefficient: 0.92



Recover the signal source of interest

Impose the null sensitivity

Dalal S., Sekihara K., and Nagarajan S., "Modified Beamformers for Coherent Source Region Suppression," IEEE TBME Vol.53, pp.1357-1363, July, 2006

Problems for adaptive spatial filter

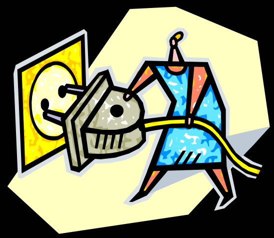
Source correlation

Background brain interference

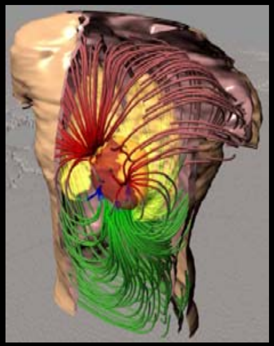
Interferences affecting to MEG sensor data

Low-rank interference

Electronics



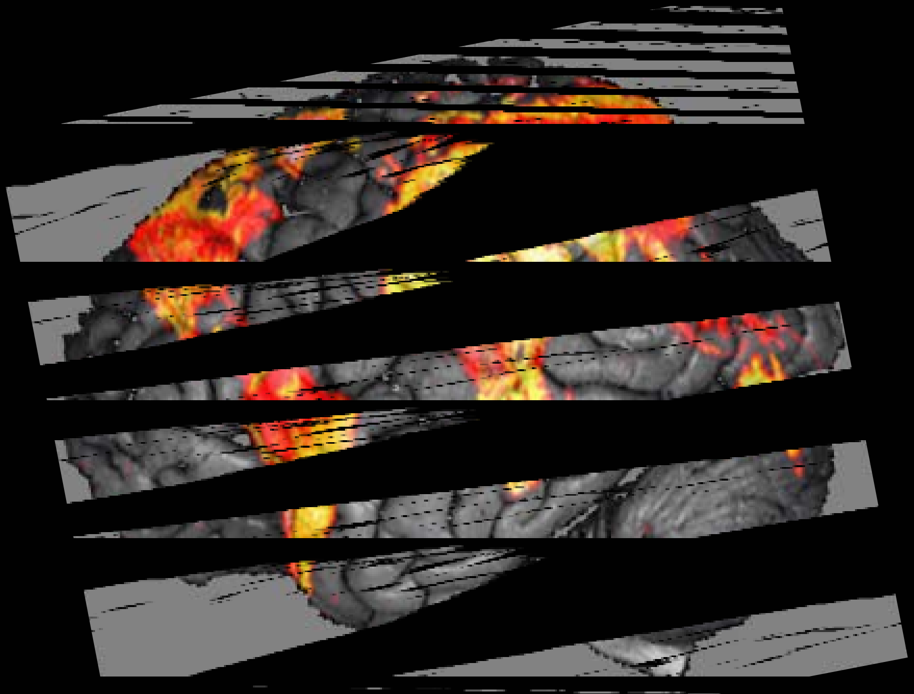
Heartbeat



Eyeblinks

Dental & Jaw Muscles

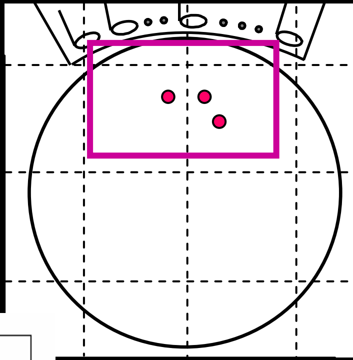
High-rank interference

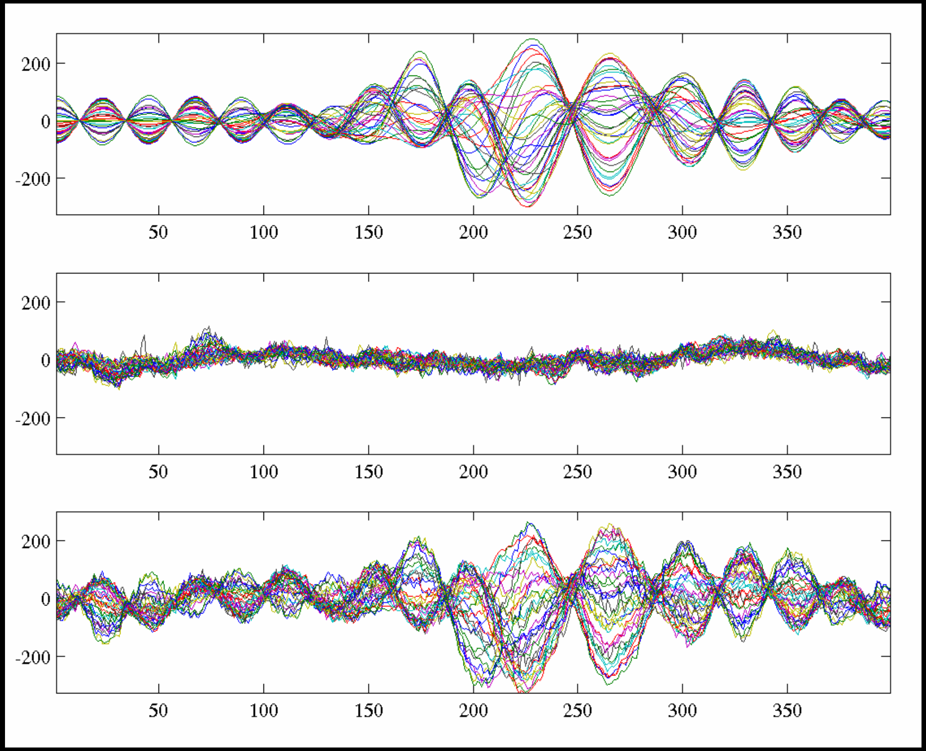


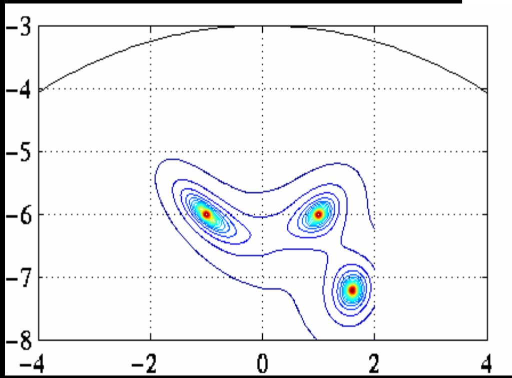
Spontaneous brain activity

no specific spatial, temporal, and frequency patterns

Influence of background source activity
(Influence of brain noise)







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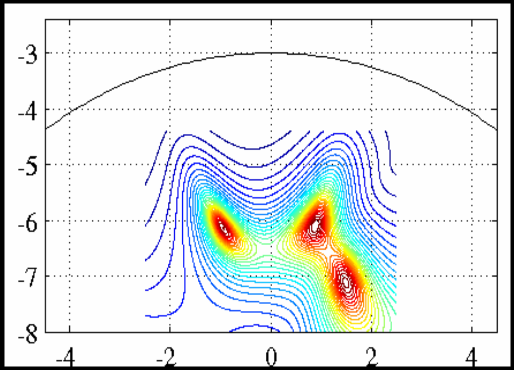

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Background activity causes a severe blur in the final reconstruction.

Prewhitening adaptive spatial filter

Data model

Task: $\boldsymbol{b}(t) = \boldsymbol{b}_S(t) + \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

Control: $\boldsymbol{b}_C(t) = \boldsymbol{b}_I(t) + \boldsymbol{n}(t)$

Covariance matrix relations

Task: $\boldsymbol{R} = \boldsymbol{R}_S + \boldsymbol{R}_{i+n}$

Control: $\boldsymbol{R}_C = \boldsymbol{R}_{i+n}$

Problem How to obtain source reconstruction free from the influence of $\boldsymbol{b}_I(t)$

Data covariance: $\boldsymbol{R} = \left\langle \boldsymbol{b}(t)\boldsymbol{b}^T(t) \right\rangle$

Signal covariance: $\boldsymbol{R}_S = \left\langle \boldsymbol{b}_S(t)\boldsymbol{b}_S^T(t) \right\rangle$

Interference plus noise covariance: $\boldsymbol{R}_{i+n} = \left\langle (\boldsymbol{b}_I(t) + \boldsymbol{n}(t))(\boldsymbol{b}_I(t) + \boldsymbol{n}(t))^T \right\rangle$

Prewhitening adaptive spatial filter

Using $\boldsymbol{u}_j$; signal level eigenvectors of $\tilde{\boldsymbol{R}} = \boldsymbol{R}_C^{-1/2}\boldsymbol{R}\boldsymbol{R}_C^{-1/2}$

Signal covariance estimation

$$\hat{\boldsymbol{R}}_S = \boldsymbol{R}_C^{1/2}\left[\sum_{j=1}^{Q}(\gamma_j - 1)\boldsymbol{u}_j\right]\boldsymbol{R}_C^{1/2}$$

Signal time course estimation

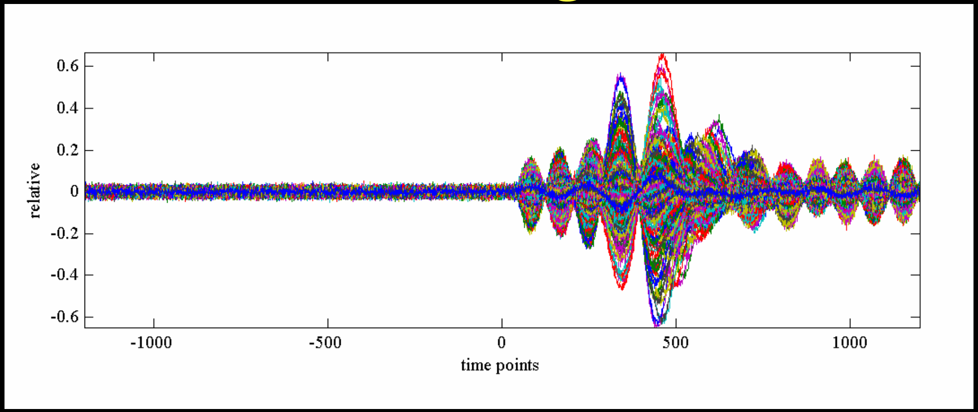
$$\hat{\boldsymbol{b}}_S(t) = \boldsymbol{R}_C^{1/2}\left[\sum_{j=1}^{Q}(\gamma_j - 1)\boldsymbol{u}_j\right]\boldsymbol{R}_C^{-1/2}\boldsymbol{b}_C(t)$$

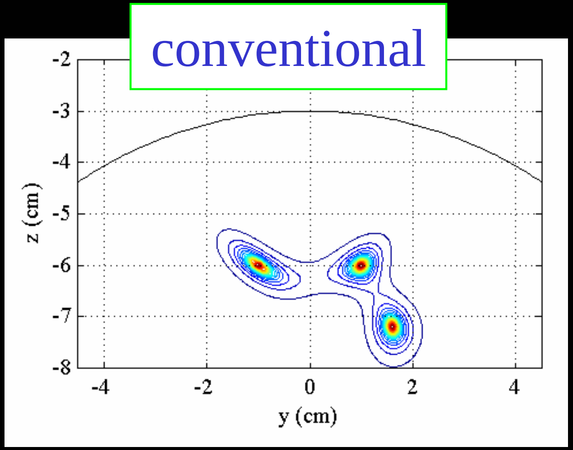
Prewhitening beamforming

$$\hat{s}_{PW}(\boldsymbol{r},t) = \frac{\boldsymbol{l}^T(\boldsymbol{r})(\hat{\boldsymbol{R}}_S + \mu\boldsymbol{I})^{-1}\hat{\boldsymbol{b}}_S(t)}{\boldsymbol{l}^T(\boldsymbol{r})(\hat{\boldsymbol{R}}_S + \mu\boldsymbol{I})^{-1}\boldsymbol{l}(\boldsymbol{r})}$$

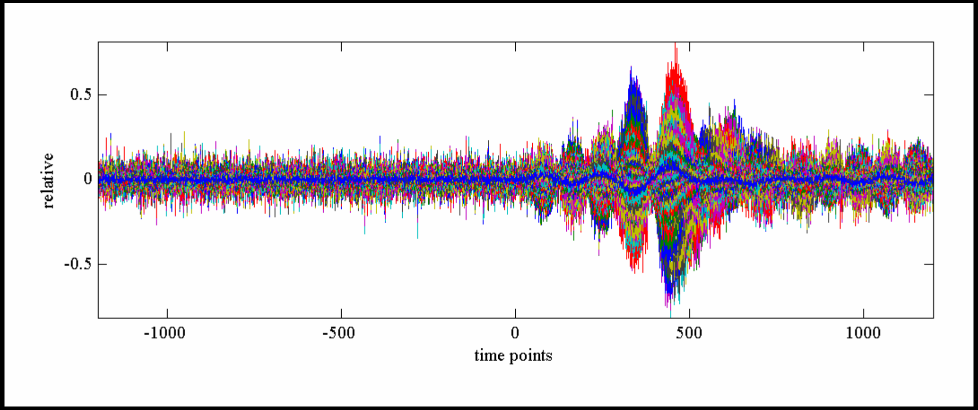
For detail: K. Sekihara et al., IEEE Transactions on Biomedical Engineering, September 2006

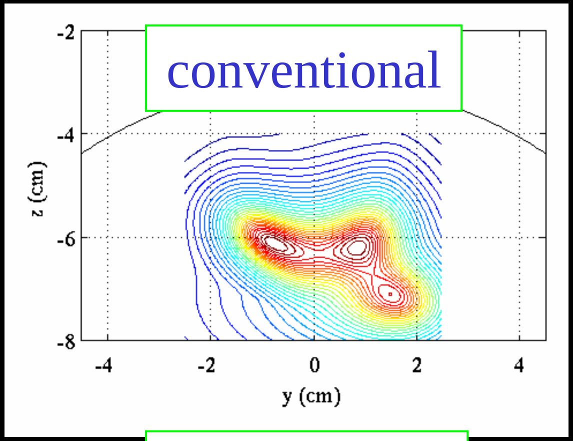
Data without background sources

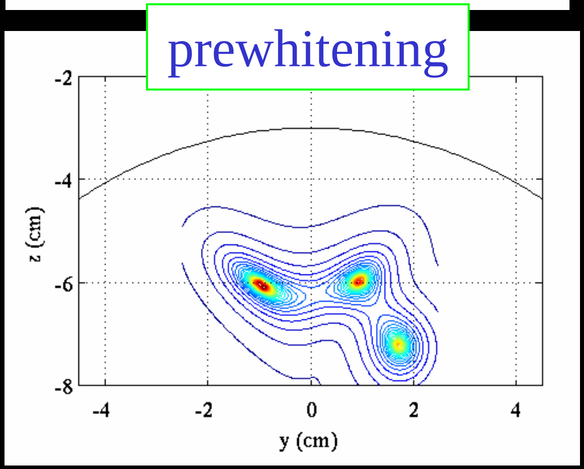




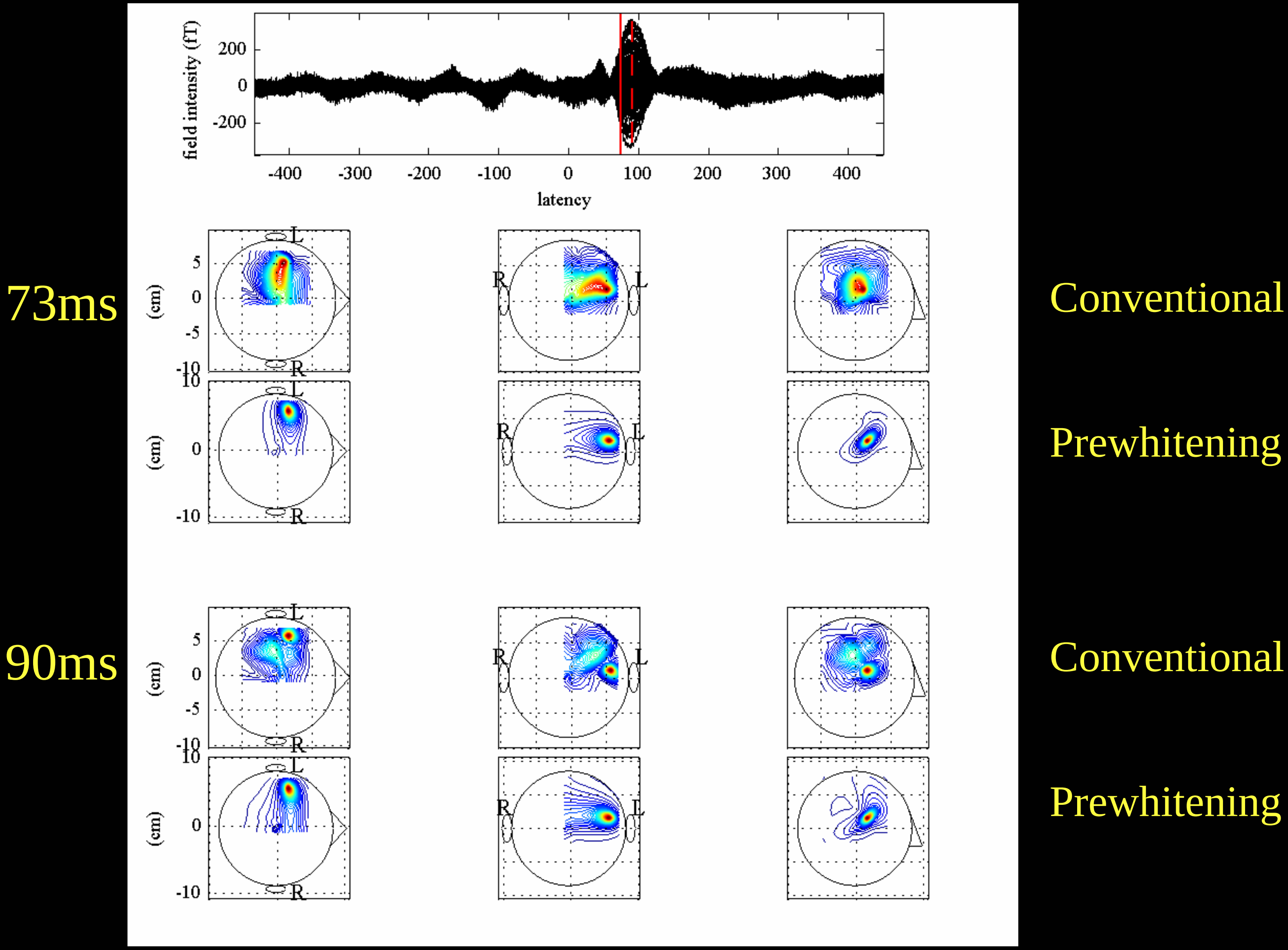
Data with background sources







Auditory evoked field



Summary

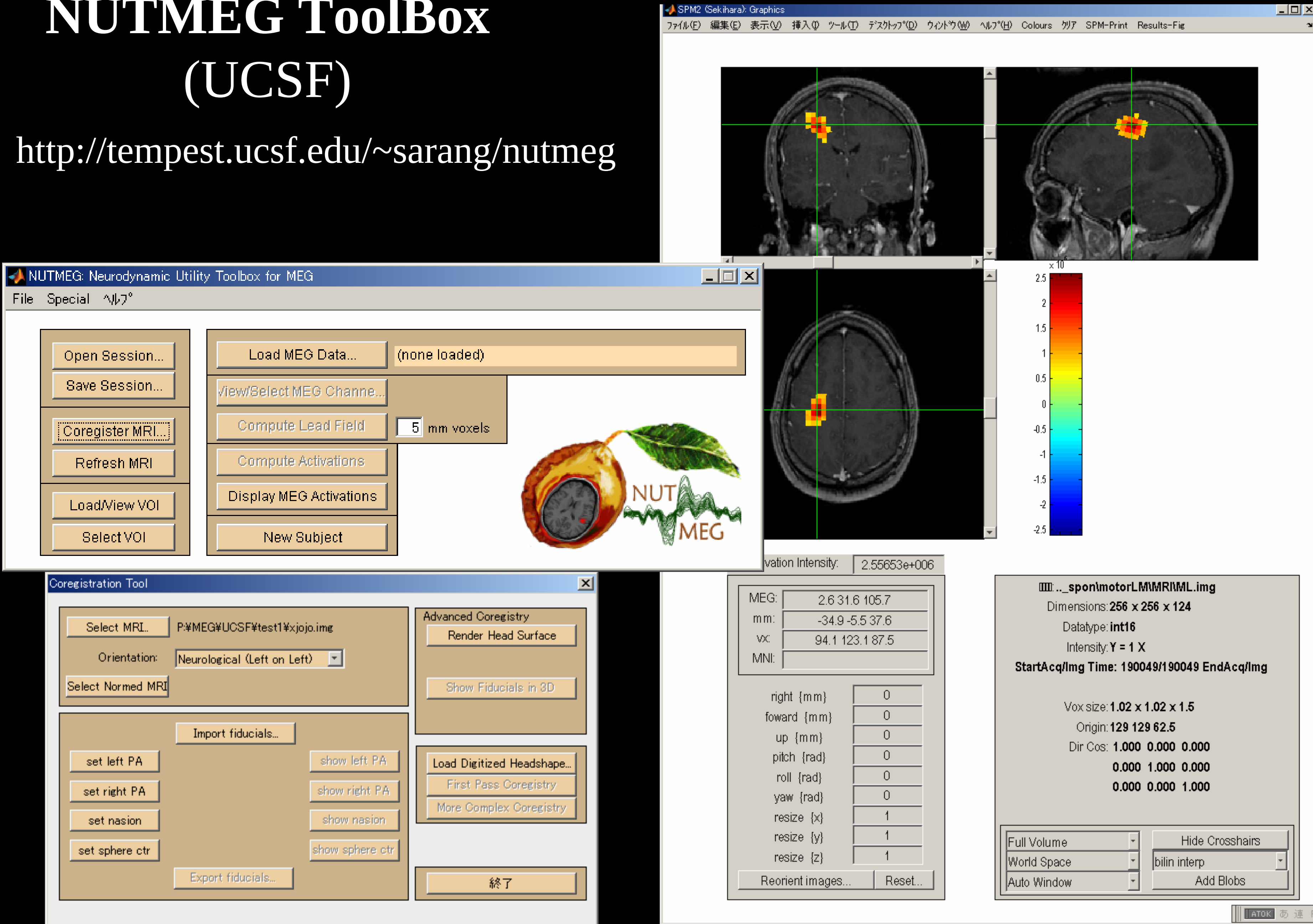
•Reviews principles of adaptive spatial filter source reconstruction.

•Discusses the comparison between adaptive and non-adaptive spatial filters.

•Discusses the following problems:
1) influence of source correlation,
2) influence of brain background interference.

NUTMEG ToolBox
(UCSF)

http://tempest.ucsf.edu/~sarang/nutmeg



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Thank you for your attention

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